The negotiation and ratification of the Kyoto Protocol has spawned a body of literature aimed at analyzing the formation of international environmental coalitions. This literature suggests that in equilibrium environmental agreements will have only a small number of signatories. As of April 2007, however, 168 countries have ratified the protocol; thirty-five of which have binding emissions reduction requirements. These thirty-five parties represent a coalition significantly larger than that predicted by the existing coalition formation models. To understand this finding, this paper develops a new game-theoretic model of international coalition formation, altering the game presented in the literature. Using this model, we attempt to explain the outcome of the Kyoto Protocol. We then investigate a method of increasing participation in future international environmental agreements. Results from the model suggest that full cooperation is a possible equilibrium if the agreement is properly specified.

1 Introduction

In 2003, the world emitted 25 billion tons of carbon dioxide into the atmosphere [6]. These emissions are byproducts of the production of goods and services necessary to sustain each country’s economy. However, most scientists agree that the release of these emissions results in increased atmospheric greenhouse gas concentrations, which lead to global warming [13]. With individual benefits and global consequences, greenhouse gas emissions are a classic example of an environmental externality. Typically, externalities are corrected through taxes, legislation, or property rights.

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2Oreskes [13] analyzes a sample of 928 papers published in peer-reviewed scientific journals between 1993 and 2003 and finds none that reject this statement.
However, without a world government, these mechanisms are not possible. Instead, policymakers have turned to international agreements to address the greenhouse gas externality.

In recent years, countries have negotiated and ratified two such international agreements to tackle the problem of climate change. The first was the United Nations Framework Convention on Climate Change (UNFCCC). This agreement entered into force in March of 1994 with 189 countries as signatories. The UNFCCC does not require countries to reduce greenhouse gas emissions; it merely encourages cooperation and information sharing [17]. The second agreement, the Kyoto Protocol, however, does specify emissions abatement targets and timetables. This agreement was adopted at the third Conference of Parties to the UNFCCC in December 1997. As of April 12, 2007, one hundred and sixty-eight countries had ratified this agreement; only thirty-five of these countries, however, are required to reduce emissions [19]. Additionally, the United States, the world's largest emitter, has yet to ratify the Kyoto Protocol.

The development of international environmental agreements has led to the creation of literature devoted to analyzing such agreements. A portion of this literature [4, 2, 3] has focused on the formation of environmental coalitions, specifically seeking to understand why these coalitions cannot sustain global cooperation. These papers develop game theoretic models to determine who will sign an environmental agreement. Typically, researchers reduce the negotiation process to two stages: a coalition formation stage followed by an emissions stage (see Figure 1). In the coalition formation stage, players choose whether or not to sign an environmental agreement. In the emissions stage, the coalition chooses abatement levels to minimize the total cost to the coalition; thus, we term these models 'Coalition Cost Minimizing Models.' In one such model with homogeneous players, Carraro and Siniscalco [4] conclude that the Nash equilibrium coalition has low membership.
Players prefer to free-ride, enjoying the benefits of other players' abatement while avoiding costly abatement themselves. In another model with heterogeneous players, Barrett [2] concludes that the maximum number of coalition members is three when he assumes constant marginal benefits and linear marginal costs of abatement.\footnote{Barrett [3] offers a more extensive overview of the literature and its results.}

These models suggest that in equilibrium environmental coalitions are small in size. However, the Kyoto Protocol was ratified by 168 countries, thirty-five of whom are required to reduce their emissions. Thus, while the Kyoto Protocol does not have full participation (the United States has not ratified), it is not small in size. In this paper, we investigate why the Kyoto Protocol does not follow the predictions of the coalition cost minimizing models. Section 2 describes the new model, which changes the coalition cost minimizing game. Section 3 provides a numerical example to help understand the results.

\section{2 Pre-Specified Abatement Model}

The primary difference between the Kyoto Protocol and the coalition cost minimizing models is in the specification of abatement levels. In the negotiation of the Kyoto Protocol, emissions reduction requirements were specified prior to the ratification stage. Thus, when the EU-15 decided whether or not to join, it knew that its emissions must be reduced to 8\% below 1990 levels for the 2008-2012 commitment period, regardless of who else ratifies the Protocol. The cost of abatement was independent of coalition membership. In the coalition cost minimizing models, abatement requirements depended on who else ratified the treaty. Thus, when a player decides to join, it is uncertain of both its cost of abatement and its environmental damages because both depend on coalition membership.
Figure 1: Game Tree, Coalition Cost Minimizing Model
2.1 Model Description

The 'Pre-Specified Abatement Model' developed for this paper is rooted in non-cooperative game theory. The game includes \( N \) heterogeneous players and two stages. In Stage Zero, emissions abatement levels are selected for each of the \( N \) players by what we term the 'Social Planner'. These abatement levels are bounded between zero and the status quo emissions level, i.e., \( \hat{a}_i \in [0, \gamma_i] \), where \( \hat{a}_i \) is the abatement level specified for player \( i \) and \( \gamma_i \) is player \( i \)'s emission level without an environmental agreement. In Stage One, each player moves sequentially and chooses whether or not to sign the environmental agreement. Specifically, each player chooses \( m_i \in \{0, 1\} \), where 1 denotes member and 0 denotes non-member. A decision to sign the agreement indicates a commitment to reduce greenhouse gas emissions by the amount the social planner has specified. We assume players comply with the agreement if they sign. Players base their decisions entirely on cost, where cost includes both the cost of abatement and the monetized damages from global emissions.

We assume the game is one of complete and perfect information. Players are deciding on a single agreement; and therefore, only one environmental coalition can form. This agreement has open membership; thus, any player can join the coalition and a coalition forms if any player does join. The agreement does not include entry into force rules, nor are any other negotiations are linked to the environmental agreement. Players are deciding on an environmental coalition only. Additionally, we do not consider transfer payments from one player to another. Finally, we assume players who do not join the coalition maintain their status quo emissions levels, i.e., coalition non-members set their abatement levels to zero. Figure 2 depicts the game tree for a two player implementation of this model.
2.2 Cost and Damage Functions

Solving this model requires the specification of abatement cost and environmental damage functions. In general, we assume the cost of abatement is increasing and convex in the level of abatement, i.e., $C'_a > 0$ and $C''_a > 0$, where $C(\cdot)$ is the cost function and $a$ is the level of abatement. However, for this game, the function used is a variation on the function presented in Nordhaus \cite{Nordhaus1994} and is given by

$$C_i(a_i) = \alpha_i a_i^2 \quad (1)$$

where $a_i$ is the level of abatement of player $i$, and $\alpha_i$ is a constant parameter specific to player $i$. Barrett \cite{Barrett2000}, de Zeeuw \cite{DeZeeuw2002}, Kaitala and Pohjola \cite{Kaitala2005}, and Na and Shin \cite{Na2006} also use abatement cost functions that are quadratic in the level of abatement.

In general, we assume that environmental damages are decreasing and convex in the level of global

\footnote{Nordhaus’s abatement function is given by $C(\mu) = b_1 \mu^{b_2} q$, where $\mu$ is the percentage reduction in greenhouse gas emissions and $q$ is global output. Nordhaus does not assume the function is quadratic, and instead chooses $b_2 = 2.887$.}
abatement (increasing and convex in the level of global emissions), i.e., $D'_A < 0 < D''_A$, where $D(\cdot)$ is the environmental damage function and $A$ is the global abatement level. The specific function used is a modification of the function presented in Goulder and Mathai [8]. We model damages as a quadratic function of global greenhouse gas emissions; Goulder and Mathai specified damages as a quadratic function of greenhouse gas concentrations. Specifically, we assume damages are

$$D_i(A) = \beta_i (\Gamma - A)^2 \quad (2)$$

where $\Gamma$ is the status quo global emission level, $A$ is the global abatement level, and $\beta_i$ is a constant parameter specific to player $i$. de Zeeuw [5] also defines damages as quadratic in abatement levels.

2.3 Subgame-Perfect Nash Equilibrium

The solution to this game is found through backwards induction. The stage one game is solved for a set of general abatement levels, $\hat{a}_i \forall i$. Using this solution, we discuss possible solutions to the stage zero game.

2.3.1 Stage One Solution

In stage one, each player decides whether or not to sign the environmental agreement by comparing the cost of joining with the cost of free-riding. We make these comparisons beginning with the last player, player $N$, and working backwards to player one. For every potential coalition formed by the first $N - 1$ players, player $N$ evaluates the following

$$\alpha_N \hat{a}_N^2 + \beta_N \left( \Gamma - \sum_{i \in M_{N-1}} \hat{a}_i - \hat{a}_N \right)^2 \leq \beta_N \left( \Gamma - \sum_{i \in M_{N-1}} \hat{a}_i \right)^2 \quad (3)$$

where $M_{N-1}$ is the set of coalition members after the first $N - 1$ players have moved. With this
inequality, we find that player $N$ joins the coalition if and only if

$$\hat{a}_N \leq \frac{2\beta_N \left( \Gamma - \sum_{i \in M_{N-1}} \hat{a}_i \right)}{\beta_N + \alpha_N}$$  \hspace{1cm} (4)$$

After assessing these inequalities for every possible coalition, we construct a function, $\Psi_N$, mapping existing coalitions to final coalitions, i.e.,

$$\Psi_N : M_{N-1} \mapsto M_{eq}$$  \hspace{1cm} (5)$$

Next, we consider player $N-1$. When player $N-1$ turn arises, she knows what the the $N-2$ players who have moved prior to her have chosen. Additionally, she can determine whether player $N$ will join, based on $\Psi_N$. Thus, player $N-1$ decides whether to sign the environmental coalition by comparing the cost of joining with the cost of free-riding, i.e., player $N-1$ considers

$$\alpha_{N-1} \hat{a}_{N-1}^2 + \beta_{N-1} \left( \Gamma - \sum_{i \in \Psi_N(M_{N-2} \cup N-1)} \hat{a}_i \right)^2 \leq \beta_{N-1} \left( \Gamma - \sum_{i \in \Psi_N(M_{N-2})} \hat{a}_i \right)^2$$  \hspace{1cm} (6)$$

where $M_{N-2}$ is the set of coalition members after the first $N-2$ players have moved. When this inequality holds, player $N-1$ will sign the environmental agreement. As with player $N$, we convert player $N-1$’s decisions into a function

$$\Psi_{N-1} : M_{N-2} \mapsto M_{eq}$$  \hspace{1cm} (7)$$

This function maps existing coalitions, $M_{N-2}$, to final coalitions, $M_{eq}$.

We iteratively repeat this procedure for players $N-2$ to player two, constructing a coalition mapping
function, $\Psi_i$, for each player. Using these functions, player one can determine the equilibrium coalition size and membership at the time she chooses whether or not to sign the environmental agreement. To make this decision, player one simply compares the cost of joining with the cost of free-riding, i.e.,

$$\alpha_1 a_1^2 + \beta_1 \left( \Gamma - \sum_{i \in \Psi_2(1)} \hat{a}_i \right)^2 \leq \beta_1 \left( \Gamma - \sum_{i \in \Psi_2(\emptyset)} \hat{a}_i \right)^2$$

(8)

If Inequality (8) holds, then player one joins the coalition, initiating a sequence of actions that results in the coalition $M_{eq} = \Psi_2(1)$. If Inequality (8) does not hold, player one chooses not to join, leading to the coalition $M_{eq} = \Psi_2(\emptyset)$.

### 2.3.2 Stage Zero Solution, Homogeneous Players

In this section, we solve the game for a special case where players are ex ante identical. Section 2.3.3 discusses the more general case where players cost and damage parameters vary. First, we find that if we follow the backwards induction procedure previously described, the equilibrium coalition size, $c_{eq}$, is a function of the pre-specified abatement level, i.e.,

$$c_{eq}(\hat{a}) = \frac{2\beta \Gamma + (\beta - \alpha) \hat{a}}{2\beta \hat{a}}$$

(9)

Furthermore, a first mover advantage exists. Thus, the first $N - c_{eq}(\hat{a})$ players choose not to join the coalition and the final $c_{eq}(\hat{a})$ players join the coalition. If players are not identical, a simple rule for determining coalition size does not exist.

Using this result, we discuss different Stage Zero strategies for the social planner. Recall that the social planner chooses the level, $\hat{a}$, that each player is required to abate. However, the rules defining this game do not specify how the social planner chooses this abatement. Thus, we consider two
different strategies. In the first strategy, we maximize individual abatement. In the second strategy, we maximize coalition size.

**Strategy 1: Players are required to abate all emissions**

First, suppose the social planner wishes to force players to remove all emissions, choosing an abatement level equal to the status quo emissions, i.e., $\hat{a} = \gamma$. The equilibrium coalition size is then

$$c_{eq}(\gamma) = \max \left\{ \frac{2\beta N + (\beta - \alpha)}{2\beta}, 0 \right\}$$

(10)

**Theorem 2.1** If players are required to abate all emissions and the cost parameter is sufficiently large, i.e., if $\alpha \geq \beta (2N + 1)$, the non-cooperative solution arises, i.e., $c_{eq} = 0$.5

**Strategy 2: Abatement is chosen to maximize coalition size**

Now, suppose the social planner’s strategy is to maximize the coalition size, i.e., we choose $\hat{a}$ such that $c_{eq} = N$. We solve Equation (9) and determine that the largest abatement level such that $c_{eq} = N$ is

$$\hat{a}(N) = \min \left\{ \frac{2\beta \Gamma}{\beta (2N - 1) + \alpha}, \gamma \right\}$$

(11)

Comparing the Solutions

We have introduced two strategies the social planner could employ. With the first strategy, we find that if players are required to abate all emissions and the damage-cost ratio is sufficiently small, the coalition size is zero. With the second strategy, we find that by reducing the required abatement level, global cooperation is possible. Thus, we have described two agreements: one is "narrow, but deep" and the other is "broad, but shallow." Which agreement is preferable?

5The proof of this Theorem and all subsequent Theorems are found in the Appendix.
Theorem 2.2 If the damage-cost ratio is less than one, i.e., \( \beta < \alpha \), global abatement is maximized when individual abatement is chosen to ensure global cooperation.

Theorem 2.3 If the damage parameter is small enough, i.e., \( \beta < \left( \frac{N-2}{N} \right) \alpha \), global cost is minimized when individual abatement is chosen to ensure global cooperation.

Thus, if the social planner wishes to maximize global abatement or minimize global cost, the preferred strategy is to set the pre-specified abatement level as defined in Equation (11) and ensure global cooperation.

We are also concerned with how this solution compares to the solution from the existing models in literature. Using the cost and damage functions previously defined in a coalition cost minimizing model, we find that the equilibrium coalition size is less than or equal to three. Additionally, each player abates

\[
a = \frac{c\beta \Gamma}{\alpha + c^2 \beta}
\]

Theorem 2.4 If the number of players is sufficiently large, i.e., \( N > \frac{9(\alpha - \beta)}{2\alpha} \), global abatement is larger in a pre-specified abatement model with abatement level chosen to ensure global cooperation than in a coalition cost minimizing model.

2.3.3 Stage Zero Solution, Heterogeneous Players

Now, we consider the more general case where players differ in costs and damages. In the previous subsection, we show that with identical players specifying abatement to maximize coalition size results in higher global abatement (see Theorem 2.2) and lower global cost (see Theorem 2.3) than requiring players to abate all emissions. Thus, we focus on a social planner who wishes to maximize coalition size in this subsection.
With heterogeneous players, players will choose to join a coalition provided their abatement requirement is not too large. Specifically, each player, \(i\), will join a coalition \(M\) if and only if

\[
\hat{a}_i \leq \frac{2\beta_i \left( \Gamma - \sum_{j \in M} \hat{a}_j \right)}{\alpha_i + \beta_i}
\]  

(13)

We assume that the social planner is interested in choosing the maximum abatement levels such that full cooperation is sustainable. Thus, we set (13) to hold with equality. This assumption results in a system of \(N\) equations and \(N\) unknowns. Since these equations are linear, we can use matrix algebra to find the abatement levels, \(\hat{a}_i\). Thus, we solve

\[
\begin{bmatrix}
\alpha_1 + \beta_1 & 2\beta_1 & 2\beta_1 & \cdots \\
2\beta_2 & \alpha_2 + \beta_2 & 2\beta_2 & \cdots \\
2\beta_3 & 2\beta_3 & \alpha_3 + \beta_3 & \cdots \\
2\beta_4 & 2\beta_4 & 2\beta_4 & \alpha_4 + \beta_4 & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots
\end{bmatrix}
\begin{bmatrix}
\hat{a}_1 \\
\hat{a}_2 \\
\hat{a}_3 \\
\hat{a}_4 \\
\vdots
\end{bmatrix}
= \begin{bmatrix}
2\beta_1 \Gamma \\
2\beta_2 \Gamma \\
2\beta_3 \Gamma \\
2\beta_4 \Gamma \\
\vdots
\end{bmatrix}
\]  

(14)

For any number of players, this system of equations results in a closed form solution specifying an abatement level for each player. For example, in a three player game, global cooperation is sustained if each player’s abatement is

\[
\hat{a}_i = \frac{2\beta_i \prod_{j \neq i} (\alpha_j - \beta_j) \Gamma}{\alpha_1 \alpha_2 \alpha_3 + \sum_{j=1}^3 \beta_j \prod_{k \neq j} \alpha_k - 3 \sum_{j=1}^3 \alpha_j \prod_{k \neq j} \beta_k + 5 \beta_1 \beta_2 \beta_3} \text{ for } i = 1, 2, 3
\]  

(15)
To help elucidate the results, we include a numerical example in this section. First, we analyze the Kyoto protocol, offering insights into why some players did not ratify the treaty. Then, we discuss a re-specification of the agreement that results in global cooperation. Finally, we include a sensitivity analysis on our results. In the first part of the discussion, we limit the number of players for this the numerical simulation to four: the United States (USA), the European Union (EU), the other countries in the Organization for Economic Cooperation and Development (OOECD), and Russia and the Economies in Transition (EIT). These four players encompass all countries who had a binding emissions reduction requirement in the Kyoto Protocol, i.e., the Annex-B countries. Later, we add China as a fifth player.

3 Numerical Example

The model requires four parameters: the status quo emission level, $\gamma_i$, the Kyoto specified abatement level, $\hat{a}^K_i$, the cost parameter, $\alpha_i$, and the damage parameter, $\beta_i$. The estimation of each parameter is discussed in this section and the results included in Table 1.

### 3.1 Parameter Estimation

The first parameter is the status quo emissions level, $\gamma_i$, measured in billion tons $CO_2$. This parameter represents the amount of carbon dioxide that each player emits into the atmosphere absent any
environmental agreement. Because environmental agreements, such as the Kyoto Protocol, focus on emissions abatement in future years, we want $\gamma_i$ to represent emissions levels in future years. We have chosen to use projected 2010 emissions levels to coincide with the first Kyoto commitment period (2008-2012). These projections are from the 2006 International Energy Outlook [7].

3.1.2 Kyoto Specified Abatement Level

The Kyoto Protocol specifies a percentage reduction in emissions below 1990 levels for each country. We use these percentages, accessed from the United Nations Framework Convention on Climate Change [18], in combination with 1990 emissions levels, as presented in the International Energy Annual 2003 [6], to compute the amount of emissions each country is allowed under the Kyoto Protocol. We then subtract this quantity from the status quo emissions level to compute the required abatement for each country. Individual country abatement requirements are aggregated to determine the abatement requirement, measured in billion tons $CO_2$, for each player.

3.1.3 Cost of Abatement

We base the cost of abatement parameter, measured in billion $ per (metric ton $CO_2)^2$, on estimates by Tol [15]. Tol models abatement cost as quadratic in the level of abatement and assumes that reducing emissions by 1% costs 0.02% of regional GDP. We compute regional GDP for each player by aggregating the 1997 country specific GDP, as reported by the World Resources Institute and listed in Tol [16].

3.1.4 Damages from Global Emissions

We use Peck and Teisberg’s CETA model [14] to estimate the damage parameter, measured in billion $ per (metric ton $CO_2)^2$, for each player. The CETA model disaggregates damages into
market and non-market components. Both types of damages are quadratic functions of regional temperature increases. Regional temperature rise is calculated using global temperature increase from an implementation of Nordhaus’ RICE model [12] combined with Peck and Teisberg’s regional scaling parameters.

3.2 Evaluating the Kyoto Protocol

The Kyoto Protocol was ratified by thirty-five countries, but not by the United States or Australia. With the model developed in this paper, we can offer insights as to why this occurred. Using Inequality (13), we can compute the maximum amount of abatement to which a player will agree as a function of the abatement levels of other coalition members. We first look at the United States. Based on the parameters in Table 1, we find that the United States will join \{EIT, EU, OOECD\} if and only if its abatement level is less than or equal to 1.57 billion tons CO$_2$. The US’s Kyoto specified abatement level, however, was 1.66 billion tons CO$_2$; thus, abatement is too large to induce cooperation by the United States.

Next, we consider the European Union. At the time the EU ratified the Kyoto Protocol, the USA had already announced its decision not to participate. Thus, the EU was considering whether to join \{EIT, OOECD\}. We find that the EU will cooperate if and only if its abatement level is less than or equal to 0.97 billion tons CO$_2$. The EU’s Kyoto specified abatement level is 0.82 billion tons CO$_2$; thus, abatement is low enough to induce cooperation on the Kyoto Protocol by the EU. The other OECD countries will join \{EIT,EU\} if and only if its abatement level is less than or equal to 0.65 billion tons CO$_2$. The OOECD’s Kyoto specified abatement level is 1.03 billion tons CO$_2$; thus, the theory predicts OOECD will not join (Australia did not join; the other OOECD countries did). Finally, the Economies in Transition join \{EU,OOECD\} if and only if its abatement
level is less than or equal to 1.06 billion tons CO$_2$. The EIT’s Kyoto specified abatement level is 0.20 billion tons CO$_2$; thus, EIT prefers to join.

### 3.3 Re-specifying the Kyoto Protocol

We have shown that the Kyoto Protocol did not achieve full cooperation because the USA and the OOECD’s abatement levels were too high to induce cooperation. Here, we solve the linear system of equations defined by Equation (14) for four players and use the resulting equations to re-specify Kyoto abatement levels such that global cooperation is an equilibrium. Table 2 compares the re-specified abatement levels for each player with the Kyoto abatement levels.
3.4 Comparing to Coalition Cost Minimizing Solution

By reducing abatement requirements, we have shown that global cooperation is possible, but is this preferred? We are again comparing a broad but shallow agreement to one that is narrow but deep; which agreement is better? We answer these questions by comparing the Kyoto Protocol, the re-specified agreement, and an implementation of the Coalition Cost Minimizing Model. We compare these three solutions in terms of the equilibrium coalition (Table 3), abatement (Figure 3), and cost (Figure 4). When we aggregate the information provided in these Tables and Figures, we find that global abatement is largest and global cost is smallest in the re-specified agreement. Additionally, global abatement in the coalition cost minimizing solution is larger than in the Kyoto Protocol. Thus, pre-specifying abatement can result in a "better" solution than allowing coalitions to minimize cost, provided abatement levels are properly chosen.
3.5 Including China

Now, suppose we want to include China to the environmental agreement. Here we investigate how abatement levels and costs change with the inclusion of a new player. We solve the linear system of equations defined in Equation (14) to find the maximum abatement levels that sustain full cooperation for a five player game. We again compare the equilibrium coalition (Table 3), abatement (Figure 5), and cost (Figure 6) among the three solutions. As in the four player game, we find that global abatement is largest and global cost is smallest in the re-specified agreement with five players.
In addition to comparing the results among the three model solutions, we compare global abatement (Figure 7) and global cost (Figure 8) between the four and five player games.\textsuperscript{6} From these Figures, we can see that China’s participation in the negotiation process results in higher abatement and lower cost.

3.6 Sensitivity Analysis

The discussion in the previous section is utilizes the parameters in Table 1. However, these parameters are highly uncertain. Thus, in this section, we investigate how the results change with respect to changes in the parameter values. We include five alternative scenarios, each corresponding to variations in parameter estimates. In the first two scenarios, High Cost and Low Cost, we assume that each player’s cost parameter is 1.25 (High) or 0.75 (Low) times as large as the base value. In the next two scenarios, High Damage and Low Damage, we multiply all damage parameters by either 1.25 (High) or 0.75 (Low). In the final scenario, we multiply each player’s damage parameter by a uniform random number between 0.5 and 1.5. The specific multipliers chosen are: 0.73 (USA),

\textsuperscript{6}The global cost for the four-player game includes the cost of China free-riding on the various coalitions.
Figure 7: Global Abatement, Four and Five Player Games

Figure 8: Global Cost, Four and Five Player Games
Using these alternative parameters, we again analyze the Kyoto Protocol. Table 4 lists the coalition members for the base case and each of the five alternative scenarios. A player’s inclusion in the list of coalition members indicates that its abatement level was small enough to induce cooperation for the parameter values assumed in that scenario. From this table, we see that with High Damages or Low Costs more cooperation occurs.

Next, we consider the sensitivity of the re-specified agreement to changes in parameter values. We compare both individual abatement (Figure 9) and individual cost (Figure 10) between the base case and the five alternate scenarios. In general, we find that abatement is larger when costs are low or damages are high. The fluctuation in abatement levels for the various scenarios suggests the importance of accurately determining cost and damage parameters. Incorrect parameters can result in over-specifying abatement which will reduce coalition size. Finally, costs are larger when damages or costs are high.

### 4 Conclusions

Most game theoretic models of international environmental negotiations in the existing literature reduce the game to two stages: a coalition formation stage followed by an abatement stage. Additionally, these models assume that the coalition chooses abatement to minimize cost in the second
Figure 9: Individual Abatement, Alternative Scenarios

Figure 10: Individual Cost, Alternative Scenarios
stage. The solution to these models suggests that the equilibrium coalition will contain three or fewer players. However, the Kyoto Protocol was ratified by 168 countries, thirty-five of whom have binding emissions abatement requirements.

In this paper, we alter the game presented in the existing literature to more closely resemble the negotiation of the Kyoto Protocol. This new game also reduces the negotiation process to two stages; however, the first stage is an abatement stage and the second is a coalition formation stage. In this game, we find that if the social planner requires players to abate all emissions, a coalition of zero players will form when costs are high. However, the social planner can reduce individual abatement requirements to a level that ensures global cooperation. We find that sustaining global cooperation in the new game results in the largest global abatement and lowest global cost. Additionally, this solution results in more abatement than the Coalition Cost Minimizing Models found in the existing literature.

In a numerical implementation of the model, we find that the United States’ Kyoto specified abatement level exceeds its maximum acceptable amount. Thus, the US’ decision not to ratify Kyoto is not surprising. Extensions of the numerical example calculate abatement levels that allow for global cooperation in a game with only Annex-B countries as well as a game that includes China. The results from the numerical experiment suggest that reducing abatement levels to ensure global cooperation can lead to lower global cost than the Coalition Cost Minimizing solution.

The work presented in this paper relies on several assumptions. First, we assume that players are rational and make decisions based entirely on cost. In reality, a player’s decision to join an environmental coalition may depend on political or other considerations. Next, we assume that non-
members emissions levels are unaffected by the formation of a coalition. We ignore the possibility that players outside the coalition could increase or decrease their emissions in response to the coalition’s abatement. Finally, we assume costs and damages are known with perfect certainty. In actuality, the cost and damage parameters are highly uncertain. However, their values are necessary in order to properly specify full cooperation inducing abatement levels. The sensitivity analysis included in this paper suggests that varying parameter values can affect the global cooperation ensuring abatement level and the coalition membership for a fixed set of abatement levels (the Kyoto requirements). Future research, both on incorporating uncertainty and assessing costs and damages, is necessary.
References


Appendix

This Appendix provides the proofs of all theorems presented in this paper.

Proof of Theorem 2.1:

We want to show that if players are required to abate all emissions and the cost parameter is sufficiently large, i.e., if \( \alpha \geq \beta (2N + 1) \), the non-cooperative solution arises, i.e., \( c_{eq} = 0 \). First, we define the equilibrium coalition size when \( \hat{a} = \gamma \), i.e.,

\[
\hat{a} = \frac{2\beta N + (\beta - \alpha)}{2\beta}.
\]

From this equation we see that the coalition size is zero if and only if \( 2\beta N + (\beta - \alpha) \leq 0 \). Thus, when \( \alpha \geq \beta (2N + 1) \), the equilibrium coalition has zero members, \( c_{eq} = 0 \).

Proof of Theorem 2.2:

We want to show that if the damage-cost ratio is less than one, \( \alpha > \beta \), global abatement is maximized when individual abatement is chosen to ensure global cooperation. First, we observe that if we want \( i \) players to cooperate, the following inequality must hold,

\[
\hat{a} \leq \frac{2\beta \Gamma}{(2i - 1)\beta + \alpha}.
\]

Global abatement for a particular coalition size is maximized if we set this inequality to equality. Now, to maximize global abatement we choose the coalition size with the largest global abatement, i.e.,

\[
\max \left\{ 0, \frac{2\beta \Gamma}{\beta + \alpha}, 2 \frac{2\beta \Gamma}{3\beta + \alpha}, \cdots, i \frac{2\beta \Gamma}{(2i - 1)\beta + \alpha}, \cdots, N \frac{2\beta \Gamma}{(2N - 1)\beta + \alpha} \right\}
\]
We want to show that $A_i \leq A_{i+1}$ for all $i$, where the subscript indicates the coalition size. We do this by contradiction. Assume $A_i > A_{i+1}$, i.e.,

$$i \left[ \frac{2\beta \Gamma}{(2i-1)\beta + \alpha} \right] > (i+1) \left[ \frac{2\beta \Gamma}{(2i+1)\beta + \alpha} \right]$$

This inequality reduces to $\beta > \alpha$. By assumption, we have $\alpha > \beta$; thus, we have a contradiction.

Global abatement increases in coalition size, and thus is maximized by choosing abatement to ensure global cooperation.

**Proof of Theorem 2.3:**

We want to show that if $\beta < \left( \frac{N-2}{N} \right) \alpha$ global cost is minimized when individual abatement is chosen to ensure global cooperation. First, we show that for a fixed coalition size, $j$, global cost is decreasing in the individual abatement level. Global cost when the coalition has $j$ members is

$$T_g(j) = j\alpha a^2 + N\beta(\Gamma - ja)^2$$

Thus, global cost is decreasing in $a$ if and only if $-N\beta \Gamma + (\alpha + Nj\beta)a < 0$. This inequality holds when $a < \frac{N\beta j}{\alpha + Nj\beta}$. For the coalition to contain only $j$ players, however, $a \leq \frac{2\beta \Gamma}{(2j-1)\beta + \alpha}$. Thus, global cost is decreasing in $a$ for coalition of size $j$ if

$$\frac{2\beta \Gamma}{(2j-1)\beta + \alpha} < \frac{N\beta j}{\alpha + Nj\beta}$$

This inequality holds if $\beta < \left( \frac{N-2}{N} \right) \alpha$. Now, we assume that for every coalition of size $j$, $a = \frac{2\beta \Gamma}{(2j-1)\beta + \alpha}$ and show that increasing coalition size decreases global cost, i.e., $T_g(j + 1) < T_g(j)$.
Substituting in for \( a \), this inequality becomes

\[
\frac{4(j + 1)\alpha\beta\Gamma^2 + N\beta(\alpha - \beta)^2\Gamma^2}{[2(j + 1)\beta - \beta + \alpha]^2} < \frac{4j\alpha\beta^2\Gamma^2 + N\beta(\alpha - \beta)^2\Gamma^2}{[2j\beta - \beta + \alpha]^2}
\]

After manipulating this inequality, we find that \( T_g(j + 1) < T_g(j) \) if and only if \( N(\alpha + 2j\beta) > \alpha \), which always holds. Thus, global cost is decreasing in coalition size.

**Proof of Theorem 2.4:**

We want to show that if the number of players is sufficiently large, i.e., \( N > \frac{9(\alpha - \beta)}{2\alpha} \), global abatement is larger when pre-specified to ensure global cooperation than when chosen to minimize coalition cost. If we choose abatement such that full cooperation is an equilibrium, global abatement is

\[
A^N = \frac{2\beta N\Gamma}{2\beta N + \alpha - \beta}
\]

If we minimize coalition cost, global abatement is

\[
A^* = \frac{c^2\beta\Gamma}{\alpha + c^2\beta}
\]

We want to show that \( A^N \geq A^* \). Manipulating the global abatement equations, we find that \( A^N \geq A^* \) if and only if \( \alpha \beta (2N - c^2) + c^2\beta^2 \geq 0 \). This inequality holds if \( 2N - c^2 \geq 0 \). In the coalition cost minimizing model, we found that \( c \leq 3 \), thus a sufficient condition for \( A^N \geq A^* \) is that \( N \geq \frac{9}{2} \).7

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7 Abatement in the cost-minimizing solution, \( A^* \), is increasing in coalition size; thus, it is sufficient to consider \( c = 3 \) when determining whether \( A^N \geq A^* \)