

# Transmission grid representations in power system models – A PTDF-based approach

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## Overview

In many liberalized electricity markets around the world the market clears without considering the physical characteristics of power transmission<sup>1</sup>. The transmission grid capacity limits possible transactions between generators and consumers at different locations. Due to the specific characteristics of AC transmission networks a power injection at a certain node has an impact on several transmission lines, the so-called loop flows. Traditionally, one approach to abstract from this behaviour has been to calculate net transfer capacities (NTC), which represent the available transfer capacity for commercial trades in between two zones and are valid for a certain period of time (e.g. [7]).

However, the classical NTC-approach has shortcomings. First, loop flows are taken into account via applying a “worst-case” scenario in the sense that physical transfer capacities are reduced to account for potential transfer limitations stemming from other power flows and further to maintain the n-1 stability criteria. This leads to an inefficient utilization of the transmission grid, because in many operating points the restriction to the worst case scenario is not binding. Second, from this approach it is not possible to analyse how transfer zones/countries are influenced by commercial trades of third parties. In the future such parties might react via the introduction of Flexible AC Transmission System (FACTS) devices, which will have a direct influence on cross-zonal power flows. Finally, there is now sound method for calculating future NTC values to use within long-term power system models.

Due to the fact that transmission grid modelling based on a full-grid representation is computationally demanding, a number of approaches have been developed to tailor the methodology of technically oriented grid studies (e.g. [5][6][8]) to the needs of power market analysis (cf. [1][2][3][4][9]). Most of these approaches derive grid equivalents based on a certain base operating-point. However, the more the outcome of the market differs from this base case, the greater the errors in terms of actual power flows versus the ones calculated in the simplified model. The current approach tries to overcome this problem by using knowledge from market operation in the grid simplification method.

## Methods

In a first step the  $l$ -by- $n$  Power Transfer Distribution Matrix  $\Phi$  is being calculated for a transmission grid containing each line of the original system, whereas  $l$  means the number of lines and  $n$  the number of nodes in the system. The components of  $\Phi$  are derived using the standard method of multiplying the branch susceptance matrix and the inverse of the bus susceptance matrix of the linearized system.

Second, the nodes in the original grid are grouped into prize zones in which we will observe a single electricity price. Those zones are typically all nodes within a country. All intra-zonal lines in  $\Phi$  are then removed and are not considered any further in the analysis. In that sense it is assumed that those lines are ideal and do not constitute any power flow limitation. If some intra-zonal lines in the original system are frequently congested during normal operation they can be kept in the matrix as well.

Then the remaining matrix  $\Phi$  is used to calculate the so called technology-specific PTDF matrix

$$\Phi_{lgz} = \frac{\sum_{n \in \mathbb{N}_g \cap \mathbb{N}_z} \Phi_{ln} \cdot Q_{ng}}{\sum_{n \in \mathbb{N}_g \cap \mathbb{N}_z} Q_{ng}} \quad \forall g, z, l \in \mathbb{X}, \quad (1)$$

whereas  $\mathbb{X}$  means the set of cross-zonal and frequently congested intra-zonal lines,  $g$  the corresponding generation technology and  $z$  the prize-zone. The sets  $\mathbb{N}_g$  and  $\mathbb{N}_z$  are the set of nodes that contain all nodes with a connected technology  $g$  and those are within prize-zone  $z$ , respectively.  $Q_{ng}$  represents the capacity of generation technology  $g$  at node  $n$ . Each element of matrix  $\Phi_{lgz}$  can be interpreted as the capacity-weighted PTDF-factor of a certain line, whereas only the nodes that are connected to the same generation technology and zone are included in the

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<sup>1</sup> Exemptions are power markets that are operated with locational market prices (e.g. PJM, ISO-NE, ISO-NY, Italy).

calculation. The idea behind this proposal is that in an electricity market all generation technologies with the same techno-economic characteristics, most notably marginal costs, have similar generation profiles. Consequently, when following this approach the resulting PTDF-factors do not change when the output of similar generators vary and thus are independent from a certain operating point. This derived PTDF matrix is then implemented in a classical linear power system model and the resulting intra-zonal power flows are compared to the results of a full DC load flow calculation.

## Results

This method has been tested on a 6-node example, which also has been used in [2],[4] and [11] to illustrate the general usability of the proposed approach. The results show that this method is able to replicate the power flows of cross-zonal lines compared to a full DC load flow calculation with high precision. In contrast to [2],[4] and [11] in this method no lines are aggregated and consequently the thermal limits of the lines in the original system can be preserved and the impact of cross-zonal grid congestion influences the market outcome accordingly. Furthermore, due to the calculation of technology-specific PTDF-factors the results are not depending on how close the actual generators output matches the corresponding output at a certain base operating-point, as it is the case in the other approaches. Similarly as in traditional linear power system models the resulting zonal market prices can be derived via the shadow price of the zonal demand constraint. In the final version of this paper we will test the proposed approach by applying it to the ENTSO-E transmission system.

## Conclusions

A novel approach has been developed in order to integrate cross-zonal transmission grid constraints into a power system model. In contrast to other proposed methods, this approach try to overcome a common problem of grid reduction methods by integrating information of power market outcomes within the grid simplification method. First results show that this approach might be appropriate to be applied in long-term power system models of a large-scale, like the ENTSO-E transmission grid. This hypothesis will be tested in the final version of this paper.

## References

- [1] Allen, E.H., J.H. Lang, and M.D. Ilic. 2008. „A Combined Equivalenced-Electric, Economic, and Market Representation of the Northeastern Power Coordinating Council U.S. Electric Power System“. IEEE Transactions on Power Systems 23 (3) (August): 896–907. doi:10.1109/TPWRS.2008.926715.
- [2] Cheng, X., and T.J. Overbye. 2005. „PTDF-Based Power System Equivalents“. IEEE Transactions on Power Systems 20 (4) (November): 1868–1876. doi:10.1109/TPWRS.2005.857013.
- [3] Shi, Di, and Daniel J. Tylavsky. 2012. „An improved bus aggregation technique for generating network equivalents“. In Power and Energy Society General Meeting, 2012 IEEE, 1–8. [http://ieeexplore.ieee.org/xpls/abs\\_all.jsp?arnumber=6344668](http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=6344668).
- [4] Papaemmanouil, Antonis, Andersson, Göran. 2011. „On the Reduction of Large Power System Models for Power Market Simulations“. 17th Power Systems Computation Conference. Zürich, Switzerland: ETH Zürich.
- [5] Jang, Wonhyeok, Saurav Mohapatra, Thomas J. Overbye, and Hao Zhu. 2013. „Line limit preserving power system equivalent“. In Power and Energy Conference at Illinois (PECI), 2013 IEEE, 206–212. [http://ieeexplore.ieee.org/xpls/abs\\_all.jsp?arnumber=6506059](http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=6506059).
- [6] J. Egerer, C. Gerbaulet, and C. Lorenz, „European electricity grid infrastructure expansion in a 2050 context“, Discussion Papers, DIW Berlin, 2013.
- [7] R. A. Rodríguez, S. Becker, G. B. Andresen, D. Heide, and M. Greiner, „Transmission needs across a fully renewable European power system“, Renewable Energy, Bd. 63, S. 467–476, März 2014.
- [8] S. Chatzivasileiadis, T. Krause, and G. Andersson, „Supergrid or local network reinforcements, and the value of controllability—An analytical approach“, in PowerTech (POWERTECH), 2013 IEEE Grenoble, 2013, S. 1–6.
- [9] H. Oh, “A New Network Reduction Methodology for Power System Planning Studies,” IEEE Trans. Power Syst., vol. 25, no. 2, May 2010.