Long-term LNG Contracts

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Abstract

Long-term contracts between exporters and importers of LNG provide a number of benefits, but also can impose costs on the contracting parties. On the one hand, long-term contracts reduce the variability of cash flows, thereby decreasing the risk and increasing the debt capacity of large, long-term, capital investments in the LNG/natural gas value chain. On the other hand, long-term contracts may limit the ability of the contracting parties to take advantage of profitable trading opportunities resulting from short-run fluctuations in market demand or supply. After developing a model that illustrates these trade-offs, we discuss why LNG contracts have historically been indexed to oil prices. Finally, we discuss how important changes in world natural gas markets have been altering the liquidity of LNG markets. On the basis of insights from the model, we argue that continuation of these trends may, in coming years, reduce the desirability of long-term contracts relative to a greater reliance on shorter-term and spot market trades.

Keywords: Long-term contracts, LNG, investment project leverage, opportunistic trades

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1. Introduction

Traditionally, LNG was almost exclusively traded under long-term contracts. However, over the last decade or so, the volume of LNG traded spot or on short-term contracts of less than four years duration has risen much faster than overall LNG trade. We shall argue that the entry of additional suppliers and demanders into the market is one important explanation for this trend.

In recent years, the relative price of oil to natural gas has also tended to drift upwards. This trend has also put pressure on long-term contracts where the price of natural gas is often indexed to the price of oil. The paper will also discuss potential explanations for the linkage between oil and LNG prices and what recent changes in the oil/gas relative price might mean for parties to long-term LNG contracts.

In investigating the costs and benefits of long-term contracts, we elucidate how the features of such contracts respond to changes in various exogenous variables. Specifically, we develop a model of the contracting environment faced by a developer of an LNG liquefaction project and its customers. We focus particularly on the effects of changes in spot market liquidity on the terms in an optimal contract between the trading partners, where “optimal” is defined as a contract giving the largest combined net present value to the trading partners.

The key idea underlying the contract model is that long term contracts result in a less volatile cash flow, which in turn increases the “bankability” of contracts. Specifically, with a less volatile cash flow, the firms are able to finance the investments with increased leverage, which brings increased tax savings. On the other hand, contracts may lead to some trades that are ex-post inefficient. An increase in LNG market liquidity has two important effects. First, the rent associated with exclusive trade between any two parties will tend to decline as each party gains access to more potential trading partners. Second, the volatility of cash flows in the absence of a long-term contract declines. Both of these developments reduce the benefits of a long-term contract. At the same time, we show that some of these consequences of increased market liquidity increase the benefits of participating in spot markets for parties that are under long-term contracts. If parties take advantage of those new trading opportunities, they further increase the liquidity of the market. This positive feedback loop means that small exogenous changes affecting the market could have substantial effects on market structure.
The model is developed in the context of stationary distributions for spot market prices and supply and demand shocks. We then show how it can be extended via indexation to oil prices to cope with non-stationary spot market prices. This allows us to discuss the role of oil indexation in long-term LNG contracts.

We end the paper by discussing possible developments in LNG markets using insights gained from the model.

1. Some recent developments in LNG markets

In order to motivate some of the subsequent analysis and discussion, we begin in this section of the paper by presenting some data on recent developments in LNG markets. Figure 1, based on data from the International Group of Liquefied Natural Gas Importers (GIIGNL), shows that spot market and short-term (less than four year duration) contract trades steadily increased in the first half of the last decade, but have stabilized at around one-quarter to one-third of total trade in the last five years.

Figure 1: Total, contracted and spot and short-term LNG trade

It is also interesting to note in Figure 1 that actual trade levels approximated the contracted volume. Evidently, much of the short-term trade must originate as parties to
long-term contracts also engage in short-term and spot market trade. This can involve swap arrangements, for example, or purchasers re-selling contracted volumes when their current demand falls short of the volume included in the contract. The model we develop later in the paper will include a description of the several ways that such short-term trades can arise from parties to a long-term contract.

In the remainder of this section, we take a closer look at the Japanese market for LNG. According to the International Gas Union World LNG Report, 2011, Japan was the largest LNG importing country (33% of marketed volume), with the second largest importer (South Korea) taking 15% of marketed volume and the third and fourth together (UK and Spain) taking another 15%. It is thus interesting to focus on the evolution of short-term and spot trade versus long-term contract trade in Japan.

Figure 2: Japanese LNG imports by origin and type

Japanese LNG imports are presented by export origin in Figure 2. Imports from the longer-term major suppliers – Indonesia, Malaysia, Australia, Brunei and the UAE – were overwhelmingly all on long-term contracts. Qatar is conspicuous for supplying a substantial amount in the last decade and also a large fraction on a short-term or spot basis in recent years. Another even more recent relatively large supplier, namely Russia
(exporting from Sakhalin), also has provided a much larger proportion of exports under short-term or spot contracts than the traditional suppliers. Most of the smaller quantities imported from a variety of countries (including re-exports) were also on a short-term or spot basis.

One reason for the increasing proportion of short-term and spot imports of LNG into Japan in 2011 was the Fukushima disaster, which resulted in a shut-down of nuclear reactors and a consequent large, and unforeseen, increase in natural gas demand for electricity generation. However, Figure 2 shows that Japan had been shifting toward a greater proportion of short-term and spot imports prior to March 11, 2011.

The increase in demand for LNG following Fukushima also raised LNG import prices. Figure 3 graphs three key natural gas price series over the last three years. These are the Henry Hub price in the US, the price at the National Balancing Point (NBP) in the UK and Platt’s Japan Korea Marker (JKM) price for LNG delivered to Japan and South Korea. Also included in Figure 3 is the Brent price for oil measured on an mmbtu basis.

Figure 3: Recent evolution of key natural gas prices

The three natural gas price series moved similarly in 2009, although the JKM price started drifting higher around the middle of that year. The Henry Hub price then declined

2 The Brent price is a better measure of world oil prices than the WTI price in the US in recent years. Pipeline constraints in North America have prevented effective arbitrage and driven a wedge between the WTI and world prices.
relative to the other prices from March 2010. Around that time, the North American market became disconnected from the rest of the world as a result of surging domestic production and a build up of inventories. By contrast, both the JKM and NBP tended to follow oil prices higher throughout 2010 and into early 2011. Following the Fukushima disaster, the JKM departed substantially from the NBP price and toward the end of 2011 and again in mid-2012 approximated the price of Brent crude on an energy-equivalent basis.

Figure 4 presents a longer-term view of Japanese LNG import prices. In these cases, the average monthly prices from key suppliers have been graphed on the vertical axes against the Brent price converted to an energy-equivalent basis on the horizontal axes. The straight line corresponds to the “energy-equivalent” price ratio. The time periods of the data are the same as in Figure 2, namely January 1998 through to June 2012. The exporting countries represented in the plots in Figure 4 are Australia (aus), Brunei (brn), Indonesia (idn), Malaysia (mys), Qatar (qat) and the United Arab Emirates (uae).

Figure 4: Japanese LNG import prices versus Brent oil price, major exporters

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3 Mark Agerton carried out the statistical analysis in this section, but he is not responsible for my interpretation and presentation of his results.
Before plotting the data, the cross-correlogram between the average monthly Brent crude price and the average price of LNG exports from each country in each month was obtained. In all cases, the LNG price lagged behind the oil price, and the number of months lag is indicated in the title on each plot. The shortest lag, of one month, was for Indonesia. Australia, Malaysia and UAE prices showed maximum correlation at a 4-month lag, while for Brunei and Qatar the maximum correlation was at a 5-month lag. This perhaps suggests that the Indonesian contracts are the most tightly tied to oil prices, with the other prices tending to key off the Indonesian contracts and perhaps each other.

In addition, for each country $i$, a simple regression equation of the form

$$p_{it}^{\text{lag}} = \alpha_i + \beta_i p_{t-k_i}^{\text{oil}} + \epsilon_{it}$$

was estimated using flexible least squares (FLS), where $p_{it}^{\text{lag}}$ represents the natural log of the average price of LNG exports from country $i$ to Japan in period $t$, $p_{t-k_i}^{\text{oil}}$ represents the natural log of the price of Brent in period $t-k_i$ and $k_i$ corresponds to the lag of maximum cross correlation identified for country $i$.

The FLS technique allows the coefficients $\alpha_i$ and $\beta_i$ in equation (1) to vary, in this case as a function of time. Rather than minimizing the sum of squared residuals in (1) as in least squares, FLS minimizes a weighted average of the sum of squared residuals and the sum of squared deviations in regression coefficients from one period to the next over the sample.

Breaks in the model structure are indicated near where the FLS coefficients achieve maximum or minimum values since at these locations the coefficients are being “pulled” in a new direction by the data. For each plot in Figure 4, two structural breaks have been identified. The month and year of these breaks is also presented in the title to each plot.

Not surprisingly, in all cases, one of the breaks occurs in the first full month after the Fukushima disaster. However, the relationship between import LNG prices and the price of oil also underwent an earlier structural change in all cases. The structural breaks thus divide the data into three time periods. Points belonging to the first time period have been represented in the plots with blue circles, points in the second period with green triangles.

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5 While power supply from alternatives to natural gas was reduced in the month of the disaster, power demand was also sharply curtailed. Hence, it is not surprising that the upward adjustment in LNG import prices took a short time to materialize.
and points in the third time period (after Fukushima) by red crosses. Since oil prices have tended to rise over time, it is also generally the case that the first time period encompasses months in the overall sample when oil prices were lowest, the second time period when oil prices were intermediate, and the third time period when oil prices were highest.

It is evident from the figures that the relationship between oil and LNG prices for all countries was close to energy parity at the lowest oil prices. As oil prices began to rise, however, the LNG prices did not rise commensurably and the scatter of points tended to follow a line indicating less than 1-1 indexation of LNG prices to the price of oil. In two cases (Indonesia and Brunei), the change in slope was sufficient to produce a structural break. In the other cases, however, the relationship at lower oil prices was already less than 1-1 by a larger amount and the initial relationship sufficed to describe the data for a longer period of time.

In each of the countries except Indonesia, the second period is characterized by a much looser relationship (marked by a greater dispersion of data about the line) between average LNG import prices and the price of oil. However, it also would appear that the structural break occurred when the LNG prices on average were adjusted upward relative to the price of oil. Furthermore, in most cases (Indonesia again being the main exception), the slope of the relationship between the prices after the adjustment was again less than 1-1, but somewhat steeper than the flatter slope that prevailed immediately prior to the upward adjustment. Finally, in the period since Fukushima, the LNG prices have been generally high, but the scatter of points looked at in isolation from the rest of the sample would not suggest a strong linear relationship between LNG prices and the price of oil (Indonesia is again the main exception where a stronger linear relationship is perhaps again in evidence). Part of the issue here, however, is that the period has been short. Additional data points may reveal more of a relationship between the two prices. In some cases, the points in the third period might be seen as an extension of the more variable scatters observed in the second period.

2. A Model of long-term LNG contracts

In this section, we develop a simple model of long-term contracting in the LNG industry. For the time being we ignore the issue of oil indexation examined at the end of the last section and suppose we are in a stationary environment where all relevant probability distributions are not varying over time. We will return to the issue of oil indexation in the next section of the paper.
2.1 The investment projects

We will consider investment a single 5-mmtpa liquefaction plant. Noting that 1 tonne of LNG contains approximately 51.322mmbtu, a 5-mmtpa/year plant would produce about $256.6 \times 10^6$ mmbtu/year (or 0.25 Tcf/year) of natural gas. Figure 5 plots real costs (in $2010) of liquefaction plants as a function of plant capacity in mmtpa.

![Fixed Cost vs Capacity](image)

Figure 5: Fixed costs of liquefaction capacity

Using the regression line in Figure 5, a 5-mmtpa plant would cost around $9.119 billion. We also assume a real operating cost of $0.28/mcf, which would give tax-deductible variable annual operating costs of around $V_X = 0.2726$ million per $10^6$ mmbtu/year.

To enable easier valuation of the importer investments, we will also assume that no additional regasification capacity is needed, but that all the LNG will be used to fuel CCGT power generation plants. In accordance with representative data for state of the art CCGT plants available at the EIA, we assume each plant has 400MW capacity and a heat rate of 6.43 mmbtu/MWh. Supposing that the plants will be used to provide intermediate load, we assume that they will be operated at a 60% load factor. Power output from each plant would then be 2.1024 TWh/year, requiring an input of $13.518 \times 10^6$ mmbtu/year of natural gas. Thus, 18 CCGT power plants would consume approximately $243.33 \times 10^6$ mmbtu/year of natural gas. An additional power plant would take the input requirements above the output of the liquefaction plant. The 5% difference between liquefaction plant
output and CCGT input demand would also allow for some LNG to be consumed in transport and regasification. For simplicity, we aggregate the 18 power plants into one importer facing the single exporter-owner of the liquefaction plant. We assume both the importer and the exporter maximize after-tax net present value of profits.

To obtain costs for the power plant investments, we again use EIA data. At a capital cost of $1.003 million/MW, a 400MW plant would require an investment of $401.2 million, or a total of $7.221 billion for 18 plants. Fixed operations and maintenance (O&M) costs of $0.01462 million/MW implies $5.848 million fixed O&M per year per plant, or $105.264 million per year for 18 plants. Variable O&M (excluding fuel) of $3.11/MWh and a heat rate of 6.43 mmbtu/MWh imply annual non-fuel variable O&M of \( V_M = 0.4837 \text{ million/} 10^6 \text{ mmbtu}. \)

Figure 6 plots shipping costs for LNG per mmbtu as a function of distance. We shall take \( S = 1.25/\text{mmbtu} \) as a representative figure.

\[
M^D = \frac{\alpha + \epsilon - P_M - V_M}{\beta}
\] (2)

Figure 6: Shipping costs/mmbtu as a function of distance

For simplicity, we will assume linear demand and supply curves for LNG. Specifically, we assume that the demand for LNG is given by
where $p_M$ is the landed price of LNG and $V_M$ is short run variable operating cost for the importing power projects. The parameter $\alpha$ is the mean intercept of the demand curve while $\beta$ is its slope. Similarly, the supply of LNG exports is given by

$$X^s = \frac{p_X - V_X - \delta - \xi}{\gamma}$$

(3)

where $p_X$ is the export net back price and $V_X$ is the short run variable cost of operating the liquefaction plant. The parameter $\delta$ is the mean intercept of the supply curve and $\gamma$ is its slope. We also allow for supply and demand to be affected by random shocks, $\varepsilon$ and $\xi$. For simplicity, we assume that the shocks shift the curves in a parallel fashion by changing their intercepts. The demand shocks could result from plant outages, or changes in other fuel prices or the demand for electricity. Supply shocks could result, for example, from plant outages, weather shocks, or strikes.

We assume that demand shocks are much more volatile than supply shocks. Specifically, we will allow the demand curve intercept to vary by $\pm 4$ while the supply curve intercept varies by $\pm 0.7$. We also set the slopes of both curves to 0.035 in absolute value, and set the demand “choke price” (the cost of fuel where demand would be zero) to $20/mmbtu$ and the minimum marginal cost of production to $1/mmbtu$. Noting that the demand price (the value measured off the demand curve) minus the supply price (the value measured off the supply curve) have to differ by the shipping cost plus the sum of variable costs of the importer and the exporter, the chosen values imply that, at the mean values of the intercepts, the volume of trade between the two parties would be $242.768 \times 10^6$ mmbtu/year. At that volume of trade, the price to the importer would be $11.02/mmbtu$ and a netback price to the exporter of $9.77/mmbtu$. The resulting market equilibrium can be represented as in Figure 7. As illustrated in Figure 7, we assume that the demand and supply shocks follow a symmetric beta distribution with a coefficient of 3.25.

If we let the profits of the importer be $\Pi$ and the price of LNG paid by the importer be $p$ then the envelope theorem implies that the demand for the natural gas input is $-\partial \Pi / \partial p$ and we can interpret the “consumer surplus” area under the input demand curve between two prices $p_0$ and $p_1$ as the change in short-run profit resulting from a change in fuel

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6 The upward sloping supply curve in effect reflects the increasing cost of obtaining feed gas for the liquefaction plant as export volumes increase.
7 Note that a price measured as $$/mmbtu translates to an equivalent of millions of dollars per $$mmbtu so the units of $$ in Figure 7 are millions of dollars.
price. Similarly, the supply curve is $\partial \Pi / \partial p$ for the supplier and the “producer surplus” area above it between two prices $p_0$ and $p_1$ is the change in short-run profit resulting from a change in output LNG price.

Figure 7: The market equilibrium

The parties to the contract can trade not only with each other but also with third parties. Specifically, we assume that there is a spot market where LNG can be bought or sold at prices that vary randomly, but which do not depend on the amount bought or sold by the two parties to our long-term contract. Let the export netback spot price available to the exporter be $p_X$ and the delivered spot price available to the importer be $p_M$.

Observe that if the spot market is well arbitrated, it would not to be possible to buy LNG at a price $p_M$ pay $S = 1.25/$mmbtu to ship it to the exporter location and sell it at a profit for $p_X$. We therefore assume that $p_M > p_X - S$.

We would also expect the delivered price to the importer to be positively correlated with the netback price available to the exporter. They need not be perfectly correlated, however, since the destination for a spot cargo from the exporter, and the point of origin...
for a spot cargo delivered to an importer, are likely to vary from one period to the next. Spot prices at the same locations are also likely to vary over time as market conditions change.

With these considerations in mind, we assume that the spot market export netback prices follow various symmetric beta distributions a mean value of $8.75 or $9.25 and standard deviations varying from $0.82 to $1.41. We also assume that the price for a spot cargo delivered to the importer can be written:

\[ P_M = P_X + V \quad (4) \]

where \( V \) also follows symmetric beta distributions independent of the distribution of \( P_X \). In the numerical examples we examine, the mean of \( V \) ranges from $1.9375 to $3.25, while the standard deviation ranges from $0.6162 to $1.2324. Notice that from the requirement that there are no arbitrage opportunities in the spot market, \( V > -S = -1.25 \). In particular, the distributions for \( V \) that have larger standard deviations cannot be combined with those with smaller mean values of \( V \) without violating this constraint. Figure 8 graphs a representative joint density function for \( P_X \) and \( P_M \).

Figure 8: Joint probability density function for the spot prices

For the distributions that we consider, \( \Pr(V > S) = \Pr(P_M > P_X + S) \) averages 0.8959, with a minimum value of 0.7043 and a maximum value of 1.00. As a result, bilateral trade
between the importing party and the exporter is likely to be preferable to spot trades for much of the time under even the least favorable distributions for \( \nu \) that we consider.\(^8\)

2.2 Financial parameters

We will use the adjusted present value approach to value the two investment projects. This assumes that the net benefit of debt can be approximated by its corporate tax benefits alone. Any personal tax benefits of equity relative to debt, which would otherwise tend to make debt less attractive, are largely offset by net positive non-tax benefits of debt resulting mainly from its benefit in helping to control agency problems between decision-makers in the firm and its shareholders.

The adjusted present value approach values the firm’s after-tax cash flows, exclusive of any tax benefits from depreciation allowances and payments associated with debt, at the all-equity rate of return. The corporate tax savings resulting from debt are then valued at the debt interest rate and added to the all-equity value of the firm. The tax benefits resulting from the depreciation allowances are valued at the risk-free rate of interest on the grounds that the allowances are known once the investment has been made (assuming any unforeseen changes in tax laws are not retroactive). We will assume an all-equity\(^9\) rate of return of 10\%, an interest rate on debt of \( r_B = 5\% \) and a risk free rate of 3\%. All projects are assumed to have a 25-year life with straight-line depreciation assumed for tax purposes. The corporate tax rate, \( \tau \), is taken to be 35\%.

In addition to the above considerations, we assume that the firms face a “value at risk” type of limitation on the debt they can hold. Denote the after-tax, but before interest, cash flow for particular values of demand (\( \varepsilon \)) and supply (\( \xi \)) shocks, and export netback (\( p_X \)) and delivered import (\( p_M \)) spot prices by \( C(\varepsilon, \xi, p_X, p_M) \). Let the amount of debt finance be \( B \) with after-tax interest cost \((1-\tau)r_BB\). We then require that the available cash flow in any year would be insufficient to cover the after-tax interest cost plus a certain fraction of the principal with a probability equal to a given level. In the following calculations, we set the amount of principal coverage in any year to be 10\% and the probability of not being able to meet the specified cash flow target to be less than 5\%. In other words, we require

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\(^8\) It also is useful to note that the correlation between \( p_X \) and \( p_M \) is related to the ranges of \( p_X \) and \( \nu \). Specifically, the variance of \( p_M \) equals the variance of \( p_X \) plus the variance of \( \nu \), while the covariance between \( p_X \) and \( p_M \) equals the variance of \( p_X \). In addition, \( p_X \) and \( \nu \) are based off the same underlying beta distribution. Hence, if the correlation between \( p_X \) and \( p_M \) is \( \rho \), the range of \( \nu \) must be \( \sqrt{(1-\rho^2)}/\rho \) times the range of \( p_X \).

\(^9\) Note that since in practice the firm will have debt, the all-equity return will be below the actual return on levered equity.
\[
\Pr\left[C(\epsilon, \xi, p_X, p_M) < 0.1B + (1 - \tau)r_B B\right] = 0.05
\]

### 2.3 Trading without a contract but under full information

It is useful first to consider two scenarios where trading occurs between the two parties in the absence of a contract. In the first scenario, discussed in this section, we assume that both parties know all relevant information about each other, including variable costs and the current values of the demand and supply shocks. This is a rather unrealistic scenario in practice, since if trades and prices depend on such information parties may not have an incentive to reveal it truthfully. In the next section, we consider the more realistic situation where the parties trade without a contract and base trades and prices solely on publicly available information about spot market prices and the shipping cost.

In the relatively unlikely case that the values of the spot prices satisfy \( p_X + S \geq p_M \), both the importer and the exporter would prefer to use the spot markets rather than trade with each other. The resulting contributions to variable profits would be

\[
\Pi^v_M = \frac{(\alpha + \epsilon - p_M - V_M)^2}{2\beta}
\]

\[
\Pi^v_X = \frac{(p_X - V_X - \delta - \xi)^2}{2\gamma}
\]

When instead \( p_X + S < p_M \), the exporter prefers bilateral trade at a net price of \( p_M - S \) to spot market trade at \( p_X \). We define two “prices” (inclusive of short-run variable costs) for the exporter and importer such that supply at the exporter price (after subtracting short-run variable cost) matches demand for the importer (after adding short-run variable cost) and the two prices differ by exactly \( S + V_X + V_M \):

\[
\tilde{p}_X = \frac{\gamma(\alpha + \epsilon - S - V_X - V_M) + \beta(\delta + \xi)}{\beta + \gamma}
\]

\[
\tilde{p}_M = \frac{\gamma(\alpha + \epsilon) + \beta(\delta + \xi + S + V_X + V_M)}{\beta + \gamma}
\]

If the spot prices satisfy \( p_X - V_X \leq \tilde{p}_X \) and \( p_M + V_M \geq \tilde{p}_M \) then both the importer and the exporter would prefer bilateral trade at the prices (8) and (9) to spot market trade. The resulting contributions to variable profits in this case would be
\[ \Pi'_{M} = \frac{(\alpha + \varepsilon - \tilde{p}_{M})^2}{2\beta} \]  

(10)

\[ \Pi'_{X} = \frac{(\tilde{p}_{X} - \delta - \xi)^2}{2\gamma} \]  

(11)

Alternatively, if \( p_{X} - V_{X} \leq \tilde{p}_{X} \) but \( p_{M} + V_{M} < \tilde{p}_{M} \), then the importer, but not the exporter, would prefer to deal in the spot market. Since we have assumed spot market supply is perfectly elastic at the prevailing spot market price, that price would then set the terms for trade between the exporter and importer. The importer would pay \( p_{M} \) and earn short-run profit (6) regardless of the source of the LNG. The demand for LNG from the importer would be given by (12).

The exporter may desire to produce more than \( M^D \) at \( p_{M} - S \) but any excess must in fact be sold spot at \( p_{X} \). The maximum production at \( p_{M} - S \) is thus

\[ X_{M}^S = \min\left( p_{M} - S - V_{X} - \delta - \xi \frac{\gamma}{\gamma}, M^D \right) \]  

(12)

and spot market supply from the exporter, if any, would be

\[ X^S = \max\left( p_{X} - V_{X} - \delta - \xi - X_{M}^S, 0 \right) \]  

(13)

The contribution to the short-run profit of the exporter in this case would be

\[ \Pi'_{X} = \left( p_{M} - S - V_{X} - \delta - \xi - \frac{\gamma}{2} X_{M}^S \right) X_{M}^S + \left( p_{X} - V_{X} - \delta - \xi - \gamma X_{M}^S \right) X_{M}^S \]  

(14)

Finally, in the case \( p_{X} - V_{X} > \tilde{p}_{X} \), we need not consider \( p_{M} + V_{M} < \tilde{p}_{M} \) since this would lead to

\[ \tilde{p}_{M} > p_{M} + V_{M} > p_{X} + S + V_{M} > \tilde{p}_{X} + S + V_{X} + V_{M} \]

which contradicts the definition of \( \tilde{p}_{X} \) and \( \tilde{p}_{M} \). Thus, \( p_{M} + V_{M} \geq \tilde{p}_{M} \) in this case, and we conclude that the importer would prefer to buy from the exporter rather than use the spot market. Also, since \( p_{X} - V_{X} > \tilde{p}_{X} \) the exporter will only trade for at least \( p_{X} \). Recall that we assume that the parties to the trade are small enough relative to the spot market as a whole that any trades from the partners have no effect on spot market prices. We conclude that, in this case, regardless of whether output is sold to the importer or to the
spot market the exporter would obtain $p_X$. The supply of output from the exporter would be given by (3) and the contribution to variable profits would be given by (7). The importer may demand more than $X_S$ at the price $p_X + S$, and, if so, will have to buy from the spot market. Product taken from the exporter would then satisfy

$$M^D_X = \min \left( \frac{\alpha + \epsilon - p_X - S - V_M}{\beta}, X_S \right)$$

and spot market purchases from the importer, if any, would be

$$M^D = \max \left( \frac{\alpha + \epsilon - p_M - V_M}{\beta} - M^D_X, 0 \right)$$

The contribution to the short-run profit of the importer in this case would be

$$\Pi^v_M = \left( \alpha + \epsilon - p_X - S - V_M - \frac{\beta}{2} M^D_X \right) M^D_X + \left( \alpha + \epsilon - \beta M^D_X \right) \frac{M^D}{2}$$

### 2.4 Trading without a contract and with partial information

In order to implement the allocations discussed in the previous section the firms would need to know the current demand and supply shocks $\epsilon$ and $\xi$, and the variable O&M costs $V_X$ and $V_M$ in addition to publicly available data. The latter would include data on spot prices and shipping costs, and measures easily inferred from observing past statistical data such as estimates of the demand and supply curves.

In this section, we examine a feasible alternative allocation and price-setting mechanism in the absence of a contract that is based solely on the latter kinds of publicly available information. Specifically, we now assume that when the prevailing spot market prices are less favorable than the parties could achieve on their own, they now set the prices to “split the difference” between $p_X$ and $p_M$, that is, in place of (8) and (9), we define

$$\hat{p}_X = \frac{p_M + p_X - S}{2}$$

$$\hat{p}_M = \frac{p_M + p_X + S}{2}$$

With this method of setting the price for trades between the exporter and importer, we can again look at the outcomes for supply, demand, bilateral trades and spot market
trades under the various possible outcomes for the demand and supply shocks and spot prices.

Once again, if the spot prices satisfy \( p_X + S \geq p_M \), both the importer and the exporter would prefer to use the spot markets rather than trade with each other and the contributions of those trades to short-run profits would again be given by (6) and (7).

For the remaining cases, where \( p_M \) exceeds \( p_X \) by more than the shipping cost \( S \), we first calculate demand and supply at the prices (18) and (19):

\[
\hat{M}^D = \frac{\alpha + \epsilon - \hat{p}_M - V_M}{\beta} \tag{20}
\]

\[
\hat{X}^S = \frac{\hat{p}_X - V_X - \delta - \xi}{\gamma} \tag{21}
\]

If \( \hat{M}^D > \hat{X}^S \), the importer would have to use the spot market to satisfy any additional demand. Possible spot market purchases would be given by

\[
M^D = \max \left( \frac{\alpha + \epsilon - p_M - V_M - \hat{X}^S}{\beta}, 0 \right) \tag{22}
\]

The contributions to short-run profits in this case would be given by

\[
\Pi^v_M = \left( \alpha + \epsilon - \hat{p}_M - V_M - \beta \frac{\hat{X}^S}{2} \right) \hat{X}^S + \left( \alpha + \epsilon - \beta \hat{X}^S - p_M - V_M \right) \frac{M^D}{2} \tag{23}
\]

\[
\Pi^v_X = \left( \hat{p}_X - V_X - \delta - \xi \right) \frac{\hat{X}^S}{2} \tag{24}
\]

Conversely, if \( \hat{M}^D < \hat{X}^S \), the exporter would have to use the spot market to dispose of any surplus supply. Possible spot market sales would be given by

\[
X^S = \max \left( \frac{p_X - V_X - \delta - \xi}{\gamma} - \hat{M}^D, 0 \right) \tag{25}
\]

The contributions to short-run profits would then be

\[
\Pi^v_M = \left( \alpha + \epsilon - p_M - V_M \right) \frac{\hat{M}^D}{2} \tag{26}
\]
$$\Pi^v_x = \left( \hat{p}_x - V_x - \delta - \xi - \gamma \frac{\hat{M}^d}{2} \right) \hat{M}^d + \left( p_x - V_x - \delta - \xi - \gamma \hat{M}^d \right) X^S \quad (27)$$

Finally, in the “knife-edge” case where $\hat{M}^d = \hat{X}^S$, the contributions to short-run profits would be

$$\Pi^V_M = \frac{(\alpha + \epsilon - \hat{p}_M - V_M)^2}{2\beta} \quad (28)$$

$$\Pi^v_x = \frac{(\hat{p}_x - V_x - \delta - \xi)^2}{2\gamma} \quad (29)$$

### 2.5 Trading under a contract

We next consider trading under a long-term contract between the exporter and importer that has the following features. There is a contract price $p$ paid by the importer for LNG delivered by the exporter to the importer’s location. The exporter thus receives a netback price of $p - S$. The contract also specifies a delivery volume $q$. The supplier is required to deliver $q$ unless both parties agree to a lesser amount. In addition, there is a take or pay clause. An importer taking $M < q$ when the export price available to the exporter in the spot market is $p_x < p$ has to pay $(p - S - p_x)(q - M) \equiv \phi(q - M)$ to the exporter. In other words, an importer not taking the full contracted supply has to compensate the exporter for any loss suffered when the export price available to the exporter is below the contract netback price $p - S$. We again assume that either party can supplement the contracted trade with spot market transactions.

We then assume that the contract terms $p$ and $q$ are chosen to maximize the sum of the expected net present values of the profits from the two investment projects. However, we also impose the incentive compatibility constraint that the expected net present value of profits obtained by each party under the contract must be non-negative and at least as good as the expected net present value of profits that party could obtain by trading without a contract but with incomplete information (the situation discussed in section 2.4).

We next discuss the spot and contracted trades that would occur for different values of the spot prices $p_x$ and $p_M$. Consider first the situation where $p_M \geq p_x + S$. Note that this also implies $p_M + \phi = p_M + p - S - p_x \geq p$. In which case the importer would prefer to take the contracted supply at price $p$ than buy spot at $p_M$ and pay $\phi$. The exporter thus will supply $q$ and if importer demand at price $p$ is less than $q$, the importer will sell the
difference spot. Such spot sales will be at a loss if $p_X + S < p$, but the loss involved would still be less than exercising the take or pay clause. The exporter or importer may also supplement sales or purchases under the contract with additional spot market transactions. The output from the exporter consumed by the importer will be

$$M^D_X = \frac{\alpha + \epsilon - p_X - S - V_M}{\beta}$$  \hspace{1cm} (30)

Possible additional spot market sales from the exporter can be written

$$X^S = \max \left( \frac{p_X - V_X - \delta - \xi}{\gamma} - q, 0 \right)$$  \hspace{1cm} (31)

and the contribution of such transactions to short-run exporter profit would be

$$\Pi_X' = \left( p - S - V_X - \delta - \xi - \frac{\gamma q}{2} \right) q + \left( p_X - V_X - \delta - \xi - \gamma q \right) \frac{X^S}{2}$$  \hspace{1cm} (32)

We then distinguish two sub-cases depending on whether or not $M^D_X$, given by (30) is smaller than $q$. When $M^D_X \geq q$, the importer may make additional spot purchases

$$M^D = \max \left( \frac{\alpha + \epsilon - p_M - V_M}{\beta} - q, 0 \right)$$  \hspace{1cm} (33)

and the resulting contribution to short-run profits would be

$$\Pi_M' = \left( \alpha + \epsilon - p - \frac{p_M - V_M}{2} \right) q + \left( \alpha + \epsilon - \beta q - p_M - V_M \right) \frac{M^D}{2}$$  \hspace{1cm} (34)

When $M^D_X < q$, the contribution to short-run profits is

$$\Pi_M' = \left( \alpha + \epsilon - p - V_M - \beta \frac{M^D_X}{2} \right) M^D_X + \left( p_X + S - p \right) \left( q - M^D_X \right)$$  \hspace{1cm} (35)

Next we consider the case where $p_M < p_X + S$, when the importer might wish to exercise the take or pay clause in the contract. If, in addition, $p_X + S > p$, however, the exporter would prefer to sell output in the spot market and fulfill the contract using spot market purchases (a swap). Note that doing so also avoids the transport cost $S$. If the importer wishes to purchase less than $q$ at the price $p$, we assume that in this case the exporter is willing to oblige. Thus, the exporter purchases
\[ X^D = \min \left( \frac{\alpha + \varepsilon - p - V_M}{\beta}, q \right) \] (36)

spot to make the swap. If \( p_M \) is low enough, the importer may also make additional spot purchases

\[ M^D = \max \left( \frac{\alpha + \varepsilon - p_M - V_M}{\beta} - X^D, 0 \right) \] (37)

while the exporter may also make spot sales in addition to the output freed up as a result of the swap

\[ X^S = \max \left( \frac{p_X - V_X - \delta - \xi}{\gamma} - X^D, 0 \right) \] (38)

The contributions to short-run profits in this case will be

\[ \Pi_M^v = \left( \alpha + \varepsilon - p - V_M - \beta \frac{X^D}{2} \right) X^D + \left( \alpha + \varepsilon - \beta X^D - p_M - V_M \right) \frac{M^D}{2} \] (39)

\[ \Pi_X^v = \left( p - p_M + p_X - V_X - \delta - \xi - \gamma \frac{M^D}{2} \right) M^D + \left( p_X - V_X - \delta - \xi - \gamma M^D \right) \frac{X^S}{2} \] (40)

Finally, in the case \( p_M < p_X + S \leq p \), the importer exercises the take or pay clause and both parties use the spot markets. The contributions to short-run profits in this case will be

\[ \Pi_M^v = \frac{(\alpha + \varepsilon - p_M - V_M)^2}{2\beta} - (p - S - p_X)q \] (41)

\[ \Pi_X^v = \frac{(p_X - V_X - \delta - \xi)^2}{2\gamma} + (p - S - p_X)q \] (42)

3. Effects of changes in the market environment

We focus on the effects of changes in the probability distributions for the spot market prices. The reason for focusing on these changes is that we expect changes in the market environment to have the largest effect on changes in the desirability of long-term contracts versus market trades. Other factors, such as the distributions of underlying demand or supply shocks, or the elasticities of demand or supply, may also affect the
The desirability of long-term contracts. However, we might expect these factors to be relatively stable over time compared to changes in the market environment.

Table 1 summarizes the solutions for the maximizing contract terms $p$ and $q$, the surpluses earned by the exporter ($X$) and importer ($M$), and the debt levels carried by the exporter and importer ($B_X$ and $B_M$ respectively), under 75 different distributions for $p_X$ and $\nu = p_M - p_X$. Specifically, for each of two possible means ($8.75/\text{mmbtu}$ and $9.25/\text{mmbtu}$) for the distribution of $p_X$ and three possible means ($1.9375/\text{mmbtu}$, $2.4375/\text{mmbtu}$ and $3.25/\text{mmbtu}$) for the distribution of $\nu$, we calculated the solutions for the optimal contract and the two different non-contract solutions for different ranges (effectively variances) of $p_X$ and $\nu$.

Table 1: Average values of key variables for different means of the price distributions

<table>
<thead>
<tr>
<th>$E(p_X)$</th>
<th>8.75</th>
<th>9.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\nu)$</td>
<td>2.4375</td>
<td>3.25</td>
</tr>
<tr>
<td>Number of distributions</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>Contract price $p$ ($/\text{mmbtu}$)</td>
<td>10.68</td>
<td>10.97</td>
</tr>
<tr>
<td>Contract quantity $q$ (10⁶ mmbtu/year)</td>
<td>223.09</td>
<td>229.59</td>
</tr>
<tr>
<td>$E(\text{NPV}_X)$ under contract ($\text{m}$)</td>
<td>45.10</td>
<td>487.06</td>
</tr>
<tr>
<td>$E(\text{NPV}_X)$ full information ($\text{m}$)</td>
<td>-312.28</td>
<td>178.57</td>
</tr>
<tr>
<td>$E(\text{NPV}_X)$ public information ($\text{m}$)</td>
<td>-434.10</td>
<td>338.46</td>
</tr>
<tr>
<td>$E(\text{NPV}_M)$ under contract ($\text{m}$)</td>
<td>1547.12</td>
<td>881.00</td>
</tr>
<tr>
<td>$E(\text{NPV}_M)$ full information ($\text{m}$)</td>
<td>1662.83</td>
<td>1121.91</td>
</tr>
<tr>
<td>$E(\text{NPV}_M)$ public information ($\text{m}$)</td>
<td>1533.69</td>
<td>792.61</td>
</tr>
<tr>
<td>$B_X$ under contract ($\text{m}$)</td>
<td>5176.72</td>
<td>5490.05</td>
</tr>
<tr>
<td>$B_X$ full information ($\text{m}$)</td>
<td>3827.87</td>
<td>4435.16</td>
</tr>
<tr>
<td>$B_X$ public information ($\text{m}$)</td>
<td>3612.66</td>
<td>4016.04</td>
</tr>
<tr>
<td>$B_M$ under contract ($\text{m}$)</td>
<td>3162.26</td>
<td>2785.63</td>
</tr>
<tr>
<td>$B_M$ full information ($\text{m}$)</td>
<td>3277.38</td>
<td>3292.48</td>
</tr>
<tr>
<td>$B_M$ public information ($\text{m}$)</td>
<td>2620.52</td>
<td>2350.06</td>
</tr>
</tbody>
</table>

There are no entries in Table 1 for the case $E(p_X)=8.75$ and $E(\nu)=1.9375$ since there was no feasible solution for the contract variables $p$ and $q$ in this case. With such low spot prices evidently the bilateral arrangement between the exporter and importers under our assumed costs is uncompetitive. In the case $E(p_X)=8.75$ and $E(\nu)=2.4375$ there also were no feasible solutions for low values for the variances for $p_X$ and $\nu$, so the number of distributions in the averages was 12 instead of 15. Also, in an attempt to find more solutions for the case $E(p_X)=8.75$ and $E(\nu)=2.4375$ we tried additional variance values, and that lead to 18 pairs of distributions instead of 15 in the case $E(p_X)=9.25$ and $E(\nu)=2.4375$.

Focusing first on the contract terms, we see that an increase in the mean of spot market prices tends to raise both the contract price and the contract volume. Since an increase in $E(p_X)$ holding $E(\nu)$ fixed also increases $E(p_M)$ by the same amount, we can see that an
increase in 50¢ in the general level of spot prices raises the contract price by a smaller amount of approximately 45¢. The higher spot market prices make it more desirable for the importer to buy from the exporter and joint profits are maximized by trading a slight reduction in the *relative* price (between the contract and the spot market) for an increased volume of trade between the two parties to the contract. An increase of 50¢ in the spread between the means of the spot price distributions holding $E(p_X)$ fixed, by contrast raises the contract price on average by about 20¢ or only half the amount of an increase in $E(p_X)$.

For the full set of solutions underlying the averages in Table 1, contract volume averaged $232\times10^6$ mmbtu/year, with a standard deviation of $5.6\times10^6$ mmbtu/year and a range from $220.2-241.8\times10^6$ mmbtu/year. Recall that a 5 mmtpa LNG plant would produce about $257\times10^6$ mmbtu/year, while 18 CCGT plants would consume about $243\times10^6$ mmbtu/year. From the averages in Table 1, we also can see that an increase of 50¢ in $E(p_X)$ (and $E(p_M)$) increased contract volume (holding other variables fixed) about $10^7$ mmbtu/year, while an increase of 80¢ in $E(v)$ increased $q$ about $5\times10^6$ mmbtu/year. The results for $p$ and $q$ imply that higher spot market prices, and a bigger gap between the spot prices available to the importer and the netback prices available to the exporter, make the bilateral trade more valuable.

Turning next to the expected net present values obtained under the contract and the two different non-contract solutions observe that strictly positive values imply the expected return on the investments exceed the required rates used to discount the cash flow components. The expected net present value amounts for the exporter in Table 1 under the contract solutions range from around 0.5% to almost 14% of the up-front investment cost of $9,119 million. For the importer, the expected net present value amounts range from around 1.9% up to more than 21% of the up-front investment costs of $7,221 million. The exporter expected net present value under the contracts increases with $E(p_X)$ or $E(v)$, while the expected net present value of the importer decreases. Not surprisingly, higher spot prices on average tend to favor the exporter and lower ones the importer. It also is interesting that the average exporter expected net present values when $E(p_X)=8.75$ and $E(v)=2.4375$ are negative in the no-contract solutions. In these cases where average spot prices are very low, trade between the exporter and importer would not occur in the absence of a contract. Furthermore, while the incentive compatibility constraint requires the contract solution to be at least as good as the no-contract solution using public information alone, in fact the expected net present values for both parties are strictly larger under the optimal contract solutions.
The final six rows of Table 1 present average debt levels for the exporter and importer. Exporter debt under the contract solution ranges from one-third to more than 43% higher than debt under the public information non-contract solutions. The corresponding percentage differences for the importer range from 16–20% higher. Furthermore, both the importer and the exporter take on more debt under the contract than under the no-contract public information solutions. As hypothesized at the outset, the tendency for the contract to stabilize cash flows in the face of random shocks allows the investing parties to carry more debt.

The sum of the debt carried by the exporter and the importer is also always higher under the contract solution than under the full information non-contract solutions. Unlike the case of the public information non-contract solution, however, the importer typically carries more debt under the full information solutions than under the contract solutions. It is just that the absolute difference is greater for the exporter, so the sum of debt levels is also greater.

The higher debt levels under the contract solutions would, by themselves, tend to raise expected net present values from the investment projects because of the assumed tax benefits from debt finance. Comparing these implied differences with the actual differences in expected net present values, we find that the implied differences exceed the actual ones. Also as hypothesized at the outset of the paper, the contract solutions impose some other losses that partially, but not completely, offset the gain from being able to carry higher debt levels. Since the full information solutions allow for the highest value ex-post trades in every circumstance, they represent the most efficient allocation of production by the exporter and the most efficient sourcing of LNG for the importer. We conclude that the trades forced by the contract solutions are less efficient, but the benefits of higher debt levels under the contract solutions more than offset the resulting losses from inefficient trades.

The above discussion has focused on the averages of the different variables under each assumed pair of values for the means of the spot price distributions. The solutions also vary in response to changes in the variances of the two spot price distributions. The effects of changes in variances are, however, generally highly non-linear.\textsuperscript{10}

\textsuperscript{10} The non-linearities made it difficult to find the optimal contract solutions for many pairs of spot price distributions. We found the pattern search algorithm in MatLab most effective. Derivative based search algorithms tended to get stuck at local maximums.
Rather than present the results from all 75 sets of spot price distributions in a table, we summarized the effects of changes in variances by estimating and then plotting a set of regression surfaces. Specifically, for each pair of values for the means of $p_X$ and $\nu$, one can view the solutions for the contract price $p$ and contract volume $q$, and other outcomes of interest, as non-linear functions of the standard deviations of the two distributions $\sigma(p_X)$ and $\sigma(\nu)$. Each function can be approximated by a Taylor expansion, which is then used to interpolate values for the outcome of interest for values of $\sigma(p_X)$ and $\sigma(\nu)$ other than the ones that we explicitly solved for. For example, the 15 solutions for the contract price $p$ when $E(p_X)=8.75$ and $E(\nu)=3.25$ and $\sigma(p_X)$ and $\sigma(\nu)$ take on 15 different pairs of values can be regressed onto $\sigma(p_X)$, $\sigma(\nu)$, $\sigma^2(p_X)$, $\sigma^2(\nu)$, $\sigma(p_X)\sigma(\nu)$ and so forth to obtain the approximation to the true non-linear surface. In practice, we found that we needed to allow for at least a third order Taylor expansion in terms of $\sigma(p_X)$ and $\sigma(\nu)$ to get a reasonable approximation to the solution values.

Figure 9 graphs the approximate solution for the optimal contract price $p$ as a function of $\sigma(p_X)$ and $\sigma(\nu)$ and for the different values of $E(p_X)$ and $E(\nu)$. Figure 10 graphs the corresponding solutions for contract volume $q$.

![Figure 9: Approximate contract prices ($/mmbtu$)](image-url)

The effects of changes in $\sigma(p_X)$ and $\sigma(\nu)$ on $p$ are, at their largest, similar in magnitude to the effects of changes in $E(\nu)$. Specifically, changes of 30–60¢ in $\sigma(p_X)$ and $\sigma(\nu)$ (holding
E(\(p_X\)) and E(\(\nu\)) fixed) altered \(p\) by at most 25\(\epsilon\), but in several cases the changes in \(p\) are much smaller. For E(\(p_X\))=8.75, increases in both \(\sigma(p_X)\) and \(\sigma(\nu)\) tend to raise \(p\) initially, but continuing increases in both \(\sigma(p_X)\) and \(\sigma(\nu)\) then decrease \(p\). However, offsetting changes in \(\sigma(p_X)\) and \(\sigma(\nu)\) tend to leave \(p\) relatively unchanged. For E(\(p_X\))=9.25 and E(\(\nu\))=1.9375, the contract price is a saddle point in \(\sigma(p_X)\) and \(\sigma(\nu)\): increases in \(\sigma(p_X)\) holding \(\sigma(\nu)\) fixed tend to increase \(p\) at first and then decrease it; while increases in \(\sigma(\nu)\) holding \(\sigma(p_X)\) fixed tend to decrease \(p\) at first and then increase it. For E(\(p_X\))=9.25 and E(\(\nu\))=2.4375, increases in \(\sigma(\nu)\) holding \(\sigma(p_X)\) fixed tend to decrease \(p\) at first and then increase \(p\) while \(\sigma(p_X)\) holding \(\sigma(\nu)\) fixed tend to increase and then decrease \(p\) at low levels of \(\sigma(\nu)\), but increase and then decrease \(p\) at high levels of \(\sigma(\nu)\). Finally, for E(\(p_X\))=9.25 and E(\(\nu\))=3.25, increases in \(\sigma(\nu)\) holding \(\sigma(p_X)\) increase \(p\) at low and high values of \(\sigma(\nu)\), but decrease it for a range of intermediate values of \(\sigma(\nu)\). Increases in \(\sigma(p_X)\) tend to slightly reduce \(p\) at low values of \(\sigma(\nu)\), but increase it at higher values of \(\sigma(\nu)\).

Figure 10: Approximate contract volumes (10^6 mmbtu/year)

The non-linear effects of changes in \(\sigma(p_X)\) and \(\sigma(\nu)\) are also reflected in the optimal contract volumes graphed in Figure 10. Except when E(\(p_X\))=8.75, E(\(\nu\))=2.4375, and for low \(\sigma(p_X)\) when E(\(p_X\))=9.25, E(\(\nu\))=3.25, an increase in the standard deviation \(\sigma(p_X)\) of spot prices tends to reduce contract volume. An increase in the standard deviation of the
spot price gap $\sigma(v)$ tends to increase the contract volume at low levels of $E(v)$ but reduce contract volume at higher levels of $E(v)$.

Changes in the standard deviations of spot prices have complicated non-linear effects on the contract terms as a result of a number of offsetting factors. Higher spot price variability provides more opportunities for contracting parties to take advantage of favorable spot market trades. However, the contract also limits the parties from taking advantage of some profitable spot market trades and an increase in spot price variance can raise the cost of such forgone opportunities. Increased variability of spot market prices also raises the variability of cash flows, thereby reducing the debt capacity of the investment projects and increasing the benefits of a contract.

![Figure 11: Ex-post trading inefficiencies relative to the full information solution](image)

We observed when discussing the summary results in Table 1 that the contract solutions allow the firms to carry additional debt relative to the no-contract solutions. We can calculate the addition to the expected net present values of the investment projects resulting from the tax benefits associated with this additional debt alone. When we do so, we find that this difference exceeds the actual gap between the sum of the expected net present values for the contract solutions and the no-contract full information solutions in particular. The latter reflect the maximum profits that can be jointly earned by the two parties given the values of the various shocks. The gap therefore reflects the ex-post
trading inefficiencies resulting from the contract terms. Figure 11 expresses these trading inefficiencies as a proportion of the sum of expected net present values under the full information no-contract solution.

The proportional gaps graphed in Figure 11 are quite small, ranging from a low of around 2% to a high of around 6.5%. The fact that these inefficiencies are relatively small is what allows the contract solution to deliver higher rents overall despite its ex-post trading inefficiencies. The benefits of the extra debt more than compensate for losses resulting from these trading inefficiencies.

Comparing Figures 10 and 11, we can see that, roughly speaking, the trading inefficiencies tend to be relatively low where the contract volumes are relatively high and vice versa. This is consistent with the idea that ex-post trading inefficiencies are a significant cost of the contract solution. By reducing contract volumes in situations where such inefficiencies are more likely, the contracting parties are free to increase their use of ex-post profitable spot market trading opportunities and limit those costs.

Figures 12–14 shed some more light on these issues by graphing variations in expected spot market trading under the optimal contracts applying in different regimes. Figure 12 graphs expected spot market purchases minus spot market sales by the importer, while Figure 13 expected spot market sales minus spot market purchases. Figure 14 sums
expected spot market transactions under optimal contracts regardless of whether they are a sale or purchase, and regardless of the whether the contracting party is the exporter or the importer, and divides the result by the optimal contract volume.

Figure 13: Net spot market sales by the exporter under optimal contracts

Figure 14: Gross spot market transactions relative to contracted volumes
Recall that when spot prices available to the importer are low relative to the contract price $p$ the importer may supplement contracted trade with spot purchases, while the exporter may also fulfill contract obligations using a swap. Conversely, when spot market prices are high relative to the contract price $p$ the exporter may supplement sales under the contract with spot market sales, while the importer may choose to sell some of the contracted supply spot rather than use it itself. Demand will be based on the opportunity cost of using the natural gas, not on the contracted price.

Both importer net spot market purchases and exporter net spot market sales increase substantially as the average gap $E(v)$ between spot prices available to importers and the netback price available to the exporter decreases. When the mean gap decreases, the probability that the parties to the contract will be able to find profitable spot market trades rises. Generally, reduced variability of the gap $\sigma(v)$ decreases spot market transactions. A reduction in $\sigma(p_X)$ also tends to decrease net spot market transactions, but the effect is much weaker. The optionality inherent in supplementing contract trades with spot market transactions is exercised more when spot market prices are more variable.

Figure 14 shows that, for $E(p_X)=9.25$ and $E(v)=1.9375$, as $\sigma(v)$ approaches the boundary where $p_M + S \leq p_X$ with non-zero probability the total volume of spot market transactions from either party to the contract can exceed 75% of the contracted volume. Of course, larger values of $\sigma(v)$ are not possible without giving rise to situations where the no arbitrage condition for the spot market is violated.

Figure 15: Additional exporter plus importer debt under the contract solutions
Figure 15 illustrates the main reason why the contract solutions yield higher expected net present value overall despite the limitations they impose on ex-post efficient spot market trades. Because the contracts make cash flows less variable than under the no-contract solutions, the contract solutions allow roughly 30% more debt than the public information no-contract solutions.\footnote{The contract solutions also allow extra debt relative to the full information no-contract solutions, but the differential is lower.}

Comparing the ranges of values covered in each of the graphs in Figure 15, it is evident that the additional debt under the contract solutions is not very sensitive to changes in the means $E(p_X)$ and $E(\nu)$. Since the contours are relatively steep vertical lines, it also is evident that changes in $\sigma(p_X)$ have a stronger effect on excess debt than changes in the variability $\sigma(\nu)$ of the gap between $p_M$ and $p_X$. The sensitivity of additional debt under the contract solutions to spot price variability is consistent with the hypothesis that the contract allows more debt by stabilizing cash flows.

Figure 16: Contract solution premiums relative to the public information equilibriums

A remarkable feature of the graphs in Figure 15 relative to those in Figures 9 and 10, and to some extent also Figures 11-14, is that the function is much more linear. Furthermore, this relative linearity carries over to Figure 16, which graphs the proportional increase in $E(NPV_X + NPV_M)$ under the contract solutions relative to the public information no-
contract equilibriums. Indeed, the general shapes of the functions graphed in Figures 15 and 16 are quite similar. While changes in the variances of the spot price distributions have complicated effects on spot market trading opportunities, non-linear changes in the contract price $p$ and volume $q$ evidently allow cash flows to change much more linearly in response to changes in $\sigma(p_X)$ and $\sigma(v)$. As a result, the additional debt afforded by a contract also changes in a more linear fashion.\(^{12}\)

From Figure 16 (or the numerical results in Table 1) we can also see that the expected net present value of the exporter plus importer projects under the contract solution is on average around 30% higher than the corresponding partial information spot market solution.\(^{13}\) The advantage of a contract is not much affected by the general level $E(p_X)$ of spot market prices, but a reduction in the average gap $E(v)$ between $p_M$ and $p_X$ noticeably reduces the benefits of a contract.

As we saw above, a reduction in the average gap $E(v)$ also reduces contracted volume and substantially increases spot market trading by parties to a contract. These are key implications of the model. A more liquid spot market that decreases the mean gap between importer and exporter netback prices decreases the relative superiority of the contract solution, decreases contracted trading volume and increases spot market trading by parties to a contract.

Figure 16 also reveals that a decrease in $\sigma(p_X)$ also substantially reduces the benefits of a contract, but the effect of $\sigma(v)$ is weak and generally more ambiguous. Comparing Figures 15 and 16, the key reason for the reduction in the benefits of a contract as $\sigma(p_X)$ declines is that reduced variability of spot market prices also reduces the advantage of a contract in allowing the firms to carry extra debt. Thus, if a more liquid spot market also decreases spot price variability, the relative superiority of the contract solution will also decline.

We can summarize the above results as follows. The contract solutions yield a higher joint expected net present value for the project participants predominantly because they allow for higher debt levels. Inefficiencies arising from contract trades limiting the ability

\(^{12}\) Along these lines, it is also noteworthy that the cubic approximations to the functions graphed in Figures 15 and fit the calculated values in each region much more accurately. The lowest $R^2$ of the regressions used to calculate the graphs in Figure 15 is 0.9989 and in two of the regions equals 1.0 to four decimal places. For the estimated functions underlying Figure 16, the lowest $R^2$ is 0.9994 and in three of the five regions $R^2$ equals 1.0 to four decimal places.

\(^{13}\) The numerical results also reveal that $E(NPV_X + NPV_M)$ under the contract solutions is approximately 12% higher than the corresponding sum under the spot market solutions based on all private as well as public information.
of parties to exploit spot market opportunities are not large, with changes in contract terms and supplemental spot market trades helping keep these under control. Generally speaking, a smaller gap between spot prices available to the exporter and the importer, and lower variability of spot market prices, reduces the advantages of the contract over spot market trades. A reduction in the average gap between importer and exporter spot market prices also encourages substantially more spot market trading by parties to the contract. Increased spot market trading in turn should make investors less concerned about not having their trades covered by contracts. On the other hand, a decrease in the variability of the gap between importer and exporter prices tends to decrease the amount of spot trading by parties to a contract.

Figure 17: Total rent under the optimal contract solutions

Figure 17 graphs the total rents, or sum of expected net present values, accruing to the two parties to the optimal contract solutions for different pairs of distributions for $p_X$ and $v$. While a reduction in the average gap $E(v)$ between $p_M$ and $p_X$ reduces the superiority of a contract relative to a no-contract solution, it nevertheless raises the overall rents accruing to the parties to a contract. As noted above, an important explanation for the sum of expected net present values to rise as $E(v)$ declines is that it allows for many more opportunities to exploit optionality inherent in having values of $p_X$ and $p_M$ that are closer together and closer also to the contract price $p$. 
Higher average spot prices in general, \( E(p_X) \), also increase the total rents accruing under an optimal contract. Since changes in the spot price distributions affect neither the costs of the exporter nor the demand curve for natural gas by the importer, the bilateral trade becomes more valuable as spot market prices increase. Total rent also increases as \( \sigma(p_X) \) increases and, except at very low values for \( \sigma(\nu) \) when \( E(p_X) = 8.75 \) and \( E(\nu) = 3.25 \), also as \( \sigma(\nu) \) increases.

**Figure 18:** Exporter share of total expected net present value under the contract solutions

Figure 18 graphs the share of overall rent under the contract that accrues to the exporter. This is again much more non-linear as a function of \( \sigma(p_X) \) and \( \sigma(\nu) \). In fact, the graphs in Figure 18 look similar to the corresponding graphs of the optimal contract prices in Figure 9. This is perhaps not surprising given that an increase in the contract price \( p \) will redistribute rents from the importer to the exporter.

4. **Oil indexation**

The above analysis assumed \( p_X \) and \( p_M \) followed stationary distributions. In practice, nominal natural gas prices will change over time both with inflation and as the real prices of energy commodities change. We saw this in Figures 3 and 4 above.

It is also evident, however, that the prices of energy commodities are closely related and, in particular, that natural gas prices are related to the price of oil. In fact, a number of papers have found that although natural gas prices and oil prices are integrated of order 1,
a regression of the logarithm of a natural gas price against the logarithm of the oil price produces a residual that is stationary in levels or can be made so if one also includes a limited number of other variables that allow for gradual changes in technology or other factors that affect the substitutability between natural gas and oil.

Suppose that both parties to a contract are confident that the relative price between $p_X$ or $p_M$ and the price of oil is likely to be a stationary probability distribution. For example, \( \ln p_X = \alpha + \beta \ln p_{oil} + \epsilon_X \) and \( \ln p_M = \alpha + \beta \ln p_{oil} + \epsilon_M \) with \( \epsilon_X \) and \( \epsilon_M \) stationary. Such a belief may be rational if natural gas and oil are often substitutes or complements in production (depending on how E&P resources are allocated) and also substitutes in consumption (with inter-fuel competition including switching of fuel input using the same capital equipment, and competition in output between goods or services using different energy sources as productive inputs).

Using the same relationship to index the contract price \( p \) to \( p_{oil} \) the analysis could be applied to derive an indexed contract price \( p_\epsilon \) based on the residual stationary shocks \( \epsilon_X \) and \( \epsilon_M \) in place of the original spot prices as used above. The indexed contract price in the example would then be \( \ln p = \alpha + \beta \ln p_{oil} + \ln p_\epsilon \).

One can think of the non-stationary part of the oil and natural gas prices in the above example as a “long-run” relative price that tends to recur over time. As has been found empirically, however, this long-run relationship could shift over time if technological changes alter the substitutability between the commodities on either the supply or the demand sides of the market. When that happens the indexation formula needs to change or the price distributions that underlie the derivation of the contract price \( p \) would cease to be stationary. Without a change in the formula, the contract price may become increasingly sub-optimal, giving parties to the contract an incentive to breach and use spot markets instead, even if penalties for breach of contract are incurred. Such a change in long-run relative prices appears to have happened in the cases examined in Figure 4.

5. Concluding comments

As more firms begin to consume LNG, and more producers enter the market, the average distance between any two potential trading partners will decline. In addition, the overall elasticity of supply or demand facing any one party will tend to increase.

Technological change, especially with regard to exploiting unconventional sources of supply, will also tend to raise supply elasticities. Furthermore, the use of natural gas in a wider range of applications may also raise demand elasticities.
As a result of all these tendencies, we would expect the variability of spot market prices to decline over time. We also might expect spot market prices facing exporters and importers to become more highly correlated. The model suggests that both of these factors will tend to increase spot market participation and reduce volumes traded under contract, further raising spot market liquidity.

Changes in the long-run relationship between oil and natural gas prices will also prompt contract renegotiations and allow parties to previous contracts to increase their exposure to spot market prices as part of the renegotiation process. Accordingly, we can foresee an evolution of world LNG markets toward larger volumes being traded on short-term contracts or sold as spot cargoes, swaps and other such arrangements, while lower volumes are sold under long-term contracts.