ON INTER- AND INTRA-GENERATIONAL EQUITIES IN ENERGY-CLIMATE POLICY MODELING*

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Akira Maeda[1] and Makiko Nagaya[2]
1: College of Arts and Sciences, The University of Tokyo
3-8-1 Komaba, Meguro, Tokyo 153-8902, Japan; maeda@global.c.u-tokyo.ac.jp
Telephone: +81 3 5465 7740; Fax: +81 3 5465 7732 (corresponding author)
2: Graduate School of Economics, Kyoto University
Yoshida-honmachi, Sakyo, Kyoto 606-8501, Japan; makiko.nagaya@gmail.com

Abstract: Model parameters used in energy-climate policy modeling play important roles in the calculation of future scenarios. Among these parameters, the elasticity of marginal utility, another name of which is the coefficient of relative inequality-aversion in the Atkinson's sense, and social time preference are understood to represent intra- and inter-generational equities, respectively, and thus are closely related to an ethical aspect of the modeling as well as to each other. This paper investigates relationship between these two equity concepts, the interchangeability of the two as model parameters, and its economic implications.

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1. Introduction

Most energy-climate policy assessment models including "integrated assessment models" are based on the Ramsey model. Its basic structure formulates the economic dynamics as an optimization model. Its objective function describes social welfare that is defined as a weighted sum of instantaneous utilities of consumption stream. On one hand, the weights are called discount factors, the feature of which is characterized by social time preference, ρ. On the other hand, a typical instantaneous utility function has the form of CRRA, constant relative risk aversion, with the parameter of the elasticity of marginal utility, η. These two features indicate that it is a pair of parameters (η, ρ) that determines one of the most important parts of energy-climate policy modeling.

Needless to say, there is a long list of studies in the economics literature that contributed to theoretical foundations regarding η and ρ. Thanks to their contributions, the rationale of utilizing these parameters has been well established as an economic theory. On the other hand, in practices of the policy modeling community, setting actual values for these parameters is not an easy task: It is hard to say that there is a well established methodology or procedure for it so far. It is because parameter setting must be consistent to the real economic data as well as ethical or philosophical considerations regarding equity among economic agents. In particular, climate change has an exceptionally long time horizon, and thus relates to intergenerational equity issues to which the setting of social time preference is crucial. This fact has led to a lot of controversies among climate change policy analysts. According to Dasgupta (2008), the model of Cline (1992) used the pair of parameters as $(\eta , \rho) = (1.5, 0\%)$ while Nordhaus (1994) used (1, 3%). In the well known study of Stern (2006), it was set as (1, 0%), however. These differences in parameter values in fact are considered to be one of primal sources of different calculation results out of their models.

Out of the two parameters, adopting specific numbers for ρ is much more controversial than for η in policy modeling practices. The role of ρ is to determine the weights for the stream of instantaneous utilities of generations, and thus is closely related to ethical considerations regarding intergenerational equity. At the same time, it is related to market interest rates (or capital rents) in theory. As a result, when one chooses a specific number for ρ, (s)he sometimes faces a puzzle that is not easy to solve in light of ethical considerations, consistency with actual data, and other factors to be considered in economic theory. In the recent literature, providing a survey of related studies, Dasgupta (2008), cited above, illustrates that there are various constraints in setting the value of ρ for a fixed η.

In the context of climate policy modeling, while the role of ρ in models has drawn many researchers' attention, the role of η does not seem to have been treated in a similar manner so far. As is mentioned above, however, η being together with ρ determines the representation of social welfare function. Thus, the role might carry the same importance with that of ρ. Emphasizing this point, Buchholz and Schumacher (2010) examined the interchangeability between η and ρ. In particular, they indicated that the role of ρ as a determinant of intergenerational equity can be replaced by the adjustment of η to some extent.

The study of Buchholz and Schumacher definitely contributed to a rich understanding of the relationship between intergenerational equity and social time preference. On the other hand, however, it failed to examine the role of η itself in contrast to that of ρ because it only focused to η's supplemental role to ρ:

It is not restricted to Buchholz and Schumacher: It is usual that a significant treatment equal with that of ρ is not given to η in the context of climate policy modeling. However, we should recall that η has a different name, that is, "relative inequality-aversion coefficient" due to Atkinson (1970). The concept behind is that η represents social preferences over uneven income distributions. In the Atkinson's sense, η is considered to be a parameter for intragenerational equity, which deserves to an equal treatment with ρ in both ethical and technical respects.

The purpose of this paper is to examine the interchangeability between η and ρ to explore insights regarding intra- and intergenerational equities as well as implications to parameter setting techniques in climate policy modeling. The sections that follow are organized as follows: the next section reviews the role of ρ in representing intergenerational equity in dynamic economic models, while Section 3 reviews Atkinson's discussion of the role of η in representing intragenerational equity. Section 4 as a main part of this paper provides definitions, propositions, and implications regarding the interchangeability between η and ρ. The final section concludes our discussion.

2. Intergenerational equity concept

This section reviews the concept of equity behind social time preference ρ as a preliminary to the sections that follow.

Consider economic agents, each of which represents one generation. Generations are represented by t. Let $U_t(C_t)$ denote the utility from general consumption for generation t. Social welfare function is described as follows:

$$W\left(\{C_t\}_{t=0}^{\infty}\right) \equiv \int_0^{\infty} U(C_t) D_t dt$$

where the set of $\{D_t\}$ represents weights such that $\int_0^{\infty} D_t dt = 1$ holds.

The stream of $\{D_t\}$ can take any functional form, but it is known that the only possible function that is time-consistent and dependent only on time is exponential. That is,

$$D_t \equiv \rho e^{-\rho t}.$$

Due to this fact, social time preference ρ is interpreted as the parameter that adjusts the degree of intergenerational equity.

If we only require the feature of time-consistency, the functional form does not have to be exponential. A possible form is the following one:

$$D_t \propto e^{-\int_0^t \rho(C(s))ds}.$$

This type of endogenous discounting is called the Uzawa-Epstein utility representation of habit formation, due to Uzawa (1968), Epstein and Hynes(1983), and Epstein(1987).

On the contrary, if we remove the requirement of time-consistency, we may allow the following form of discounting factor:

$$D_t \propto e^{-\rho(t)}.$$

This is known as “generalized hyperbolic discounting.” It is also known that there are some relatives of this type of discounting.

3. Intragenerational equity concept

This section reviews Atkinson’s measure of inequality-aversion that is another name of η. Atkinson (1970) examined methodologies that measure the degree of uneven income distribution for a cross section of the economy. That is, he explored the following topic: When there are two distribution functions $f(y)$ and $g(y)$ of disposable income y, there must be a measure that tells us which distribution is better than the other.

Consider a cross section of the economy where individual disposable income is represented by *y* and the utility accruing from the consumption of *y* is represented by *u*(*y*). Moreover, represents a distribution of *y*. The weighted average of utilities is described as follows:

$$f(y)$$

.

This is equivalent to the following:

.

$$W \equiv \int_0^{\bar{y}} u(y) f(y) dy$$

Borrowing the idea of the expected utility theory, we know that the degree of preference for stochastic dominance over income distributions is substituted by the magnitude of expected utility, . The concept of "equally distributed equivalent", , which is comparable to the concept of certainty equivalent, is helpful for developing the measure as follows:

$$W = \left(\int_0^{\bar{y}} f(y) dy \right) \cdot \int_0^{\bar{y}} u(y) \left(\frac{f(y)}{\int_0^{\bar{y}} f(y) dy} \right) dy \equiv \left(\int_0^{\bar{y}} f(y) dy \right) \cdot E^f \left[u(y) \right].$$

∃ s.t. and where *u* must be increasing and concave in *y*. A measure of uneven income distribution is defined as follows:

$E^f \left[u(y) \right]$. Th

This is known as the "Atkinson measure." According to the expected utility theory, the following measures y_{EDE} represent degrees of risk averse attitude of an economic agent.

relative risk aversion,

absolute risk aversion.

Following the same construction with the expected utility theory, we can interpret these risk aversion coefficients as measures of the degree of preference for inequality-aversion. Thus, we can introduce a utility function of the following form:

$$y_{EDE} \le \mu \qquad u(y_{EDE}) = \int_0^{\bar{y}} u(y) \left(\frac{f(y)}{\int_0^{\bar{y}} f(y) dy} \right) dy \quad \text{and} \quad \mu = \int_0^{\bar{y}} y \cdot \left(\frac{f(y)}{\int_0^{\bar{y}} f(y) dy} \right) dy$$

,

$$I = 1 - \frac{y_{EDE}}{\mu}$$

and consider the parameter η to represent a coefficient of constant relative inequality-aversion. Notice that the above form of utility function has a quite nice property in that it can be easily summed up to the same form of aggregated utility function. That is, when we consider the following *U*(*C*):

$$R_R(x) = -\frac{u''(x)}{u'(x)} x$$

$$R_A(x) = -\frac{u''(x)}{u'(x)}$$

$$u(c) \equiv \frac{c^{1-\eta}}{1-\eta}$$

$$U(C) \equiv \max_{\{c\}} \int_0^{\bar{c}} \frac{c^{1-\eta}}{1-\eta} f(c)\,dc \quad s.t.\, C = \int_0^{\bar{c}} c \cdot f(c)\,dc\,,$$

,

then it has the same functional form such that:

.

$$U(C) = \frac{C^{1-\eta}}{1-\eta}$$

This is shown as follows:
Define *L* as:

.

First-order necessary conditions (FONCs) indicate:

or .

$$L \equiv \int_0^{\bar{c}} \frac{c^{1-\eta}}{1-\eta} f(c)\,dc + \lambda \cdot \left[C - \int_0^{\bar{c}} c \cdot f(c)\,dc \right].$$

Thus, we have:

or .

Due to the envelope theorem, we have:

$$\frac{\partial L}{\partial c} = c^{-\eta} - \lambda = 0 \qquad c = \lambda^{-1/\eta}$$

,

leading to:

.

Because of this nice property for aggregation, even if we consider a single representative economic agent for one generation, the parameter η can be understood to represent the degree of preference for inequality-aversion for the generation as a whole.

$$C = \lambda^{-1/\eta} \int_0^{\bar{c}} f(c)\,dc \qquad \lambda \propto C^{-\eta}$$

In short, the discussions in current and previous sections are summarized as follows:

$$\frac{d}{dC} U(C) = \frac{\partial}{\partial C} L = \lambda\,.$$

Consider a typical social welfare function:

$W \equiv$ where and . (1)

This function explicitly includes two concepts of equity. The one is intergenerational, and is represented by social time preference ρ while the other is intragenerational, being represented by Atkinson's coefficient of relative inequality-aversion η. Per the former, the following statements are equivalent:

$$U'(C) \propto C^{-\eta}$$

- The value of ρ is larger (smaller).
- The present generation regards their future generations as being less (more) important than ever.

Per the latter, the following statements are equivalent:

- The value of η is larger (smaller).
- The preference of each generation is more (less) inequality-averse.
- Each generation is less (more) patient with an unequal distribution of wealth.

$$\int_0^{\infty} u(c_t) L_t e^{-\rho t}\,dt \qquad L_t = e^{nt} \quad . \qquad u(c) = \frac{1}{1-\eta} c^{1-\eta}$$

4. Interchangeability between two equity parameters

Discussions in Sections 2 and 3 indicate that η and ρ together comprise a complete set of parameters that defines a social welfare function with exponential discounting. Also, it is indicated that η and ρ have symmetric shares to describe two equity concepts on the welfare. A natural question will be thus whether we are able to replace one's role with another as parameters. This section works on this question.

Consider a fixed set of η and ρ for the definition of social welfare (1). Suppose that we perturb η by adding to the current level. Due to this perturbation, the optimal solution for the consumption path will change. It leads to a slight $\Delta\eta$ deviation from the original path. Suppose further that we perturb ρ as well by adding to the current constant level along time *t*. This second perturbation creates another change in the optimal consumption path. If we can find appropriate magnitudes and shape along time horizon for and so that the changes in consumption path caused by the perturbations can cancel out, then we can regard these two parameters as $\left\{\delta\rho_t\right\}_{t=0}^{\infty}$ being interchangeable. More specifically, we introduce definitions regarding interchangeability as below.

To explicitly include perturbations in the analytical framework, let us introduce the following representation of social welfare function:

(2)

An optimal consumption path is described as follows: $\Delta\eta$ $\left\{\delta\rho_t\right\}_{t=0}^{\infty}$

where represents a feasible set of consumption path and the superscript * indicates optimal solutions. Then, interchangeability is defined as follows:

Definition (Interchangeability between η andρ)

If there exists a pair of and that satisfies

≡, (3)

$$W\left(\left\{c_t\right\}_{t=0}^{\infty},\eta,\rho,\Delta\eta,\left\{\delta\rho_t\right\}_{t=0}^{\infty}\right)\equiv\int_0^{\infty}c_t^{1-(\eta+\Delta\eta)}L_t e^{-\int_0^t(\rho+\delta\rho_\tau)d\tau}dt$$

then it is said that η *is interchangeable with time-varying social time preference ().*

Moreover, if is constant in time, i.e., , then it is said that η *and* ρ *are completely interchangeable with each*

$$\left\{c_t^*\left(\eta,\rho,\Delta\eta,\left\{\delta\rho_t\right\}_{t=0}^{\infty}\right)\right\}_{t=0}^{\infty}\equiv\arg\max_{\left\{c_t\right\}_{t=0}^{\infty}\in\mathcal{F}}W\left(\left\{c_t\right\}_{t=0}^{\infty},\eta,\rho,\Delta\eta,\left\{\delta\rho_t\right\}_{t=0}^{\infty}\right)$$

$\mathcal{F}$ *other.*

Noting that = = , the condition (3) leads to the following:

$\Delta\eta(\neq 0)$ $\left\{\delta\rho_t\right\}_{t=0}^{\infty}\left(\neq\left\{0\right\}_{t=0}^{\infty}\right)$

$$\arg\max_{\left\{c_t\right\}\in\mathcal{F}}W\left(\left\{c_t\right\}_{t=0}^{\infty},\eta,\rho,0,\left\{0\right\}_{t=0}^{\infty}\right)\quad\arg\max_{\left\{c_t\right\}\in\mathcal{F}}W\left(\left\{c_t\right\}_{t=0}^{\infty},\eta,\rho,\Delta\eta,\left\{\delta\rho_t\right\}_{t=0}^{\infty}\right),\qquad(3$$

$\rho_t\equiv\rho+\delta\rho_t$).

$\left\{\delta\rho_t\right\}_{t=0}^{\infty}$ $\delta\rho_t=\delta\rho\ \forall t$

$c_t^{1-\eta}e^{-(\rho-n)t}$ $e^{(1-\eta)\cdot\ln c_t}e^{-(\rho-n)t}=$ $e^{nt}\cdot e^{(1-\eta)\cdot\ln c_t-\rho t}$, the

$$\left(1-\eta\right)\cdot\ln c_t^* - \rho t = \left(1-\eta-\Delta\eta\right)\cdot\ln c_t^* - \rho t - \int_0^t \delta\rho_\tau d\tau ,$$

,
equivalently,

$$\Delta\eta\cdot\ln c_t^* + \int_0^t \delta\rho_\tau d\tau \equiv 0$$

$\forall t.$ (4)

Taking a derivative with respect to time, we finally obtain the following condition:

$\forall t$ (5)

$$\frac{\dot{c}_t^*}{c_t^*}\Delta\eta + \delta\rho_t \equiv 0$$

Observing the equation (5), we can address the following propositions:

Proposition 1

If per-capita consumption keeps rising or declining in equilibrium, η *is interchangeable with time-varying social time preference. If per-capita consumption stays constant in equilibrium, then* η *is never interchangeable with any form of social time preference.*

Proposition 2

η *and* ρ *are completely interchangeable with each other if and only if per-capita consumption rises at a constant rate in equilibrium.*

Notice that the following optimality condition—known as the Euler equation—holds for any Ramsey-type model that maximizes the welfare (1):

$$\frac{\dot{c}_t^*}{c_t^*} = \left(1/\eta\right)\cdot\left(r_t^* - \rho\right).$$ (6)

Using Equation (6), Equation (5) is replaced by the following condition:

$$\left(\frac{r_t^*}{\rho}-1\right)\cdot\frac{\Delta\eta}{\eta}+\frac{\delta\rho_t}{\rho}\equiv 0$$ (7)

It is typical in a growing economy that $r_t^* \geq \rho$ holds. When $r_t^* > \rho$, $\Delta\eta/\eta \begin{Bmatrix} > \\ < \end{Bmatrix} 0$ indicates $\delta\rho_t/\rho \begin{Bmatrix} < \\ > \end{Bmatrix} 0$. Also, in a neoclassical economic growth theory, r_t^* is convergent to ρ. With the condition of $r_t^* = \rho$ thereafter, $\delta\rho_t/\rho = 0$ must hold, indicating that there is no interchangeability between the two thereafter.

On the other hands, if r_t^* happens to be constant, the complete interchangeability holds. Such case happens when a production function for one-sector model setting $f\left(k\right)$ is the AK model in the framework of endogenous growth theory, namely, $f\left(k\right)\equiv Ak$. It is easy to see that with the AK model. with the AK model.

The above considerations are summarized as the following corollaries.

Corollary 1 (from Prop.2)
With a one-sector production function, η *and* ρ *are completely interchangeable with each other if and only if the production function is reduced to be the AK model.*

Propositions and Corollaries presented above provide us implications regarding equity considerations. As we have seen in Sections 2 and 3, changes in values of η and ρ are viewed as changes in social preferences over intra- and intergenerational equities. Proposition 1 basically points out the following:

(i) In the economy in which per-capita consumption keeps growing, allowing the present generation to regard their future generations as being less important than ever is equivalent to allowing each generation to be more patient with an unequal distribution of wealth. In this sense, the increase in the degree of intergenerational inequality is equivalent to the increases in the degrees of intragenerational inequality for all generations. This situation most likely occurs in a framework of neoclassical economic growth theory.

(ii) In the economy in which per-capita consumption keeps declining, allowing the present generation to regard their future generations as being less important than ever is equivalent to allowing each generation to be less patient with an unequal distribution of wealth. In this sense, the increase in the degree of intergenerational inequality is equivalent to the decline in the degree of intragenerational inequality. This situation, however, is not typical in a framework of neoclassical economic growth theory.

(iii) If the economy has already reached a steady state in which per-capita consumption is constant, intra- and intergenerational equity concepts are completely separated.

(iv) In addition to the above (i), if the economy is subject to increasing-returns-to-scale, the increases in the degrees of intragenerational inequality for all generations are uniform.

Another important implication of Equation (5) is related to whether   is time-varying or constant. As was discussed at the end of Section 2, constant   indicates that the resulting discounting factor still holds the exponential feature. The following corollary addresses this point.

Corollary 2 (from Prop.2)
Perturbation of η *can be cancelled out by adjustments of the degree of exponential discounting if and only if per-capita consumption rises at a constant rate in equilibrium.*

$\{\delta\rho_t\}$

$\delta\rho$

5. Conclusions

This paper examined the interchangeability between the coefficient of relative inequality-aversion η and social time preference ρ to explore insights regarding intra- and intergenerational equities as well as implications to parameter setting techniques in climate policy modeling practices. The pair of parameters (η, ρ) is a key determinant of characterizing objective functions in Ramsey-type models. When we adopt specific values for these two parameters, we must be sure that they are closely related to each other in light of ethical considerations as well as consistency to actual economic data. However, past studies in the economics literature focus on these two parameters separately, and have never treated them together. This perception motivated us to launch our study.

As was shown in Proposition 2, the necessary and sufficient conditions for these two parameters to possess the characteristics of complete interchangeability are that per-capita consumption rises at a constant rate in equilibrium. According to Corollary 1, the condition is equivalent to the one that the production function is the AK model given that the economy is assumed to consist of one-sector production. With such condition, the pair of values for (η, ρ) are not unique: for example, for a given pair of values, we will be able to obtain a new pair such as (η + Δη, 0) with which equilibrium path of consumption does not change. However, this operation of changing parameters is directly related to changes of social preferences over intra- and intergenerational equities. A more limited version of interchangeability is addressed as in Proposition 1.

Another important implication of these propositions and corollaries is connected to the properties of discounting factors. If the complete interchangeability holds, even if we make an adjustments of (η, ρ), we can keep exponential discounting. Otherwise, adjustments require an introduction of general hyperbolic discounting or endogenous discounting so as to have the same equilibrium. Findings obtained here illustrated a new perspective of policy modeling.

References

Atkinson, A.B. (1970). “On the measurement of inequality.” *Journal of Economic Theory* 2, 244-263.

Buchholz, W., J. Schumacher (2010). “Discounting and welfare analysis over time: Choosing the η.” *European Journal of Political Economy* 26, 372-385.

Cline, W.R. (1992). *The economics of global warming*. Washington, DC: Institute for International Economics.

Dasgupta, P. (2008). “Discounting climate change.” *Journal of Risk and Uncertainty* 37, 141-169.

Epstein, L. G. and J. A. Hynes(1983). “The Rate of Time Preference and Dynamic Economic Analysis,” *The Journal of Political Economy*, 91(4): 611-635.

Epstein, L. G. (1987). “A Simple Dynamic General Equilibrium Model,” *Journal of Economic Theory*, 41: 68-95.

Nordhaus, W.D. (1994). *Managing the global commons: The economics of climate change*. Cambridge, MA: MIT.

Stern, N.H., et al. (2006). *The Stern Review of the economics of climate change*. Cambridge: Cambridge University Press.
Uzawa, H. (1968). "Time Preference, the Consumption Function, and Optimum Asset Holdings," in: Wolfe, J.N. ed., *Capital and Growth: Papers in Honor of Sir John Hicks* (Aldine, Chicago, IL).