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**A MULTIOBJECTIVE E3 MODEL WITH UNCERTAIN DATA**

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**Overview**

Multiple Objective Programming models allow the analysts to explicitly consider distinct axes of evaluation of different policies, consistent with energy sustainable strategies, economic growth, social well-being and environmental concerns. This analytical tool allows assessing the environmental impacts (e.g., global warming potential and/or acidification potential), resulting from changes in the level of the economic activities sustained by distinct policies. In particular, multiple objective linear programming (MOLP) models based on the linear inter/intra industrial linkages of production have been used to study the interactions between the economy, the energy system and the environment. However, the uncertainty associated with the coefficients and parameters of these models, generally derived from Input-Output (I-O) analysis, may lead to conclusions that are not robust, in the sense they are not “immune” to data changes. The technical coefficients of the I-O matrix are not exactly known and, in general, they are estimated, being subject to a considerable level of uncertainty (Rocco and Guarata, 2002). In this context, we propose a decision support tool based on MOLP and I-O analysis, which allows the analysts/ decision makers to assess impacts on the economy, the environment and the energy system, based on the levels of activity of economic sectors, taking into account the intrinsic uncertainty in the model coefficients with the aim of identifying robust strategies.

**Methods**

I-O analysis is an analytical tool which allows evaluating the inter-relations between different economic activities, being often applied to assess energy-economic-environment impacts. In I-O analysis, it is assumed that each sector produces the output with inputs combined in fixed proportions. All the industries possess constant returns to scale and supply is infinitely elastic. The inter/intra-sector flows can be either originated from domestic production or from imports. In order to obtain the level of domestic output, competitive imports (imports with no endogenous substitute) should be deducted to final demand (private consumption, public consumption, investment and exports).

I-O analysis and linear programming (LP) are closely related. In its simplest form, with no substitute inputs, I-O analysis can be seen as a simple particular case of LP. The use of the I-O methodology in the framework of LP models allows obtaining value added information, which would not be possible to achieve with the separate use of both techniques. Inter/intra-sector relations established in I-O analysis allow designing the production possibility frontier. LP models allow choosing the optimum level of activities, which cope with a certain objective, respecting the productive relations imposed by I-O analysis.

Traditional studies, which use I-O analysis in the framework of LP, generally consider a single objective function to be maximized or minimized, usually an aggregated economic indicator. However, strategic decisions must be made in a complex and turbulent environment, characterized by a fast pace of the evolution of technology, market structures and societal needs. Real-world problems inherently incorporate multiple, conflicting and incommensurable axes of evaluation of distinct policies. In this context, mathematical

models for decision support become more representative of reality if distinct aspects of evaluation are brought into consideration. Hence, environmental, economical and social concerns should be considered explicitly, enabling to assess the trade-offs between them. Since, in a multiple objective context, there is not a prominent solution whenever the objectives functions are in conflict (as the optimal solution in single objective problems), the aim of multiple objective models is to support analysts/decision makers in identifying good compromise plans. They are understood as well-balanced solutions in face of all axes of evaluation (objective functions), that attain satisfactory goals according to the decision maker's preferences. In MOLP models the concept of optimal solution gives place to the concept of efficient (non dominated) solution. A feasible solution for a MOLP is efficient if and only if there is no other feasible solution that improves the value of an objective function, without worsening at least the value of another objective function.

MOLP models based on I-O methods have been used for economic and energy planning. Recently these models have also embodied environmental concerns (e.g., Oliveira and Antunes (2002, 2004)).

In most real-world situations, the necessary information to specify the exact model coefficients is not available, because data is scarce, difficult to obtain, uncertain, etc. In this sense, it is convenient to extend traditional mathematical programming models for decision support without assuming the exactness of the model coefficients. Lenzen (2001) refers some uncertainty sources specifically related to I-O models: linearity and proportionality of I-O model coefficients, statistical sources and aggregation of data, among others. The uncertainty modeling in I-O models is mainly based on two different approaches (Rocco and Guarata, 2002): the probabilistic approach, where the probabilistic distributions are presumably known (see West (1986)), and the interval programming approach, also known as "unknown but bounded approach", where the extreme bounds of uncertain coefficients are considered without being associated to a possibilistic (as in fuzzy sets) or a probabilistic structure. In practice, it would be necessary to consider an outstanding amount of information to estimate the probabilities associated with all the elements of the national I-O matrix. In this way, the interval approach is more indicated for uncertainty handling in I-O models (Jerrel, 1996). Therefore, an interactive procedure devoted to tackle MOLP problems with interval coefficients has been developed. This procedure is based on a surrogate problem aimed at computing solutions which minimize the worst possible deviation of each interval objective function from a specified interval goal, also considering satisfaction thresholds on the uncertain constraints (Oliveira and Antunes, 2006).

## **Results**

The method developed is aimed at providing the DM with information about robust solutions, that is, solutions which perform well, according to the multiple objective functions, in distinct scenarios underlying the interval coefficients. In this setting, robust solutions minimize the worst possible deviation of each objective function from an interval ideal solution. This interval reference point corresponds to the best individual optimum value obtained either with the most favorable set of coefficients or with the less favorable set of coefficients. The "worst case" and the "best case" scenarios are taken into account in order to perceive the risks and opportunities at stake; that is light is shed on trade-offs between objective function values and also robustness degrees in different scenarios. Therefore, the methodology provides not just detailed information associated with the efficient solutions (outputs of the activity sectors, gross added value, employment, imports and exports, private consumption, GDP, public deficit, public debt, global warming potential, acidification potential, etc.) but also a characterization regarding their robustness to the uncertain data.

## Conclusions

Traditionally the I-O modeling approach has omitted any sources of uncertainty. The inter/intra industrial linkages have generally been viewed as static and deterministic. This paper proposes a MOLP model based on I-O analysis, which allows the analysts/DMs to assess impacts on the economy at a macroeconomic level, the environment and the energy system, based on the levels of activity of industrial sectors, in a situation of uncertain data modeled through interval coefficients. The aim is to provide information regarding robust solutions, that is, efficient solutions which attain desired levels for the objective functions across a set of plausible scenarios.

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