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PUTTING DECOMPOSITION OF ENERGY USE AND POLLUTION ON A FIRM FOOTING - CLARIFICATIONS ON ZERO AND NEGATIVE VALUES AND THE RESIDUAL

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Overview

The literature on energy use and pollution emissions decomposition recently identified the logarithmic mean Divisia index (LMDI) approach as one of the most favorable (Ang 2004; Ang and Liu 2001, 2007a and 2007b). This is based mainly on four features of the method, namely its ability to handle zero and negative values, the absence of any residual term and the ease of calculation. In addition, it is invariant under time and factor reversal and fulfills aggregate consistency and proportionality.

I show how the problems with zero and negative values in decomposition can be understood by reference to some ill-defined mathematical operations and integral approximation. Referring to integral approximation, which is the basis of any decomposition analysis, I also discuss the residual in decomposition and show that the presence of a non-zero residual is natural and that requiring a zero residual as a strategy to identify optimal decomposition methods is without basis. I illustrate these findings with the LMDI, simulations allowing for comparison of the LMDI results to the values based on exact solution of the integrals involved, and application of the LMDI and other methods to several data sets (CO2 and SO2 in Sweden, CO2 in Korea).

Methods

I develop an analytically consistent formalism of decomposition techniques that is based on integral approximation and that avoids ill-defined mathematical operations such as division by zero.

This shows, that several of the motivations mentioned above to identify optimal decomposition methods actually have no basis as guidelines for the quality of a decomposition approach. The zero and negative value problems of decomposition analysis stem from ill-defined operations during the calculation of the indices (Muller 2006). These operations are the expansion of quotients, i.e. the multiplication with x/x, where x stands for some variable of interest, and subsequent application of derivation with respect to time or taking logarithms. In case of $x < 0$, this can lead to problems of division by zero or taking logarithms of zero or negative values. These problems can be avoided by avoiding ill-defined mathematical operations during the calculation of the various parts of the indices.

The residual, on the other hand, reflects the fact that any such decomposition is based on integral approximation (Trivedi 1981). The residual cannot be argued to necessarily be zero for an optimal decomposition approach. The reason being that a zero residual stemming from a truly correct approximation of the integrals involved is highly unlikely. A zero residual reflects no error in approximation, which means that largely unknown functions with information available only on their values for some discrete points (such as once a year) have been guessed correctly for any point of time. A zero residual can thus reasonably only stem from some assignment of the residual from the approximation to the various parts of the decomposition. Such an assignment is arbitrary unless further information on the underlying functions is available. Forcing the residual to be zero could in principle also involve mutually canceling terms of opposite sign in different parts of the decomposition. This could make the zero-residual decomposition even less exact than a decomposition with some non-zero residual term based on a good approximation.

I illustrate these claims on a formal level first, referring to the LMDI as an example. I then do some simple simulation, where the effect of the different underlying integral and derivative approximation readily can be assessed and compared to the values from exact calculations of the integral. Some application to real data provides further illustration.

Results

Most decomposition methods currently used are based on the choice of weights for the boundary values of the intervals for which integrals have to be approximated. Laspeyres- and Paasche-Index based methods, for example, approximate by employing step-functions given by the right- or left-hand values. The Divisia-Index with $\alpha = ½$ gives equal weights to each boundary value and thus approximates the unknown function by joining the known boundary points by a straight line. Only if more information on the underlying functions is available, some judgment if they are better captured by a step function or the straight line or some other type of approximation might be possible, thus suggesting a most adequate decomposition method or choice of weights, respectively.

On this background, the LMDI has no theoretical support and the Divisia Index with $\alpha = ½$ may be a transparent, simple and easy to calculate alternative. With this method, the residual term, although usually not zero, has not been excessively large for the cases I have calculated. For these examples, the several terms of the decomposition using the Divisia index with $\alpha = ½$ were almost identical to the results from the LMDI. In the simulations, however, the LMDI performed better for larger time intervals and the results suggest that the potential good performance of the Divisia-Index might depend on the availability of relatively short time-intervals (i.e. short enough that the functions do not change much over the range of the interval).

Conclusions

Reference to integral approximation, which is the basis of any decomposition method, and to well-defined mathematical operations only (such as avoiding division by zero), helps to clarify the residual and problems with zero and negative values in decomposition analysis. This could help to identify improved, transparent and reliable decomposition methods. The Divisia method with $\alpha = ½$ performs quite well and is transparent. The LMDI also performs well and sometimes even better (e.g. in some simulations, where the exact decomposition solution is known), but its adequacy still has to be understood, as it is not based on integral approximation. The optimal method should be chosen in relation to the data available. Increased information on this data and on how variables might develop between the discrete points available (e.g. deriving some limits on how fast it can change) could improve this choice.

References

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