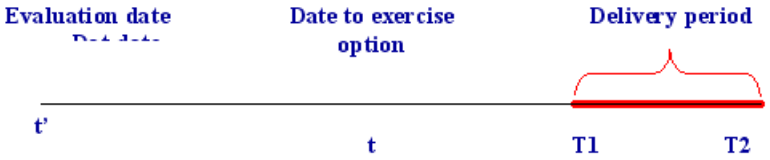
Alain Galli, Margaret Armstrong, Ridhima Tewary and Zaizhi Wang

EVALUATING ASIAN OPTIONS ON ELECTRICITY FUTURES USING A TWO-FACTOR MEAN REVERTING MODEL & A MODIFIED BOX-COX TRANSFORMATION

Cerna, Ecole des Mines de Paris, 60 Bvd Saint-Michel, 75272 Paris, France
alain.galli@ensmp.fr

Overview

In the electricity market the standard delivery periods for futures are one month, a quarter or one year. This paper focuses on how to evaluate European puts and calls on electricity futures (as shown below)



We use a two factor mean reverting model for the spot price St = exp(Xt)

$$dX_t = \alpha_3(L_t - X_t)dt + \sigma_3 dW_t^X$$
$$dL_t = \alpha_4(\lambda_4 - L_t)dt + \sigma_4 dW_t^L \qquad [1]$$

Here the second term Lt corresponds to a slowly evolving long-term mean. Day-ahead prices are considered as being a future with a maturity of 1 day, rather than as a pure spot price.

The difficulty when evaluating options with delivery periods longer than 1 day is that the pay-off involves a sum of futures which are each lognormally distributed and that the distribution of this sum is not known explicitly. Most of the methods for Asian options were developed for the geometric Brownian motion model & are difficult to apply in our case. Monte Carlo simulations can be used but as there are two Brownians, this becomes time-consuming, especially if the Greeks are also required.

Our aim was therefore to develop a simple approximation to take account of the gradual change in the shape of the distribution from lognormality toward normality as the delivery period increases or as the time lag before the exercise date of the option increases. We propose a variant on the Box-Cox transformation to account of this effect. It is then easy to evaluate European options (and also the Greeks) analytically.

Methods & Results

Using standard techniques in stochastic processes we express any future F(t', t, T1, T2) as a conditional expectation under the appropriate filtration. We give a closed form expression for options with a one-day delivery period. For longer delivery periods it is an Asian option and the density of the payoff is not known explicitly.

We illustrate how the distribution of the futures gradually changes from lognormality toward normality, depending on the start of the delivery time and on its length. Another

feature specific to options on futures is that the term structure at the time of pricing has a strong influence on the option value. The option price is quite different depending on whether the market is in backwardation or contango.
A modified version of the Box-Cox transformation is proposed which allows us to fix the mean and variance while still allowing the distribution to vary from lognormality to normality as a function of one parameter. We discuss ways of finding the optimal value of this parameter in order to reproduce the distribution of the payoff accurately.

Conclusions
We have developed a simple but efficient approximation for evaluating Asian options on futures. This method has been tested for the two-factor mean reverting model given in [1] by comparing the results with those obtained MC simulations. The work to date suggests that it would be suitable for more general models.
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