Overview

The ability to induce shifts in electricity load within a day may reduce costs for a number of reasons. Lowering daily peaks or managing foreseeable spikes in demand are useful for system security. Similarly, taking advantage of favourable weather conditions with respect to generation of renewable energy could also be achieved through successful load-shifting. One of the ways in which this outcome can be targeted is through the use of financial incentives. In designing a suitable monetary incentive or deterrent for consumption, knowledge of the expected demand response to an effective change in electricity price is key. In the context of intra-day shifting, understanding how the profile of price elasticity varies within a day is of particular relevance.

Estimation of intra-day patterns in price elasticity of electricity demand has received limited attention in the literature (Fan and Hyndman, 2011; Knaut and Paulus, 2016; Kulakov and Ziel, 2019; Chong, 2021) and has recently become topical in light of the gas shortage and soaring electricity prices in Europe. This study proposes a new econometric approach which introduces smoothness in the intra-day pattern of elasticity, and preserves this smoothness of elasticity at all times of the day. Smoothness in elasticities is argued in the paper to be characteristic of data which captures aggregated consumption behaviour (e.g. whole population, the residential sector, all office buildings in a city) as opposed to individual or household-level data. Since time of day is measured in a cyclical manner, it follows that if elasticity is smooth, it must be so in a cyclical manner. In other words, even though each day arbitrarily starts at the first measurable moment after midnight, when elasticity between midnight and this time is continuous, it can be considered to be "smooth everywhere".

Methods

The method proposed in this paper follows an econometric modelling approach not dissimilar to those in existing works. A linear model of the form

\[ q_t = c p_t + x_t' \beta + \epsilon_t, \]

in which \( q_t \) represents electricity demand at time \( t \), \( p_t \) represents price, and \( x_t \) is a vector of other variables with explanatory power on demand including an intercept. The coefficients \( c \) and \( \beta \) are to be estimated. The way in which demand and price enter the equation as \( q_t \) and \( p_t \), respectively, varies between being the natural logarithms or raw levels in the literature. For example, Tiwari and Menegaki (2019) use logarithms for both variables, Fan and Hyndman (2011) uses a log-linear model in which demand is in logarithms and price is in levels and also a log-log model, and Lijesen (2007) uses both a linear model as well as a log-log model. Price elasticity estimates are then obtained from some transformation of the estimate of \( c \) in (1), depending on whether \( q_t \) and \( p_t \) are in logs or levels.

Unlike Fan and Hyndman (2011) and Knaut and Paulus (2016), the proposed method for estimating a daily pattern in price elasticities fits a single regression equation to the data instead of performing a regression for each intra-day period. This approach makes it easy to test whether a daily pattern exists. Variations in price elasticity can be introduced by including an interaction variable between price and a dummy variable for each intra-day period as in

\[ q_{d,h} = c_0 p_{d,h} + \sum_{j=1}^{H-1} c_h \mathbb{1}(h = j) p_{d,h} + \epsilon_{d,h}, \]

where \( d \) is the day in the sample, \( H \) is the total number of periods per day, and \( h \) is the intra-day period. The function \( \mathbb{1}(\cdot) \) takes the value 1 if its argument is true and 0 otherwise. The equivalence between subscripts in (1) and (2) is given by \( t = (d - 1) \times H + h \). The estimated price elasticity at the \( h \)-th intra-day period is then computed using \( c_0 \) and \( c_h \), with the elasticity at the \( H \)-th period computed using only \( c_0 \). Testing the joint significance of \( c_1, \ldots, c_{H-1} \) is effectively checking for evidence of variation in price elasticity throughout the day. Note, however, that (2) does not impose smoothness on elasticities even if \( H \to \infty \).
An alternative way of introducing easily-testable variation in price elasticities throughout the day is through the model

\[ q_t = c p_t + \sum_{s=1}^{S} \left[ \gamma_s \sin \left( \frac{2\pi(t - H)}{H} \right) + \delta_s \cos \left( \frac{2\pi(t - H)}{H} \right) \right] p_t + \beta' x_t + e_t, \quad (3) \]

where \( S \) can be selected using the data. In this application, the value of \( S \) is selected by recursively increasing the value from \( S = 0 \) until some \( S = S^* \) such that \( \gamma_{S^*} + 1 \) and \( \delta_{S^*} + 1 \) are both not statistically significant. If \( S > 0 \), then price elasticity throughout the day is not constant. This Fourier approximation estimates a repeated pattern of elasticity within each day and ensures that elasticity is smooth throughout the sample.

All three models are estimated by two-stage least squares using \( p_{t-1} \) as an instrument for \( p_t \).

**Results**

Using hourly data from the German wholesale market for the year 2016 which is obtained from the European Power Exchange (EPEX), price elasticity is estimated by (i) fitting model (1) to the whole sample (constant elasticity), (ii) fitting model (2), and (iii) fitting model (3). Since prices in this sample fall below 0, the logarithm of price is not defined for the whole sample, and the log-log specification is not feasible. All models are estimated with linear (demand and price in levels) and log-linear (logarithm of demand and price in levels) specifications.

The joint tests from (2) and (3) both indicate that there is variation in price elasticity of electricity demand within the day. Estimates of price elasticities using both models exhibit similar patterns, with a range of values which includes the constant estimate from (1). The pattern of estimates from the dummy variable model described in (2) is noticeably less smooth that those of (3), especially between 1500–2100hrs. Electricity demand is found to be most elastic between 2100hrs and midnight, and least elastic around 0700–0800hrs and between 1800–2000hrs. Intra-day elasticities from (2) and (3) fall within similar ranges of price elasticity whether a linear or log-linear specification is used.

**Conclusions**

This paper proposes a new and simple way to estimate smooth variations across the day in price elasticity of electricity demand. The proposed method also provides an easy way to test whether price elasticities are constant throughout the day or whether there is evidence of variation.

Applying the approach to German wholesale electricity data provides estimates for price elasticities which vary throughout the day and this variation is found to be statistically significant.