# Prospects for a second "peak" in U.S. oil supplies

Roberto Ferreira da Cunha[1]

## EXTENDED ABSTRACT

This paper investigates prospective long-term scenarios for supplies of unconventional oil and gas in the United States. We do so through the "Long-term Oil and Gas Images" (LOGIMA) model; a novel, geological, Hotelling-type, extraction-exploration model of non-renewable resources. The proposed scenarios describe U.S. unconventional oil and gas producers' profit-maximizing responses to alternative expectations of future global average oil prices. Our results show that U.S oil and gas supplies have room to increase over the next decade and could possibly reach a second "peak" before XXXX, triggered by the depletion of unconventional plays. The anticipated average of global oil prices, will affect the rate of drillings, which ultimately will set the precise timing of the second peak.

**JEL classification numbers:** Q3 and Q4.

**Keywords**: Hotelling rule, Geological constraints, Unconventional oil and natural gas.

[1] Berkeley Research Group. Corresponding author: rcunha@thinkbrg.com. Rua da Quitanda 86 – 2° andar, Centro. 20081-920, Rio de Janeiro, RJ, Brazil.

# 1 . INTRODUCTION

The aggregate evolution of unconventional oil and gas supplies in the United States stems from the independent drilling decisions of numerous small, price-taking producers. Each well drilled constitutes an investment decision, based on the producer's anticipated present value of net revenues generated by recoverable resources. Two sets of variables broadly govern their valuation exercise: "above-ground" economic variables provide market price, taxes, capital and operating cost conditions, whereas "below-ground" technical and geological variables determine the recoverable volumes from wells, such as initial well productivity and pressure decline rates.

Whereas economic variables are typically beyond control of small producers, technical and geological variables are more manageable. Beyond their individual estimations, the productivity of wells is subject to "learning-by-doing" and "spillover" effects. Producers gradually acquire information about the configuration of underground formations, both by drilling their own wells and by observing the behavior of competitors. This accumulation of technical and geological expertise enhances their ability to select the next drilling location, moving towards new underground areas with higher recoverability of hydrocarbons.

Nonetheless, the natural underground distribution of resource density imposes a limit on how much well productivity can increase. Surveys from the U.S. Geological Service show that the spatial distribution of resource concentration, throughout formations, is approximately log-normal (USGS, 2012). Therefore, "sweet spot" portions within formations can contain up to 100 times more resources than the lowest density areas. As a result, once the richest areas have been located and exploited, new wells drilled will necessarily display a lower productivity, which will eventually lead aggregate supplies to decline.

Therefore, the productivity of drilling efforts tends to evolve under an approximate bell-shaped pattern. Initial wells are drilled, approximately, in ascending order of productivity, as producers progressively infer the configuration of underground formations. When the sweet spot has been located, productivity reaches its highest possible point. Productivity then begins to decline, as the previously acquired information allows the drilling of remaining lower density areas in decreasing order of resource recoverability. The number of drillings required to achieve the localization of sweet spot areas is a function of the prevailing state of technological progress.

This paper proposes a new model to capture the bell-shaped evolution of effort productivity. The "Long-term Oil and Gas Images" (LOGIMA) model is a geological, Hotelling-type model that describes the optimal quantities to be supplied under divergent assumptions about prices. It is based on Okullo, Reynes and Hofkes' (2015) ("ORH") model, which accounts for convex drilling costs and pressure decline inside wells. However, the ORH model features a linear relationship between exploratory effort and reserve additions, which leads to a strictly declining equilibrium for reserve additions, when initial reserves are low.

In the case of unconventional supplies, reserve additions have increased over the past ten years, whereas initial reserves were low. Increasing reserve additions under low initial reserves can be explained by the inability of producers to locate the most productive drilling sites when cumulative drillings is low. Labeled by Uhler (1976) as the "information effect", under limited geological information and heterogeneous sites, producers are unable to locate the highest quality areas without the drilling of wells. That is, producers must acquire geological knowledge through the drilling of wells, making drilling productivity a function of cumulative drillings.

LOGIMA accounts for this property, by extending the ORH model with the introduction of the exploration-discovery framework of Pindick (1978), featuring instead a novel *bell-shaped* formulation for the effort productivity function $f(S)$. The model's equilibrium features bell-shaped trajectories both for quantities supplied and reserve additions. Furthermore, the bell-shaped trajectory of supplies takes place with a lag relative to the trajectory of additions. Finally the level in which the peak of bell-shaped additions takes place is higher than the level of the peak in supplies – matching known empirical stylized facts for oil and natural gas supplies.

We calibrated LOGIMA using data from the U.S. Energy Information Administration (EIA) for seven unconventional oil and gas plays in the U.S. Following that, we simulated three potential scenarios for unconventional oil and gas supplies in the United States, between 2020 and 2040, and under three different levels of anticipated international oil benchmark prices ($40, $70 and $100 per barrel). Our calibration showed that oil reserve additions displayed a clear *increasing* trend on all the plays, over the period between 2007 and 2014. Furthermore, all the scenarios showed unconventional oil and gas supplies would not have room to increase beyond 2040.

The remainder of this paper is structured as follows: Section 2 presents a literature review of Hotelling-type models for oil and natural gas supplies, and provides the context for the proposed formulation of LOGIMA. Section 3 details the formulation of LOGIMA model, and derives its optimality conditions. Section 4 describes the equilibrium of the model. Section 5 describes the calibration effort for unconventional oil and gas supplies in the U.S. Section 6 provides the resultant, out-of-sample scenarios for unconventional oil and gas supplies from the United States, to the year 2040. Section 7 draws all these points together into a conclusion.

## 2. Literature review

The state-of-the-art literature on non-renewable resource economics currently depicts bell-shaped trajectories for quantities, alongside strictly *declining* trajectories for reserve additions. Holland's (2008) work was the first, in recent times, to discuss the occurrence of bell-shaped supply trajectories in equilibrium, and offered four extensions to Hotelling's basic model that generated production peaks in market equilibrium: (1) demand increases (2) technological progress (Slade, 1982), (3) reserve degradation (Pindyck, 1978) and (4) the need to produce in different geographical sites.

More recently, Okullo, Reynes and Hofkes (2015) proposed the addition of a geology-based constraint to the Hotelling exploration-exploitation framework, which reflected the producers' inability to violate declining reservoir pressures, at any level of costs. This constraint limited supplies in on-line fields $q$ to a fixed share $\delta$ of the reservoir stock $R$ ($q \leq \delta \cdot R$). Consequently, increasing quantities supplied beyond $\delta \cdot R$ can only occur through new reserve additions $x$. If the initial reserve stock is low (i.e. $R(0) = 0$) and costs are strictly convex ($C_{qq} > 0$, $K_{xx} > 0$), optimal supplies become bell-shaped trajectories.

Anderson, Kellogg and Salant (2018) confirmed the need for a geological constraint (as raised by Okullo et al., 2015) by observing that oil supplies in Texas, between 1990 and 2007, did not deviate from their extant declining trend in response to price shocks. If, as believed by Holland (2008), the "bell-shape" had stemmed from cost functions, this would have been the ideal moment for producers to adjust quantities in response to changing prices. For this reason, the authors concluded that declining pressures had to be a "constraint" on the profit-maximization problem faced by producers.

Further separating the roles of geology and technology from economic variables through the effort productivity function matters, particularly in the case of unconventional oil and gas supplies. When the variation in resource concentration, between areas, is sufficiently high, the highest density "sweet spot" area will contain significantly more resources than the average. The need to discover its location by drilling wells will therefore impose declining unit costs until the sweet spot is reached, without any relationship to "above-ground" economic variables. For technical and geological reasons, the evolution of effort productivity becomes *bell-shaped*.

# 3. Long-term Oil and Gas Images (LOGIMA) Model

LOGIMA can be seen as a two-step extension of the ORH model (Okullo et al., 2015). First, we introduce the exploration-discovery framework of Pindick (1978), requiring producers to decide on drilling schedule $w(t)$ and to contend with reserve additions given by $x(t) = w(t) \cdot f(S(t))$. This formulation implies $dx/dw > 0$; however, the productivity of discoveries is adjusted by a function $f$ of the remaining level of resources $S(t)$. Our second step represents the interplay of information and depletion effects from Uhler (1976) in $f$, making it a bell-shaped function of $S(t)$ ($f_S < 0$ if $S(t)/\bar{S} \geq \bar{\bar{s}}s$, $f_S > 0$ otherwise). The objective function is as follows:

$$\max_{q,w} \pi = \int_0^\infty e^{-r \cdot t} \cdot \left( \bar{p} \cdot q - C(q) - K(w) \right) \cdot dt \tag{1}$$

Where $\bar{p}$ is the expected average market price in the future, $C(q(t))$, of the extraction schedule $q(t)$, is the variable cost function ($C_q > 0$, $C_{qq} > 0$); and $K(w(t))$, of the drilling schedule $w(t)$, is the capital cost function ($K_w > 0$, $K_{ww} > 0$). The stock $S(t)$ initially contains the natural endowment $\bar{S}$, and declines with reserve additions given $w \cdot f$. The stock $R(t)$ refers to the prevailing reserve inventory. Initially, the reserve stock is empty ($R(0) = 0$); it increases with reserve additions $w \cdot f$ and decreases with extractions $q$. Finally, producers remain subject to the geological constraint of Okullo et al. (2015).

$$\dot{S} = -w \cdot f(S), \qquad S_0 = \bar{S} \tag{2}$$

$$\dot{R} = w \cdot f(S) - q, \qquad R_0 = 0 \tag{3}$$

$$q \leq \delta \cdot R \tag{4}$$

Producers maximize (1) subject to constraints (2) to (4) and non-negativity constraints on all variables. The problem's current-value Lagrangean is given by:

$$L(\bullet) = \big(\bar{p} \cdot q - C(q) - K(w)\big) - \lambda \cdot w \cdot f + \mu \cdot (w \cdot f - q) + \gamma \cdot (\delta \cdot R - q) \tag{5}$$

The Maximum Principle of Pontryagin et al. (1962) yields:

$$\bar{p} = C_q + (\mu + \gamma) \tag{6}$$

$$K_w / f = (\mu - \lambda) \tag{7}$$

$$-(\mu - \lambda) \cdot w \cdot f_S = \dot{\lambda} - r \cdot \lambda \tag{8}$$

$$-\delta \cdot \gamma = \dot{\mu} - r \cdot \mu \tag{9}$$

$$\gamma \cdot (\delta \cdot R - q) = 0 \tag{10}$$

The terminal constraints of the problem are:

$$\lim_{t \to \infty} \beta \cdot \lambda \cdot S = 0 \tag{11}$$

$$\lim_{t \to \infty} \beta \cdot \mu \cdot R = 0 \tag{12}$$

Equation (6) indicates that, at all times, the sum of marginal cost $C_q$ and composite scarcity rent $(\mu + \gamma)$ must be equal to the anticipated price level $\bar{p}$. Equation (7) yields the optimal levels of drilling $w$. At all times, producers must select a level of drillings, such that the marginal cost of drillings $K_w$ per unit of marginal increase in additions attained $f$, be equal to the marginal benefit of reserve additions $(\mu - \lambda)$. Equation (8) states that the marginal benefit of an additional unit of resource on the stock must equal the net present value of the change in the productivity of discoveries stemming from that unit.

Equation (9) gives the dynamic efficiency conditions for $\mu$, indicating that $\mu$ grows (declines) when $\gamma$ is small (large). If (4) is binding, production takes place below the level that would be optimal in the absence of this geological constraint. Considering the complementary slackness condition (10), we see that a marginal increase in $R(t)$, when (4) is binding, must lower $\gamma$, and so on, until the point where a marginal increase would lead the constraint to be no longer binding (i.e. $\gamma = 0$). Equations (11) and (12) state that when resources (reserves) are not exhausted, when $t \to \infty$ a marginal increase in the stock will not necessarily increase profits.

## 4. EQUILIBRIUM DYNAMICS

The equilibrium of LOGIMA is bell-shaped for reserve additions $x(t)$ and production $q(t)$. Furthermore, the peak in additions precedes the peak in discoveries, and takes place at a higher level than the peak in production. This section demonstrates these results, by estimating a phase portrait of the changes in drilling $\dot{w}$ and $\dot{q}$, together with a phase portrait of additions $\dot{x}$ and production $\dot{q}$, which result from the adopted schedule of drillings. The first-order conditions imply the following optimal dynamic changes for drilling and production:

$$\dot{w} = \frac{r \cdot K_w/f - \delta \cdot \gamma}{K_{ww}/f} \tag{1}$$

$$\dot{q} = \delta \cdot (w \cdot f(S) - \delta \cdot R) \tag{2}$$

Equation (1) states that drillings will increase or decline depending on $\gamma$, which is the marginal profit generated by a marginal relaxation of the pressure-decline constraint, and the level of drillings $w$. The lower the level of reserves, the lower the production level allowed by the constraint and; therefore, the higher the profit if the constraint is marginally relaxed. Reserve levels are lowest at the beginning $(R(0) = 0)$, and towards the end of production $(t \to \infty)$. If, in between, reserves do not rise enough to lower $\gamma$ to the point of making $\dot{w}$ positive, the drilling schedule is necessarily declining in equilibrium.

Equation (2) shows the optimal change in production. Production increases only when additions $x = w \cdot f$ are high enough to exceed the prevailing ceiling level of production $\delta \cdot R$. Otherwise, additions will not compensate the pressure-led decline in supplies, causing overall production to decline. If $\bar{p}$ is sufficiently high and $C_{qq}$ sufficiently low, then the pressure decline constraint will always bind in equilibrium, and production $q$ will be dictated by the optimal drilling effort $w^*$. Thus, equilibrium becomes a balance between the marginal revenue of drillings and the increasing marginal cost with scale $(K_w > 0$ and $K_{ww} > 0)$.

The phase portrait of equations (1) and (2) is given by respectively setting $\dot{w}$ and $\dot{q}$ to zero:

$$\left(\frac{r+\delta}{\delta}\right) \cdot \frac{K_w}{f} = \bar{p} - C_q - \lambda \qquad (3)$$

$$w = \frac{q}{f} \qquad (4)$$

Equation (3) shows that the isocline of drillings is downward sloping on the $q \times w$ space[2]. Above (below) it, the optimal drilling schedule grows (declines). However, over time, the isocline bends under the influence of $f$ and $\lambda$. The bell-shaped productivity of the drilling schedule affects both the slope and the vertical intercept, causing the isocline to rotate clockwise (counterclockwise) during the ascending (descending) phase of drilling productivity, returning to its original slope when $t \to \infty$. Over time, the increasing scarcity rent $\lambda$ further lowers the vertical intercept, which is also affected by the bell-shaped evolution of well productivity.

Equation (4) shows the isocline of production is positively inclined. Above (below) the isocline production grows (declines). Over time, the isocline also bends, depending on the productivity of drillings $f$. It rotates clockwise during the ascending phase of well productivity, and counter-clockwise in the descending phase towards its original position as $t \to \infty$. Figure 1 shows the resulting four regions of phase portrait, as well as the bending "zones" within which the isoclines bend. As initial reserves are equal to zero, initial production is equal to zero and the system will start in either Region I or Region IV.

When the system begins in Region I, effort declines initially. As both isoclines bend clockwise initially, they move away from the starting point, allowing drillings to decline while production increases. When production crosses the isocline of production, it peaks and begins to decline. When the system begins in Region IV, drillings will increase initially, until the bending isocline of effort places it in Region I. From that point onwards, similar dynamics to the trajectories that started in Region I will ensue. Depending on the parameters, effort may cross its isocline on the descending phase, momentarily increasing, until resuming its declining trend.

---

[2] This can be seen by taking total derivatives with respect to time on (3), dividing by $dq$ and noting from (2) and (9) that $\frac{dS}{dq} = \frac{\partial S}{\partial q} = 0$ and $\frac{d\lambda}{dq} = \frac{\partial \lambda}{\partial q} = 0$:

$$\left(\frac{r+\delta}{\delta}\right) \cdot \frac{K_{ww}}{f^2} \cdot \frac{dw}{dq} = -C_{qq} \le 0$$

The equilibrium of additions can be seen by rewriting the model with $x$ as a decision variable instead of $w$. Consequently, $K(w)$ becomes $K(x/f) = K(x, f)$, with $K_x > 0$, $K_{xx} > 0$, $K_f < 0$ and $K_S > 0$ ($K_S < 0$) in the ascending (descending) phase of $f$. This means capital costs, measured as a function of desired reserve additions, are convex with additions. Nonetheless, it is still affected by the drilling productivity function. Capital costs still decline as well productivity increases. As a result, during the ascending phase of productivity, costs decline as the resource stock declines. Conversely, in the descending phase, costs increase as the stock declines:

$$\dot{x} = \frac{rK_x - \delta\gamma + K_f \cdot f_S}{K_{xx}} \tag{5}$$

Equation (5) shows that the sign of $\dot{x}$ is affected by the joint interaction of $\delta \cdot \gamma$ and $K_f \cdot f_S$. The impact of $\delta \cdot \gamma$ is highest in the initial and late periods, when the reserve base is lowest. The term $K_f \cdot f_S$ is positive (negative) in the ascending (descending) phase, while $f_S$ is also initially increasing as the stock depletes. If $K_f \cdot f_S$ is high enough to make $\dot{x} > 0$ from the start; additions will initially rise at an increasing rate. This means that, relative to the ORH model, the consideration of a bell-shaped $f$ can explain how additions may initially increase even when initial reserves are low. Differentiating the definition $x = w \cdot f(S)$ yields:

$$\frac{\dot{x}}{x} = \frac{\dot{w}}{w} - w \cdot f_S \tag{6}$$

Equation (6) shows that reserve additions increase (decrease) when $\dot{w}$ is higher (lower) than $w^2 \cdot f_S$. In the ascending phase, $f_S$ is negative ($f_S < 0 \Rightarrow w^2 \cdot f_S < 0$), and therefore additions will increase if drillings increase. Additions can still increase, even if drillings decrease, provided the decline level is not higher than $w^2 \cdot f_S$. Conversely, in the descending phase, ($f_S > 0 \Rightarrow w^2 \cdot f_S > 0$) additions decline if drillings decline, and can decline even if drillings increase, provided the increase remains below $w^2 \cdot f_S$. As a result, a declining equilibrium for drillings can generate a bell-shaped equilibrium trajectory for reserve additions.

Finally, the peak in additions takes place prior to the peak in production, and at a higher level. If the initial reserve base is equal to zero, and parameters $\bar{p}$ and $C_{qq}$ are such that it is always optimal to produce at capacity, production can only increase if additions increase initially, and can only decrease when additions fall short of production ($\dot{q} \propto x - q$). Therefore, production can only peak when additions are in decline; going from being higher to being lower than production. As the equilibrium trajectories of additions and production are bell-shaped, the peak in additions must occur prior to, and exceed, the peak in production.
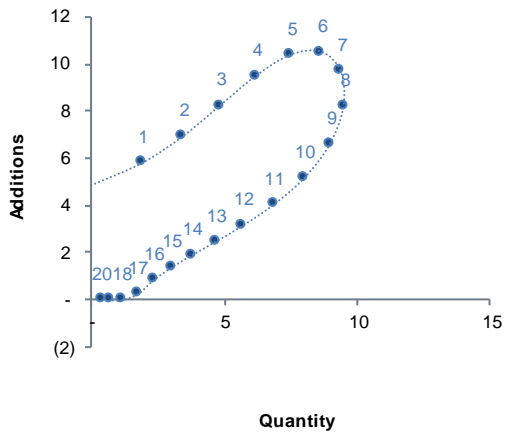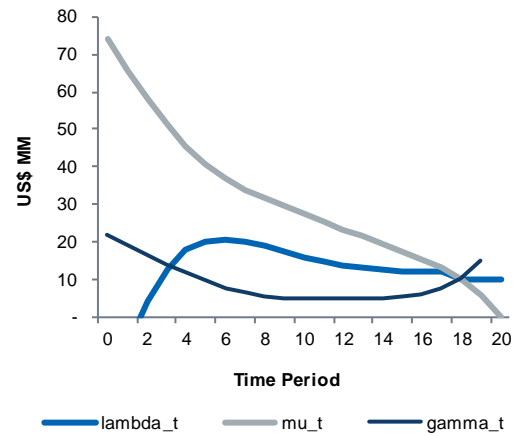
—9—

**A - Quantity and Price schedules**
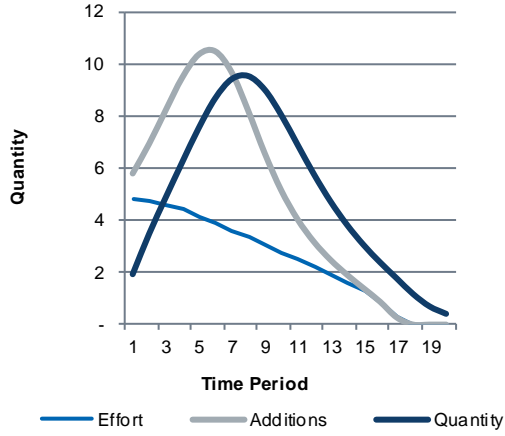


**B - Phase diagram between q and w**
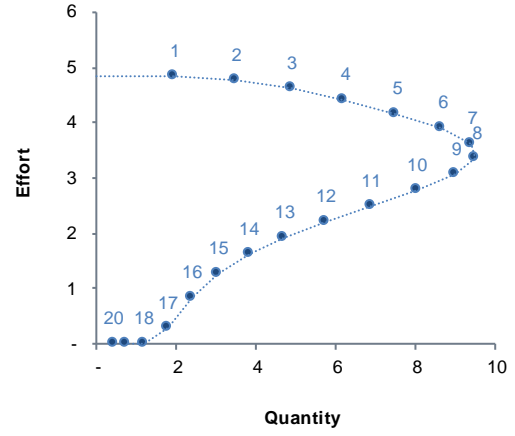


**C - Phase diagram between q and x**



**D - Shadow Prices**



**E - Bending isoclines on x lead discoveries to peak between time periods 3 and 4**
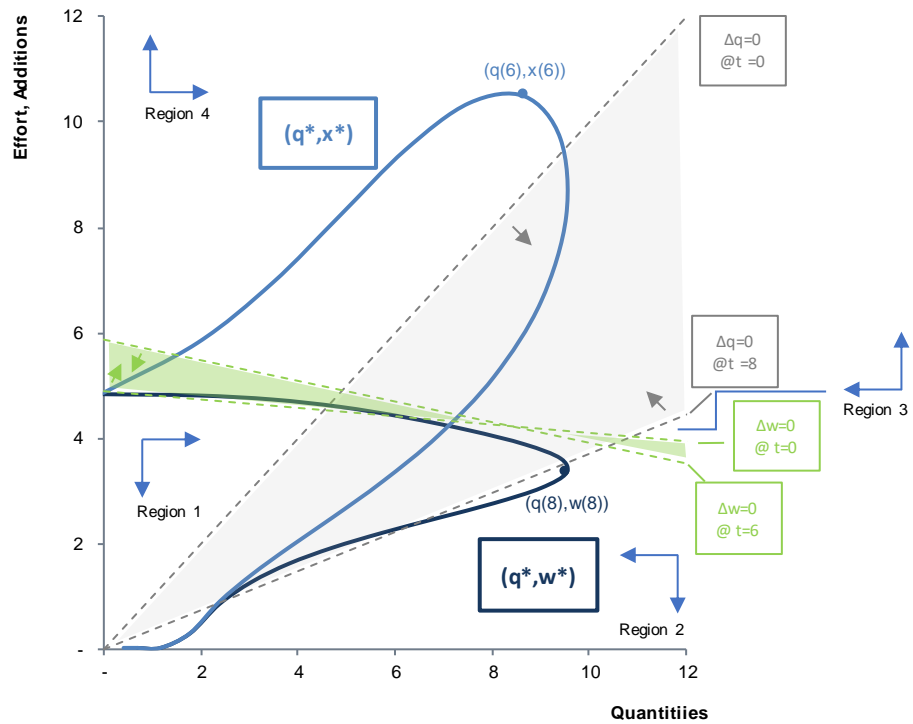


**Figure 1 – Proposed model's equilibrium.**
Source: own elaboration.

# 5 . CALIBRATION TO U.S. UNCONVENTIONAL OIL AND GAS

To come in final paper

# 6. PROSPECTIVE SCENARIOS FOR UNCONVENTIONAL SUPPLIES TO 2040

To come in final paper

# 7 . CONCLUSIONS

This paper investigated potential long-term scenarios for supplies of unconventional oil in the United States. Conventional oil supplies in the U.S. notoriously displayed a "peak" in 1970, at 9.637 mmpbd, and then declined in most years, until they reached 5,000 in 2008. From that point onwards, unconventional supplies reversed this trend, causing supplies to reach 10,990 in 2018, and exceeding the 1970 peak for the first time. This increase, over that period, stemmed primarily from increasing productivity of drilling rigs: the observed average productivity of one drilling rig increased from 0.5 million barrels of oil, per year, in 2008 to 5.7 in 2018.

The works of Okullo, Reynes and Hofkes (2015) (ORH model) showed that pressure decline constraints, specific to nonrenewable resources in liquid and gaseous states, were considered within a complete Hotelling-type exploration-extraction framework. However, the equilibrium result also works for both, under low initial reserves, when it concerns optimal additions, which begin at their maximum and follow a strictly decreasing schedule. This equilibrium trajectory precludes the model from explaining the empirically observed increasing trajectory of reserve additions in U.S. unconventional oil supplies over the last ten years.

This paper proposes the "Long-term Oil and Gas Images" (LOGIMA) model: a geological, Hotelling-type, extraction-exploration model of non-renewable resources. The model introduces a second geological constraint to the ORH model, in order to account for bell-shaped reserve additions and quantities simultaneously. Our proposed extension successfully demonstrated bell-shaped reserve additions; thus, providing a solution to the assessment of plays in early stages of development, when reserve additions initially increase. This was achieved by postulating a bell-shaped evolution of the productivity of exploratory efforts.

Until this point, models assumed producers were able to locate the most productive drilling sites at first. In reality, the probability of a large discovery is very low, initially. Only as cumulative discoveries increase, producers gain more information about the location of natural, geological sweet spots (i.e. the cluster with the largest resource concentration). Thus, the odds of drilling productive wells increases. Once the sweet spot cluster has been located, only less good wells remain. From then onwards, accumulated information approximately allows remaining wells to be drilled in a decreasing order of resource concentration.

We demonstrated that the consideration of a bell-shaped relationship, between exploratory effort and reserve additions in the ORH model, is sufficient to generate bell-shaped trajectories simultaneously in quantities and additions. In the absence of pressure decline constraints, both trajectories would occur at the same time. In LOGIMA, pressure decline constraints do not occasion bell-shaped trajectories in quantities; but rather, they introduce a lag, relative to the bell-shaped trajectory of reserve additions. Perfect bell-shaped trajectories will occur, if all exogenous parameters remain constant over time.

Changing conditions within the global oil and gas market affect the long-term price that producers expect, leading the observed evolution of quantities and addition to deviate from perfect bell-shaped trajectories. The drop in global oil prices that has occurred in 2014, has led the number of active drilling rigs to drop significantly in response. Our calibration efforts suggest that the observed evolution, between 2007 and 2014, was compatible with a long-term, expected price, which was equal to the prevailing average WTI price for the period. The drop in drilling activity was compatible with lower expectations about the price of oil.

Going forward, cumulative oil production from the seven unconventional plays stands at approximately 20 Bbls of oil (including liquids), which represents only about 12% of the estimated 146 billion barrels of resource endowment from these plays. If we take natural gas into account as well, in energy equivalent terms, cumulative production stood at 59 Bboe at the end of 2018, representing 18% of the overall endowment of these plays, of 323 billion barrels. The remaining plays, which were not surveyed in the dataset, are estimated to contain an additional 28 Bboe of oil and liquids, and 85 Bboe including natural gas.

By all measures, cumulative production remains in its very early stages. This means geological and technical understanding is likely to still have room to increase; thus opening the possibility for well productivity to continue to increase in the future. Higher productivity would arise from further progression towards geological sweet spots, in combination with advanced technological progress. Under the parameters that we have considered, the foreseeable gains in well productivity would be enough to sustain U.S. unconventional oil supplies for anticipated prices of above $40 per barrel. Supplies are unlikely to reach their peak, sometime before XXXX.

Therefore, this paper contributes to the body of literature on non-renewable resource economics, through its fresh look at the role of natural geological sweet spots in the build-up of oil and natural gas supplies. The asymmetric distribution of reservoir sizes concentrates oil and gas resources into a limited number of reservoirs. Initially, a lack of geological information calls for a higher number of drillings, in order to expedite the build-up of discoveries. Under our proposed model, producers phase their exploratory effort optimally, in order to locate the largest deposits at the least cost; which explains the ascending side of "bell-shaped" supply trajectories.

## 8. BIBLIOGRAPHY

Anderson, S. T.; Kellogg, R. & Salant, S. W.. Hotelling Under Pressure. Journal of Political Economy 126, no. 3, June 2018, 984-1026.

Arps, J. & others. Analysis of decline curves Transactions of the AIME, Society of Petroleum Engineers, 1945, 160, 228-247.

Arps, J.; Mortada, M.; Smith, A. & others. Relationship between proved reserves and exploratory effort Journal of Petroleum Technology, Society of Petroleum Engineers, 1971, 23, 671-675.

Baihly, J. D.; Altman, R. M.; Malpani, R.; Luo, F. & others. Shale gas production decline trend comparison over time and basins SPE annual technical conference and exhibition, 2010.

Baker Hugues. North America Rotary Rig Count (Jan 2000 - Current) 2017. Retrieved from https://www.bakerhughes.com/.

British Petroleum (BP). Statistical review of world energy. British Petroleum Inc., London, UK, 2017. Retrieved from https://www.bp.com/en/global/corporate/energy-economics/statistical-review-of-world-energy.html.

Cairns, R. D. & Davis, G. A.. Adelman's Rule and the Petroleum Firm. The Energy Journal, International Association for Energy Economics, 2001, 22, pp. 31-54.

EIA (2013). Assumption to Annual Energy Outlook 2013. U.S. Energy Information Administration, Washington, DC, 2013.

EIA (2015). Assumptions to Annual Energy Outlook 2015. U.S. Energy Information Administration, Washington, DC, 2015, 60-62.

EIA (2017). Drilling Productivity Report. U.S. Energy Information Administration, Washington DC, 2017. Retrieved from https://www.eia.gov/petroleum/drilling/.

EIA (2017a). Assumptions to the Annual Energy Outlook 2016. U.S. Energy Information Administration, Washington DC, 2017.

EIA (2017b). Explanatory notes and sources to the Drilling Productivity Report. U.S. Energy Information Administration, Washington DC, 2017.

Herfindahl, O. C.. Depletion and economic theory. Extractive resources and taxation. Univ. of Wisconsin Press Madison, 1967, 63-90.

Hilaire, J.; Bauer, N. & Brecha, R. J.. Boom or bust? Mapping out the known unknowns of global shale gas production potential. Energy Economics, Elsevier, 2015, 49, 581-587.

Holland, S. P.. Modeling Peak Oil. The Energy Journal, International Association for Energy Economics, 2008, 29, 61-79.

Hotelling, H.. The Economics of Exhaustible Resources. Journal of Political Economy, The University of Chicago Press, 1931, 39, 137-175.

Hubbert, M. King. "Nuclear energy and the fossil fuel." Drilling and production practice. American Petroleum Institute, 1956.

IHS CERA. Upstream Capital Costs Index. IHS Cambridge Energy Research Associates, 2017. Retrieved from https://ihsmarkit.com/Info/cera/ihsindexes/index.html.

Laherrere, J.. Forecasting future production from past discovery. International Journal of Global Energy Issues, Inderscience Publishers, 2001, 18, 218-238.

Laherrère, J.. Future of oil supplies. Energy, Exploration & Exploitation, Multi Science Publishing, 2003, 21, 227-267.

Leonard, D. & Van Long, N.. Optimal control theory and static optimization in economics. Cambridge University Press, 1992.

Livernois, J.. On the Empirical Significance of the Hotelling Rule. Review of Environmental Economics and Policy, 2009, 3, 22-41.

Nome, S. & Johnston, P.. From Shale to Shining Shale. Deutsche Bank Research, Deutsche Bank Research, Deutsche Bank Research, 2008.

Nystad, A. N.. Rate sensitivity and the optimal choice of production capacity of petroleum reservoirs. Energy Economics, 1987, 9, 37 – 45.

Okullo, S. J.; Reynes, F. & Hofkes, M. W.. Modeling peak oil and the geological constraints on oil production. Resource and Energy Economics, 2015, 40, 36 – 56.

Pindyck, R. S.. The Optimal Exploration and Production of Nonrenewable Resources. Journal of Political Economy, The University of Chicago Press, 1978, 86, 841-861.

Pontryagin, L.; Boltyanskii, V. & Gamkrelidze, R.. The mathematical theory of optimal processes. New York, 1962.

Rehrl, T. & Friedrich, R.. Modelling long-term oil price and extraction with a Hubbert approach: The LOPEX model. Energy Policy, 2006, 34, 2413-2428.

Reyes, F.; Okullo, S. J. & Hofkes, M. W.. How does economic theory explain the Hubbert peak oil model?. USAEE Working Paper, 2010.

Samuelson, P. & Nordhaus, W.. Microeconomics. Mcgraw-hill book company, Oklahoma city, 2001.

Slade, M. E. & Thille, H.. Whither Hotelling: tests of the theory of exhaustible resources. Annu. Rev. Resour. Econ., Annual Reviews, 2009, 1, 239-260.

Slade, M. E.. Trends in natural-resource commodity prices: An analysis of the time domain. Journal of Environmental Economics and Management, 1982, 9, 122-137.

Thompson, A. C.. The Hotelling Principle, backwardation of futures prices and the values of developed petroleum reserves – the production constraint hypothesis. Resource and Energy Economics, 2001, 23, 133 – 156.

Uhler, R. S.. Costs and supply in petroleum exploration: the case of Alberta. Canadian Journal of Economics, JSTOR, 1976, 72-90.

USGS Variability of distributions of well-scale estimated ultimate recovery for continuous (unconventional) oil and gas resources in the United States. U.S. Geological Service, U.S. Geological Survey, 2012. Retrieved from https://pubs.usgs.gov/of/2012/1118/OF12-1118.pdf.