The pricing of univariate European Crack spread option with jumps

Abstract

Pricing crack spread options is an interesting field because the future prices with commodities as the underlying assets have some specifics features. The occurrence of jumps is one of specific characteristic of commodity prices due to supply and demand shocks which coincides with the arrival of important new information. Recent study of Kyriakou et al. (2016), by adopting the model of Heston (1993) and Bates (1996) to price the crude oil futures vanilla options, shows that their options perform well. Another important distinctive feature of commodity prices is the existence of cointegration relations. Some studies find that the commodity spread option is influenced by the existence of cointegration (see Duan and Theriault (2007), Nakajima and Ohashi (2012), and Farkas et. al. (2017)). Hence, in this paper we aim to investigate the pricing of European crack spread options on the cointegrated underlying assets.

We extent the model from Dempster et. al. (2008) by addressing the jumps characteristic then we derive the option pricing formula based on our extended model. In this model the crack spread model directly (known as univariate model) instead of using two underlying assets (known as explicit model). Even though univariate approach ignores the correlation between the price movement of the two underlying assets, Dempster et. al. (2008) argues that if cointegration exists between those two underlying assets then the spread should be modelled directly. Furthermore, some studies also find that the univariate model performs better than the explicit model when implemented to real data (see Mahringer and Prokopczuk (2015) and Aba and Goard (2016)).

In order to test the cointegration relation, we apply Johansen test to investigate co-integration relationship among these commodity markets, we find that this relation exist. We also test the mean reversion in the risk neutral measure by estimating the regression parameter of one year crack spread as long-end futures spread and the one month crack spread as the short-end futures spread. The result indicates that the spot crack spread is mean reverting. Since the cointegration test suggests the existence of long-term equilibrium of crude and heating oil, the mean-reversion of the crack spread does not occur as a result of separate mean reversion of crude and heating oil prices but by the cointegration between these two prices.

The parameters of our proposed model is estimated by calibrating the market data. We use the end of trading day call and put option prices of heating oil crack spread options from the New York Merchantile Exchange (NYMEX) from February 2010 to April 2011. We use an older period of observation because there was no trading in recent periods. We define two groups of time to maturity: 60 days or less as a short term option and more than 60 days as a long term option. For each group of time to maturity, we try to minimise the sum square of errors of our proposed model and market data. We also calibrate the market data based on other existing models: the Bachelier model (ABM), Black-Scholes model (GBM), 1-factor Schwartz model, and Demspter model. By comparing our proposed model with other models, we find that our model performs better in capturing market prices, both for call and put options.

The average signed and unsigned percentage errors is used to compare the prices of market and model. We calculate this percentage errors on both group of time to maturity and three groups of moneyness; out-the-money (OTM), at-the-money (ATM), and in-the-money (ITM). For OTM options, all models on average under-price options compared to market prices. This

indicates the low demand of these options. ITM options are modelled well by these five models. For ATM options, the five models perform better on longer expiry.

Keywords: Cointegration, Crack spread options, Energy market, Options on oil futures, Options pricing

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