# ‘Coefficient clubs’ and underlying energy demand trends for panel data

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## 1 Introduction & sketch of methodology

A number of existing energy demand studies estimate panel data models of the form:

$$q_{it} = \mu_i + \mu_t + \beta_P p_{it} + \beta_Y y_{it} + u_{it} \tag{1}$$

Where $q$ is a measure of energy consumption, $p$ a measure of prices and $y$ a measure of income - often all expressed in naural logarithms. $\mu_i$, $\mu_t$, $\beta_P$ and $\beta_Y$ are parameters to be estimated. The time-based fixed effects (FE) in these models ($\mu_t$) have, in a number of applications, been used as a proxy for a panel variant of the underlying energy demand trend (UEDT). The UEDT concept was initially cast within a pure time-series data context, using a structural time series modeling framework to obtain underlying demand trends as an unobserved component to be estimated. The single equation specification, as it has appeared in previous literature, can be expressed (in state space form) as:

$$q_t = \mu_t + \beta_1 p_t + \beta_2 y_t + \eta_t \tag{2a}$$

$$\mu_t = \mu_{t-1} + v_t \tag{2b}$$

One distinguishing feature of this specification is that the time-varying intercept $\mu_t$ is formally time-varying, with the current value of the intercept being some evolution of itself in the previous period, typically via a random walk process. The same however cannot be said for the $\mu_t$ fixed effect terms in the panel model in Eq. (1), which are independent of each other in each period by construction.

This model structure in Eq. (2a) and Eq. (2b) can be extended with relative ease to multivariate settings that are amenable to panel data modeling. Restrictions on certain model parameters can be used to essentially mimic the typical assumptions adopted when modeling panel data, in particular the assumption of common parameters to all panel members. It is therefore possible to devise a panel data modeling framework in state space form that directly inherits the spirit of the UEDT as originally outlined in early time-series based applications, without losing the assumption of common coefficients as adopted in most existing panel data studies.

With this in mind, the objective of this study is to evaluate the scope for using state space models for the purpose of estimating panel UEDT's. To be more precise, we have two competing methods: the FE estimator is the 'reigning champion', and is well understood, trusted and widely applied; state-space methods are the 'contender'. Moreover, the reigning champion only approximates the model the time-varying effects we are interested in, while state-space models provide a more formal means to model them explicitly. It is additionally thought that the data-dimensions, especially along the $T$ dimension for the contender mean that it should not be able to compete against the champion i.e. that the methods cannot compete in the same weight class.

The steps involved in the study are:

- Compare and contrast the FE and state-space techniques for the purpose of recovering time-varying intercept effects (e.g. a UEDT style effect) under the following true data generating process (d.g.p.):

$$q_{it} = \mu_t + \beta_{Pt} p_{it} + \beta_{Yt} y_{it} \tag{3}$$

  i.e. constant coefficients on two exogenous variables, and a time-varying common intercept term. The above d.g.p. will be simulated many times over in a Monte Carlo exercise, snd the FE and state-space models will be pit against each other in a contest to provide the most accurate and reliable estimates of key model parameters.

- The comparisons discussed above will be conducted under modest panel dimensions, both with respect to $T$ and $N$. This is to provide a fair and objective evaluation of the potential to apply state-space based approaches to the types of datasets we still see widely in practice. These dimensions will run from small (to the point that most editors may simply desk-reject the paper), through medium (where referees are likely to reject) to moderately large.[1] The following values will be considered: $T = 5, 10, 15, 20, 25, 30$ and $N = 5, 10, 15, 20, 25, 30$.

- Points of comparison will include both individual parameter accuracy as well as overall model accuracy.

Besides the fact the state-space approach can in principle replicate FE estimates, another perhaps more appealing feature of this technique is its flexibility to extend from a

[1] Large dimensional datasets are not explicitly considered, since different complications can emerge in the presence of high-dimensional data.

time-varying intercept into a fully time-varying framework. The simulations will therefore also be conducted using a fully time-varying d.g.p. e.g.:

$$q_{it} = \alpha_i + \alpha_t + \beta_{Pt}p_{it} + \beta_{Yt}y_{it} \tag{4}$$

### 1.1 From common coefficients to 'coefficient clubs'.

The maintained assumption in much empirical research, is that the price and income elasticities are common to all panel members. This assumption can be relaxed, and a yet more general panel representation might be given by:

$$q_{it} = \mu_t^{(\kappa_j)} + \beta_{1t}^{(\kappa_k)}p_{it} + \beta_{2t}^{(\kappa_m)}y_{it} + \eta_{it} \tag{5}$$

$\kappa_j$, $\kappa_k$ and $\kappa_m$ are identifier functions used to denote membership/clustering of coefficients into clubs with $j \in J$, $k \in K$ and $m \in M$, and $\{J, K, M\} \leq N$. While full details are not provided here (owing to space considerations), it is possible to identify club membership using a relatively simple detection mechanism, that is relatively cheap in terms of implementation time and with a high degree of accuracy. Note that the detection mechanism also operates as a simple test for panel membership, not just panel clustering. Specifically if $J = K = M = N$, then we have perfect heterogeneity and the panel members might best be handled as individual time-series. In any other case, there would be evidence of some non-trivial panel structure or coefficient clubbing. We might imagine a-priori that $J$, $K$ and $M$ will typically be small, both relative to $N$ and in absolute terms. Lastly note that when $J = K = M = 1$, there is only one club e.g. parameters are common across all panel members.

## 2 Preliminary Results

The table contained at the end of this extended abstract summarizes part of the base simulation results, for combinations of $N$ and $T$ ranging from 5 to 30 in increments of 5. Panel A shows summary results for the first data generating process using the two models. Without exception, the state-space model (i) is *at least* as likely as FE models to provide estimates of the time-varying intercept in which the true values lie within the estimated confidence intervals and (ii) provides *more accurate point estimates*, more often than FE models. The simple conclusion being, you are more likely to have more precise estimates with equally reliable confidence intervals, even in small data samples, using a simple state-space structure instead of an FE estimator.

Panel B extends the analysis to the case of fully time-varying panel data specifications. Unsurprisingly the state-space approach outperforms the FE model, but not only this we can observe it has generally strong performance in very modest data dimensions. The implications of the results in this table are not only that state-space methods would outperform FE models–that is an intuitive conclusion–but more importantly that they appear to have quite pleasing overall accuracy. With data dimensions of just $T = 15$ and $N = 15$ or above, the state-space approach will provide estimates that both imply significance where it should, and have the true coefficient values in the confidence interval, in upwards of 74% of all replications. This gets closer to 90% when $T = N = 30$, and should improve further still as samples continue to grow in either dimension. These sorts of results are not implied by asymptotics, nor (arguably) are they consistent with the consensus understanding carried by most empirical researchers.

Having established some evidence to support the potential viability for state-space models to be used in panel applications, a simple empirical exercise is done to model time varying features in a dataset on industrial energy demand for the OECD 17 (a dataset previously studied using FE models). The empirical application also exploits a panel-clustering mechanism to club the coefficients in the spirit discussed above. Results are reported in Figures 1 and 2. Figure 1 shows that there is some visible heterogeneity in income elasticities of industrial energy demand with evidence of both coefficient clubbing and time-varying parameters. The number of identified clubs is much less than the number of panel members.[2]

For the UEDT's in Figure 2, we see more heterogeneity. Notable features are (i) the number of UEDT clubs is still less than the number of panel members and (ii) club membership for the UEDT is not the same for income elasticities as for the UEDT (or for price elasticities, not reported here). Thus, two types of heterogeneity are uncovered, first being the variation over time, and second being the pattern of club allocations. The patterns of coefficient club membership will likely help to inform policy evaluation and design, and help prioritize which nations deserve what scale of policy response, and at what time.

[2]This toy example could be varied in many ways, in line with recent variables, transformations etc. that have appeared in recent research, but for the sake of the current illustration it is preferred to keep things more simple.

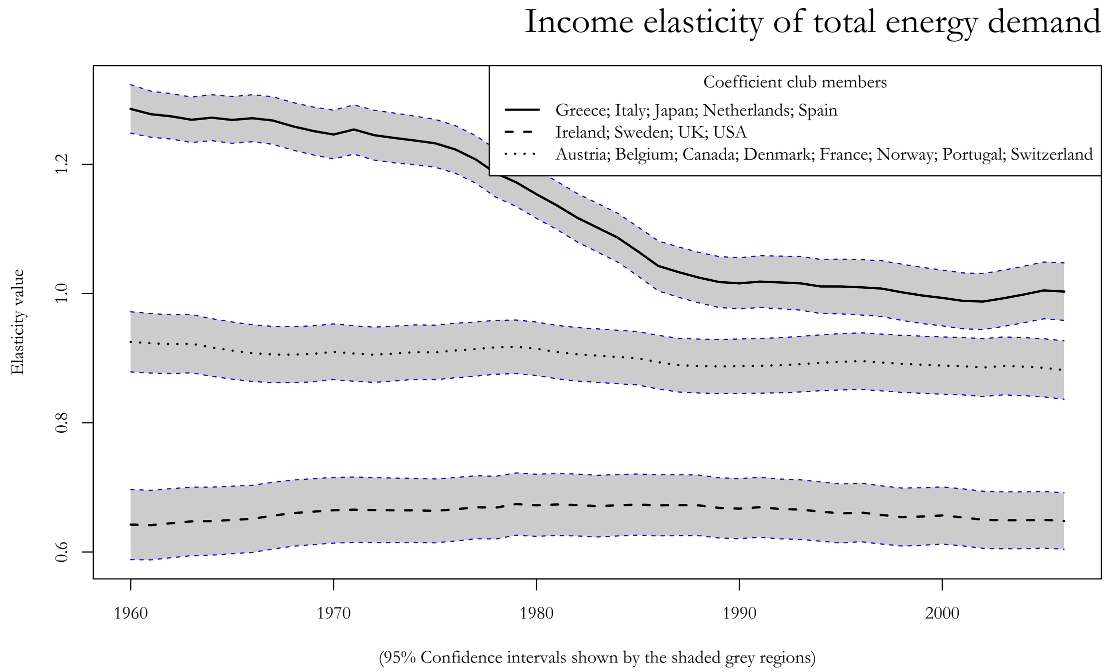


Figure 1: Estimated coefficient clubs for the income elasticity of energy demand for total energy among the 'OECD 17' countries, 1960-2006. N=17, T=47, $\kappa$=3.

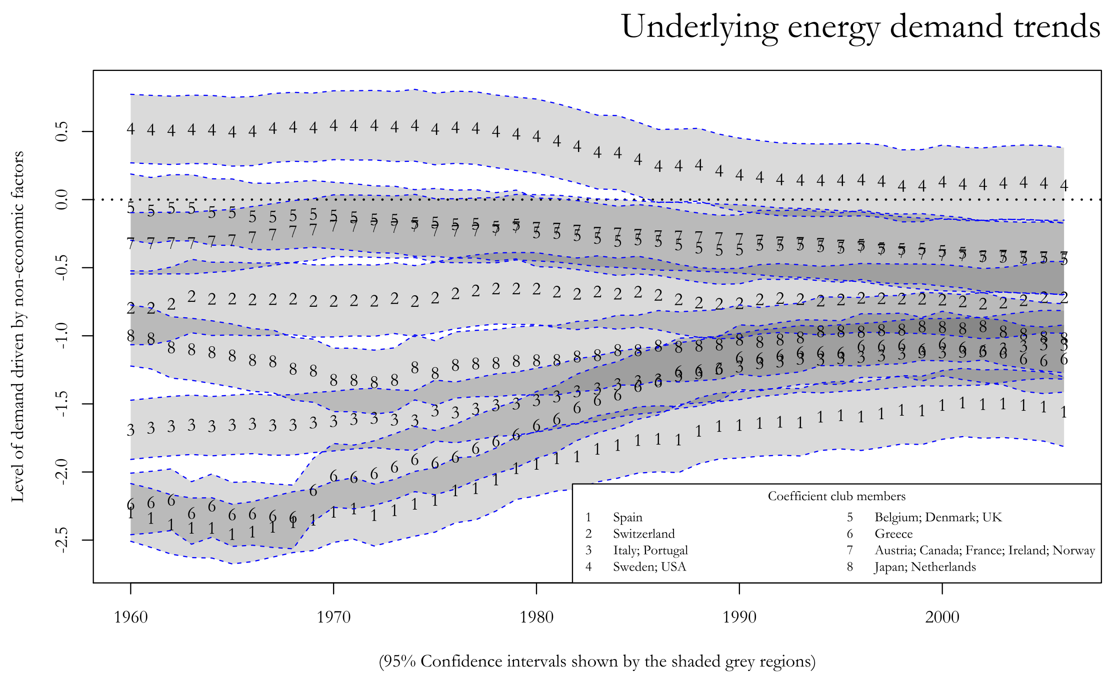


Figure 2: Estimated coefficient clubs for the underlying energy demand trends (UEDTs) for total energy among the 'OECD 17' countries, 1960-2006. N=17, T=47, $\kappa$=8.

**Panel A: True d.g.p.** $Y^*_{it} = \alpha_t + \beta_1 X_{1it} + \beta_2 X_{2it}$

**FE Coverage and significance:** $\alpha_t$

| N | Length of time series $T$ 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|
| 5 | 0.36 | 0.50 | 0.52 | 0.55 | 0.48 | 0.51 |
| 10 | 0.56 | 0.61 | 0.68 | 0.67 | 0.69 | 0.66 |
| 15 | 0.58 | 0.68 | 0.73 | 0.80 | 0.75 | 0.73 |
| 20 | 0.67 | 0.70 | 0.75 | 0.78 | 0.78 | 0.76 |
| 25 | 0.65 | 0.77 | 0.77 | 0.79 | 0.80 | 0.81 |
| 30 | 0.70 | 0.74 | 0.80 | 0.81 | 0.86 | 0.85 |

**TVP Coverage and significance:** $\alpha_t$

| N | Length of time series $T$ 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|
| 5 | 0.37 | 0.43 | 0.45 | 0.48 | 0.47 | 0.52 |
| 10 | 0.61 | 0.68 | 0.71 | 0.74 | 0.78 | 0.78 |
| 15 | 0.65 | 0.73 | 0.77 | 0.83 | 0.81 | 0.79 |
| 20 | 0.72 | 0.76 | 0.79 | 0.80 | 0.83 | 0.84 |
| 25 | 0.72 | 0.79 | 0.80 | 0.82 | 0.84 | 0.84 |
| 30 | 0.76 | 0.79 | 0.82 | 0.84 | 0.85 | 0.86 |

**FE Relative accuracy score:** $\alpha_t$

| N | Length of time series $T$ 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|
| 5 | 0.45 | 0.38 | 0.42 | 0.36 | 0.35 | 0.31 |
| 10 | 0.42 | 0.41 | 0.38 | 0.32 | 0.32 | 0.32 |
| 15 | 0.42 | 0.36 | 0.36 | 0.37 | 0.33 | 0.34 |
| 20 | 0.41 | 0.40 | 0.38 | 0.37 | 0.34 | 0.33 |
| 25 | 0.41 | 0.43 | 0.39 | 0.38 | 0.36 | 0.35 |
| 30 | 0.43 | 0.42 | 0.41 | 0.37 | 0.40 | 0.37 |

**TVP Relative accuracy score:** $\alpha_t$

| N | Length of time series $T$ 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|
| 5 | 0.55 | 0.62 | 0.58 | 0.64 | 0.65 | 0.60 |
| 10 | 0.58 | 0.59 | 0.62 | 0.68 | 0.68 | 0.68 |
| 15 | 0.58 | 0.64 | 0.64 | 0.63 | 0.67 | 0.66 |
| 20 | 0.59 | 0.60 | 0.62 | 0.63 | 0.66 | 0.67 |
| 25 | 0.59 | 0.57 | 0.61 | 0.62 | 0.64 | 0.65 |
| 30 | 0.57 | 0.58 | 0.59 | 0.63 | 0.60 | 0.63 |

**Panel B: True d.g.p.** $Y^*_{it} = \alpha_t + \beta_{1t} X_{1it} + \beta_{2t} X_{2it}$

**FE Coverage and significance:** $\alpha_t$

| N | Length of time series $T$ 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|
| 5 | 0.19 | 0.43 | 0.41 | 0.41 | 0.39 | 0.47 |
| 10 | 0.20 | 0.56 | 0.50 | 0.54 | 0.53 | 0.48 |
| 15 | 0.19 | 0.57 | 0.53 | 0.52 | 0.59 | 0.43 |
| 20 | 0.19 | 0.38 | 0.60 | 0.60 | 0.54 | 0.71 |
| 25 | 0.30 | 0.48 | 0.48 | 0.63 | 0.52 | 0.64 |
| 30 | 0.35 | 0.43 | 0.49 | 0.43 | 0.52 | 0.57 |

**TVP Coverage and significance:** $\alpha_t$

| N | Length of time series $T$ 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|
| 5 | 0.32 | 0.41 | 0.47 | 0.47 | 0.48 | 0.55 |
| 10 | 0.58 | 0.65 | 0.70 | 0.72 | 0.76 | 0.77 |
| 15 | 0.61 | 0.70 | 0.75 | 0.82 | 0.80 | 0.78 |
| 20 | 0.68 | 0.75 | 0.78 | 0.80 | 0.82 | 0.84 |
| 25 | 0.68 | 0.77 | 0.80 | 0.82 | 0.83 | 0.84 |
| 30 | 0.72 | 0.78 | 0.81 | 0.83 | 0.84 | 0.86 |

**FE Coverage and significance:** $\beta_{1t}$

| N | Length of time series $T$ 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|
| 5 | 0.21 | 0.03 | 0.05 | 0.13 | 0.33 | 0.25 |
| 10 | 0.09 | 0.22 | 0.41 | 0.14 | 0.06 | 0.22 |
| 15 | 0.19 | 0.32 | 0.20 | 0.08 | 0.13 | 0.03 |
| 20 | 0.40 | 0.18 | 0.22 | 0.25 | 0.09 | 0.10 |
| 25 | 0.11 | 0.21 | 0.19 | 0.13 | 0.07 | 0.08 |
| 30 | 0.25 | 0.11 | 0.03 | 0.02 | 0.05 | 0.01 |

**TVP Coverage and significance:** $\beta_{1t}$

| N | Length of time series $T$ 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|
| 5 | 0.68 | 0.69 | 0.71 | 0.74 | 0.84 | 0.72 |
| 10 | 0.75 | 0.68 | 0.78 | 0.77 | 0.86 | 0.85 |
| 15 | 0.80 | 0.68 | 0.79 | 0.84 | 0.78 | 0.86 |
| 20 | 0.84 | 0.81 | 0.74 | 0.81 | 0.84 | 0.75 |
| 25 | 0.76 | 0.79 | 0.81 | 0.74 | 0.87 | 0.82 |
| 30 | 0.77 | 0.79 | 0.82 | 0.88 | 0.87 | 0.86 |