

# *Multi-product Supply Function Equilibria*

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## **Overview**

In power markets many heterogenous but closely related goods are traded: power delivered at different time periods, and locations, and different product types such as active power, reactive power, balancing power, and different types of ancillary services. We study oligopolistic competition in such general setting.

The operation of those markets is not straightforward as those products are related both for consumers and producers. For consumers goods can be substitutes or complements. For instance demand can be shifted across demand periods (substitution), and both active power and reserve power are complements to the provision of a reliable energy supply. On the producers side there can be economies or diseconomies of scope reactive power and might be a by-product from the production of active power (economies of scope), while providing reserve capacity comes at an opportunity for not being able to produce in the spot market, that is production is rivalrous (diseconomies of scope).

In practice two approaches are used to study those markets: (1) A complex auction in which firms submit their bids for all (bundles) of goods, and the auctioneer co-optimizes across produce categories. One such market would be the PJM market. (2) Several simple auctions which operate in parallel, and where firms submit bids for individual goods. In this paper we focus on the functioning of the first type of market. This approach is often used in many European Power Exchanges.

## **Methods**

We consider a procurement auction with two heterogeneous divisible goods and a uniform price for each good. Firms submit a “bid-surface” for each good, in which they state their production output as a function of all the prices in the market.. Demand is assumed to be stochastic. We solve for Nash equilibria.

## **Results**

We show that the payoffs and the allocation of underlying goods are invariant to bundling. A symmetric equilibrium is determined by a partial differential equation, and through a single price pair and quantity pair, only a single supply surface passes.

For quadratic costs and linear demand, bundles can be chosen such that it is equivalent to trade divisible packages in two separate auctions. Such packaging simplifies both the analysis of and the operation of a multi-product auction.

## **Conclusions**

Our paper provides a tractable model to study oligopolistic competition in multi-unit auctions.

## **References**

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