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COMPARING PAY-AS-BID AND SYSTEM MARGIONAL PRICING AUCTION IN AN OLIGAPOLY ELECTRICITY MARKET

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Overview
This paper concerns the comparison between a pay-as-bid (PAB) auction and a system marginal pricing (SMP) auction in power market. Both PAB and SMP have been used in the balancing market for electric power worldwide. Before 2001, almost all power markets were organized based on SMP auction. In 2001, the UK market switched from a SMP rule to the PAB auction. After the California energy crisis, a similar switch was put into consideration by many markets including the emerging power market in China. Some researchers suggested that the SMP auction should be in favor of PAB auction based on experience from auction theory and experimental evidence[1-3]. In this paper, the SMP auction and PAB auction is compared within a Supply Function Equilibrium (SFE) framework.

Methods

The SFE framework was introduced by Klemperer and Meyer [4] and developed by Green and Newbery [5]. Traditionally, the SFE framework is used only for SMP rule. In this paper, it was extended to consider the PAB auction.

For deriving the SFE under SMP rule, a firm choose its supply function \( q_j(p) \) to maximize its profit function as below:

\[
\pi_i(p,t) = p(D(p,t) - \sum_{j \neq i} q_j(p)) - C_i(D(p,t) - \sum_{j \neq i} q_j(p))
\]

By solving the first-order conditions derived from equation (1), a SFE can be solved as follows:

\[
q_i(p) = \left[ p - C'_i(q_i(p)) \right] \left( \frac{dD}{dp} + \sum_{j \neq i} dq_j \right)
\]

Assuming a linear demand function \( D(p,t) = N(t) - \gamma p \), proper cost functions and linear or affine supply function form, one can solve the Linear SFE (LSFE) or Affine SFE (ASFE).

To extend the SFE framework into PAB auction, we can change the profit function into the following form:

\[
\pi_i(p,t) = \frac{1}{2} p(D(p,t) - \sum_{j \neq i} q_j(p)) - C_i(D(p,t) - \sum_{j \neq i} q_j(p))
\]

The demand function then can be expressed as

\[
D(p) = N(t) - \gamma Avg(p), \text{ where } Avg(p) = \frac{\int_0^p pdq(p)}{q(p^*)}
\]

is genration weighted average price under PAB rule.

Based on the assumptions listed above, we can use the SFE framework for comparing the result of SMP auction and PAB auction in power market.
For constrain of space, the detail of deriving SFE is omitted here. For LSFE, the PAB is equal to SMP that means average price under PAB and SMP is the same. For ASFE, the average price under PAB is slightly lower than that under SMP. A numerical example based on data from Day [6] is presented in the result section.

Results

Fig. 1: Comparison of average price between PAB, SMP and PAB-SMP under LSFE

![Graph showing comparison of average price between PAB, SMP and PAB-SMP under LSFE](image1)

Fig. 2: Comparison of average price between PAB, SMP and PAB-SMP under ASFE: Blue for SMP, Red for PAB and green for PAB-SMP

![Graph showing comparison of average price between PAB, SMP and PAB-SMP under ASFE](image2)

Conclusions

Based on the SFE model and numerical example, we can conclude that the SMP and PAB auction generate almost the same market clearing output and Average price.

Our analysis showed that, under uncertain demand, the generators maximize their expected profit and use their supply function to achieve them. Different auction rules have no impact on these expected profits when giving the generators are risk neutral, that is the SMP and PAB are revenue equivalent.