# Short-term Hedging for an Electricity Retailer

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**ABSTRACT**

A dynamic global hedging procedure making use of futures contracts is developed for a retailer of the electricity market facing price, load and basis risk. Statistical models reproducing stylized facts are developed for the electricity load, the day-ahead spot price and futures prices in the Nord Pool market. These models serve as input to the hedging algorithm, which also accounts for transaction fees. Back-tests with market data from 2007 to 2012 show that the global hedging procedure provides considerable risk reduction when compared to hedging benchmarks found in the literature.

**Keywords:** Risk management, power markets, energy, load modeling, futures contracts

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## 1. INTRODUCTION

With the recent liberalization of electricity markets and the disentanglement of the vertical integration in the electricity supply chain in Nordic countries, continental Europe, North America and Australia, new risks have arisen for some of the participants of the electricity markets. One such risk-facing participant is the retailer buying from wholesalers to sell to end-users. These retailers[1] sign contracts giving them the obligation to supply electricity to consumers. Retailers often need to supply a quantity of electricity at a price that is predetermined while acquiring it at a variable market price (Von der Fehr and Hansen 2010, Johnsen 2011), exposing the retailers to price risk. Furthermore, as the quantity of electricity which must be supplied to consumers is uncertain, retailers also face load (or volumetric) risk (Deng and Oren 2006).

Electricity is not easily storable and retailers cannot build up electricity reserves upon which to draw to cover an unexpectedly high load demand or an electricity price increase. The non-storability of electric power fuels extreme price volatility as highly inelastic demand can cause spot prices to skyrocket when shortages occur. For retailers, the volatility can affect profitability since an unexpected high cost of electricity can lead to major losses. "*The profit margin for a retailer is so small in relation to the price risk that the profit margin can quickly disappear if the price risk is not hedged*" (NordREG 2010). In some cases, there was eventual bankruptcy as with the Pacific Gas and Electric Company in 2001 and Texas Commercial Energy in 2003. To prevent such events,

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1. Retailers is the term used on Nord Pool for these participants. On the US market, they are referred to as load serving entities.

some government regulatory initiatives were even implemented to force retailers to hedge their obligation to serve electricity loads. For example, the California Public Utility Commission now requires load serving entities (LSE) to use forward contracts and options (with mandatory physical settlement) to reduce their risk exposure (State of California 2004).

It is clear that deficient risk management can lead to financial hardship for retailers and developing effective hedging methodologies in the electricity market has become paramount. Different approaches, using different electricity derivatives, have been proposed in the literature. (Deng and Oren 2006) survey available derivatives and list the papers that implement methods pertaining to each. Hedging procedures can be divided into two main categories: (i) static, and (ii) dynamic. For static hedging, hedging instruments are bought at one point in time and the hedging portfolio is never rebalanced. For dynamic hedging, the composition of the hedging portfolio is adjusted through time as additional information becomes available. Dynamic hedging procedures can be divided into two sub-categories, which we refer to as *local* and *global* hedging. Local hedging procedures minimize the risk associated with the portfolio until the next rebalancing whereas global hedging procedures minimize the risk related to the terminal cash flow.

Several papers apply static hedging without considering load uncertainty. Stoft et al. (1998) describe simple hedging strategies with vanilla derivatives. Bessembinder and Lemmon (2002) identify the optimal position in forward contracts for electricity producers and retailers through an equilibrium scheme. Tanlapco et al. (2002) compare the performance of direct versus cross hedging, the latter practice consisting of the use of futures on other commodities to hedge an exposure to electricity prices. Fleten et al. (2010) optimize the static futures contract position of a hydro-power electricity producer in Nord Pool. Other papers studying static hedging incorporate load uncertainty in their model. Wagner et al. (2003) and Woo et al. (2004) investigate static hedges with forward and futures contracts under different risk constraints. Näsäkkälä and Keppo (2005) consider an agent with a time-varying load estimate and propose a mean-variance optimization scheme with forward contracts for the optimization of the static hedge ratio and the hedging time. Deng and Xu (2009) examine hedging strategies using interruptible contracts in a one-period setting. In a series of papers, Vehviläinen and Keppo (2003), Oum et al. (2006), Oum and Oren (2009), and Oum and Oren (2010) propose a static hedging procedure maximizing the expected utility of a LSE or a generator using a portfolio of derivatives. Kleindorfer and Li (2005) optimize the expected return of an electricity portfolio corrected by a risk measure (either variance or value-at-risk). The literature on dynamic hedging strategies includes some local procedures. For example, Ederington (1979) suggests to hedge an underlying asset with its futures in a way to minimize the one-period variance of the total portfolio. Byström (2003), Madaleno and Pinho (2008), Zanotti et al. (2010), Liu et al. (2010), and Torro (2011) adapt this procedure to the electricity market, but with different model specifications for the spot and futures prices. [Byström(2003)] applies one-week horizon hedges on Nord Pool, comparing conditional and unconditional hedge ratios. The unconditional version of hedge ratios outperforms the conditional models. Madaleno and Pinho (2008) and Zanotti et al. (2010) compare different correlation models for the spot and futures prices to compute optimal hedge ratios on European electricity markets. Liu et al. (2010) use copulas to represent the relationship between the spot and futures prices. Torro (2011) studies the case of early dismantlement of the hedging portfolio in the Nord Pool market. Alizadeh et al. (2008) propose a regime-switching process with stochastic volatility for the joint dynamics of the spot price of a commodity and its associated futures, allowing for the calculation of a regime dependent optimal hedge ratio.

Alternative dynamic hedging schemes are discussed in Eydeland and Wolyniec (2003). For example, there is delta hedging, a method which consists in building a portfolio with value variations that mimic those of the hedged contingent claim. Eydeland and Wolyniec (2003) apply delta hedging to achieve perfect replication when a LSE hedges the price of a fixed amount of load to be served. When perfect replication cannot be achieved, they propose local mean-variance optimization to tackle hedging problems.

Local procedures are attractive because they are simple to implement. Local risk minimization procedures are myopic however as they do not necessarily minimize the risk through the entire period of exposure (see Rémillard 2013). Global hedging procedures remedy this drawback by taking into account the outcomes of all future time periods at any point in time; they evaluate the adequacy of a hedge by looking at the terminal hedging error, i.e. at the maturity of the hedged contingent claim. The following is a non-exhaustive list of papers which study this methodology in general financial contexts. Schweizer (1995) minimizes the global quadratic hedging error in a discrete-time framework for European-type securities. Rémillard et al. (2010) extend his work for American-type derivatives. Follmer and Leukert (1999) minimize the probability of incurring a hedging shortfall. Follmer and Leukert (2000) minimize an expected function of the terminal hedging error.

There have been attempts to apply global hedging procedures in electricity markets. Goutte et al. (2013) use the global quadratic hedging scheme from Schweizer (1995) to hedge various vanilla contingent claims involving a deterministic amount of load with futures. Supposing that the market share of the retailer is a specific functional form of the price charged to customers, Hatami et al. (2009) propose a scheme for the joint optimization of the retail price and the derivatives investment policy. They only consider open-loop[2] solutions. Rocha and Kuhn (2012) consider a retailer that optimizes its global procurement cost with respect to a mean-variance objective measure over a given period of time by purchasing futures and vanilla options. They propose two simplifications to increase tractability: time steps are aggregated into sub-periods and only decision variables that are linear functions of past realized state variables are considered. Kettunen et al. (2010) use a two-dimensional binomial scenario tree representing spot price and load uncertainty to compute the optimal futures hedging portfolio of a retailer incurring electricity procurement costs while maximizing its expected terminal cash value under conditional-cash-flow-at-risk (CCFAR) constraints. Futures are assumed to be priced with a risk premium that only depends on time to maturity. Other work in the literature considers dynamic portfolio management with the perspective of electricity generators. For example, Fleten et al. (2002) propose a global portfolio optimization (both generation scheduling and financial contract management) scheme for electricity generators under scenarios built with a mix of construction and simulation. A total of 256 scenarios are considered within a five-stage stochastic program.

The current paper contributes to the literature on global hedging procedures for electricity markets and offers three main contributions. First, we develop a dynamic global hedging methodology that makes use of futures contracts for a retailer facing load risk, price risk, basis risk and transaction costs. Obtaining global solutions to such hedging problems is non-trivial and often requires advanced numerical schemes. Our model has the unique advantage that the optimal hedging strategy is computed using a dynamic program that requires neither simplifying the optimization problem (e.g. restricting admissible strategies to linear ones as in Rocha and Kuhn (2012)) nor reducing the dynamics of the state variables (e.g. using binomial trees as in Kettunen et al. (2010) or a low number of scenarios as in Fleten et al. (2002)). Furthermore, as the hedging strategies

2. An open-loop solution only uses load and futures values as well as current portfolio composition and value at the initial time step in setting the investment strategy over all rebalancing periods.

considered are of the *feedback* (and not *open-loop*) type, they consider up-to-date market information when the portfolio is rebalanced. Second, we find accurate models for the state variables that completely specify their stochastic dynamics. These models serve as input to our hedging procedure and are tailor-made to fit this purpose. We develop a reduced-form *weekly* load model which is flexible enough to capture the seasonality trends, both in level and in variance, exhibited by historical data. We also present a statistical model for the joint dynamics of the spot and futures prices which encompasses stylized facts of electricity futures returns: autocorrelation, volatility clusters and fat tails. A statistical approach using multivariate time series analysis is applied. Directly modeling futures prices enables the integration of market information in our hedging procedure. This can be seen as an improvement over the approach that models the spot price dynamics and links it to futures prices through an expected spot price, potentially applying a risk-premium. The latter approach, which is used in other global optimization schemes for electricity utilities as in Kettunen et al. (2010) and Eichhorn and Römisch (2006), can cause mismatches between forecasted and observed futures prices. Moreover, we present a seasonal model that captures some dependence between the load and futures returns. The third contribution is an empirical study which compares the performance of different hedging procedures on the Nord Pool market.

The non-quadratic global hedging procedure developed outperforms the benchmarks in reducing the risk borne by the retailers. Hedging backtests show a significant reduction in several risk metrics applied to the weekly hedging error. Considering the case of a retailer serving 1% of the Nord Pool load, the $TVAR_{1\%}$[3] is reduced from 172,900€ to 161,900€ if our load-basis model is used instead of the simple model in the delta hedging procedure in the in-sample backtest (see Table 9). When our global hedging procedure is applied, the $TVAR_{1\%}$ is further shrunk by a considerable amount to 128,100€.

The remainder of the paper is organized as follows. Section 2 presents the price and volumetric risks faced by retailers and describes the hedging procedure. Section 3 describes the data used for modeling purposes and presents the models for the electricity load, the spot price and futures prices. Section 4 describes the numerical experiments which test the efficacy of the hedging methodology. Section 5 presents concluding remarks. Some technical results, estimation details and goodness-of-fit tests are relegated to online Appendices.

## 2. RISK EXPOSURE AND HEDGING FOR RETAILERS

In this section, we describe the risks faced by a retailer and the hedging procedure it can undertake to hedge its exposure with futures contracts.

### 2.1 Risks Faced by Retailers

Consider the case of a retailer forced to supply a quantity of electricity at a predetermined price to end-users while buying it at a variable price on the market. Market conditions for a retailer might differ across different electricity markets and we look specifically at the Nordic electricity market Nord Pool. This market is chosen since it is one of the first to operate in a liberalized setup, and mature markets provide some historical data less likely to include structural changes.

3. The tail value-at risk at 1% ($TVAR_{1\%}$) is defined as the expected value of the loss given that a loss in the worse 1% possible has occurred.

Assume all electricity purchases occur on the day-ahead market.[4] In this market, the spot price is set on an hourly basis by balancing supply and demand. Suppose the retailer needs to serve the load $L_{t,d,h}$ during hour $h$ of day $d$ in week $t$, while $S_{t,d,h}$ is the Nord Pool system spot price for the corresponding period. The total load to be served during week $t$ is thus

$$L_t = \sum_{d=1}^{7} \sum_{h=1}^{24} L_{t,d,h}. \tag{1}$$

The mean price paid by a retailer for the purchase of each unit of load during week $t$ is

$$S_t^* = \frac{\sum_{d=1}^{7}\sum_{h=1}^{24} L_{t,d,h} S_{t,d,h}}{L_t}, \tag{2}$$

the load-weighted average of all hourly prices during the week.[5] Assume the retailer charges a predetermined price $\Pi_t$ for each unit of load sold to consumers at time $t$. Such a price setup is consistent with both the variable and fixed price products sold by retailers in Norway (see Von der Fehr and Hansen 2010). For the fixed price product, the price is locked for a long period, typically one to three years. For the variable price product, the retailer is allowed to change the price every week, but price changes are only effective two weeks after consumers have been notified. Thus, in both the variable and fixed price cases, the retailer does not have the flexibility of adjusting the retail price on the short-term hedging period considered herein and the price can be considered as fixed on this time horizon. As pointed out in Von der Fehr and Hansen (2010), the variable price product is the most prevalent retailing product (49.5% of households in the first quarter of 2007). The fixed price product, although less popular, still represented over 15% of households in the first quarter of 2007.

If no hedging is implemented, the retailer cash flow for weekly operations during week $t$ is

$$L_t(\Pi_t - S_t^*). \tag{3}$$

A retailer thus faces revenue uncertainty due to (i) price risk caused by the variability of the spot price $S_{t,d,h}$, and subsequently of $S_t^*$, and (ii) volumetric risk caused by randomness in the total volume $L_t$ of electricity to be served.

## 2.2 Electricity Futures Contracts

A retailer wishes to hedge its exposure to both price and volumetric risks with derivatives on the electricity markets. Different derivatives are available to hedge those risks: forward and futures contracts, options, weather derivatives and interruptible contracts. With the exception of forward and futures contracts, most derivatives on the Nord Pool market are traded over-the-counter,

4. In reality, because of load forecasting errors, some electricity might have to be purchased on the real-time market. Allowing this feature would increase the number of variables to be modeled, increasing the complexity of the problem and changing the nature of its possible solutions. The feature is disregarded in the current work.

5. In reality, the hourly spot price paid by the retailer is local and can be affected by grid congestion. We do not consider this issue here, but in further work $S^*$ could be defined as the load-weighted average of the local spot price.

**Table 1: Liquidity of Weekly Futures**

| Weeks-to-maturity | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Percentage | 96% | 90% | 61% | 29% | 16% |

*Notes*. Percentage of trading days between January 1, 2007 and December 31, 2012 with non-null trading volume of Nord Pool weekly futures on NASDAQ OMX.

**Table 2: Liquidity of Daily Futures**

| Days-to-maturity | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Percentage | 64% | 16% | 8.8% | 2.6% | 2.0% |

*Notes*. Percentage of trading days between January 1, 2007 and December 31, 2012 with non-null trading volume of Nord Pool daily futures on NASDAQ OMX.

are illiquid, and are not well-suited for dynamic hedging methodology as they may be unavailable when they are required. More liquid futures and forwards are better suited for dynamic hedging procedures.

For the Nord Pool market, futures and forward contracts are traded on the NASDAQ OMX. Futures contracts provide hedging for shorter horizons (daily and weekly), while forwards cover longer periods (months, quarters and years). The current paper focuses on short-term hedging for two main reasons. First, the possible adjustment by the retailer of the selling price for the variable price product reduces the importance of hedging for long durations as long-term increases in the electricity prices can be transferred to customers. Second, the presence of transaction costs affects the cost of hedging when there are many rebalancing periods. Tables 1 and 2 present the percentage of trading days on which non-null trading volumes occur for weekly and daily base load futures.[6] Liquidity is much higher on weekly contracts with one-, two- and three-week maturities. These derivatives will be used in this paper.

Futures on Nord Pool are cash-settled; no exchange of the underlying commodity occurs. The underlying asset $S_T$ of a weekly futures maturing at week $T$ is the arithmetic average of the Nord Pool system spot price observed during week $T$:

$$S_t = \frac{1}{7 \times 24} \sum_{d=1}^{7} \sum_{h=1}^{24} S_{t,d,h}, \tag{4}$$

where a week starts on Monday and ends on Sunday. Contracts are traded on NASDAQ OMX during weekdays until the Friday of week $T-1$. The futures is thus not traded during its maturity week.

There is a slight mismatch between the average weekly electricity price paid by the retailer, given by (2), and the underlying asset of weekly futures given by (4). The basis ratio

$$\eta_t = \frac{S_t^*}{S_t} \tag{5}$$

links the former and the latter.[7] The basis ratio represents an additional source of risk which must be taken into account by the hedging procedure.

6. Base load means that the contracts deliver electricity at all hours, in opposition to peak load contracts that only deliver electricity between 8:00am and 8:00pm on weekdays.

7. As shown in the online appendix A.1, the basis ratio $\eta$ usually revolves between 1 and 1.05. This is explained by a higher spot price during peak hours when electricity consumption is more important.

Futures contracts are marked-to-market. This means that (i) their cash flows do not occur strictly at maturity (in opposition to forward contracts) and (ii) the variation of their quote (referred to as the futures price) is reflected by the continuous transfer of funds between the margin accounts of the long and short position holders. Using futures contracts implies having to pay transaction fees and the cost of these will be accounted for in our methods.[8]

### 2.3 Hedging Procedure

Throughout this section, the retailer hedges its week $T$ exposure. A self-financing investment portfolio containing a risk-free asset[9] and futures with maturity week $T$ is set up at $t_0$ and rebalanced weekly until $T-1$. Since futures are traded during weekdays only, rebalancing occurs on Fridays at closing time. The closing price on Friday of week $t$ (or Sunday if $t=T$) of the risk-free asset is $B_t = \exp(rt)$, and the closing price of the futures is $F_{t,T}$, $t = t_0, \ldots, T$. Since futures are cash-settled, the last futures quote on Sunday of week $T$ is automatically set by the clearing house to $F_{T,T} = S_T$. The hedging procedure is summarized by the following algorithm:

At week $t_0$. An initial amount of capital $V_{t_0}$ is allocated for hedging purposes. The retailer enters into $\theta_{t_0+1}$ long positions on the futures contract. A portion $\mathscr{M}_{t_0}$ of the initial capital is placed in the margin account required by the clearing house. Another part is used to pay transaction fees $\mathscr{C}_{t_0}$. The remainder $\mathscr{B}_{t_0}$ is invested in the risk-free asset. If $V_{t_0}$ is insufficient to cover the margin call and fees, the money is borrowed. Since entering positions on futures contracts involves no immediate cash flows besides the amount placed in the margin and transaction fees,

$$V_{t_0} = \mathscr{M}_{t_0} + \mathscr{B}_{t_0} + \mathscr{C}_{t_0}.$$

Capital $\mathscr{M}_{t_0} + \mathscr{B}_{t_0} = V_{t_0} - \mathscr{C}_{t_0}$ (both inside and outside the margin) is invested (or borrowed) at the risk-free rate $r$.[10]

At week $t+1,\ t \in \{t_0, \ldots, T-2\}$. The total capital available for hedging (the sum of the amount placed in the margin account and in the risk-free asset) at week $t$ before transaction costs are paid is $V_t$. This capital accrues interest up to week $t+1$ and is now worth

$$(V_t - \mathscr{C}_t)\frac{B_{t+1}}{B_t}.$$

The futures margin account of the retailer is adjusted from marking-to-market,[11] the amount $\theta_{t+1}(F_{t+1,T} - F_{t,T})$ is deposited (withdrawn if negative) in the margin account. The total capital available for hedging at week $t+1$ is therefore

8. Transaction fees are described at http://www.nasdaqomx.com/commodities/Marketaccess/feelist/. Fixed annual fees for membership to the Exchange are disregarded in the current study. Variable fees which are proportional to the volume of futures transactions include Exchange fees (for trading positions) and Clearing fees (for clearing positions). Exchange fees are 0.004 €/MWh. Clearing fees depend on the volume of futures cleared in the most recent quarter, but they range from 0.0035 €/MWh to 0.0085 €/MWh. For illustrative purposes, a 0.004 €/MWh rate is used. Combining Exchange and Clearing fees, entering or clearing any long or short position is therefore approximated to cost 0.004 €/MWh.

9. Since this paper focuses on short-term hedging, a constant weekly risk-free rate $r$ is assumed.

10. It is assumed that the retailer can always borrow capital at the risk-free rate. Such an assumption has a limited impact; hedging errors are very insensitive to interest rates because of the short term horizon of the hedge.

11. To simplify calculations, it is assumed that the futures are marked-to-market weekly. On NASDAQ OMX, marking-to-market is in reality performed daily. However, because maturities are short-term (and therefore accumulation of interest is small), such an approximation has only a minor impact.

$$V_{t+1}=(V_t-\mathscr{C}_t)\frac{B_{t+1}}{B_t}+\theta_{t+1}(F_{t+1,T}-F_{t,T}).$$

The retailer modifies its portfolio to hold $\theta_{t+2}$ long positions on the futures contract. Transactions fees $\mathscr{C}_{t+1}$ are paid. A margin call might be made, but it does not affect the total amount $V_{t+1}-\mathscr{C}_{t+1}$ invested at the risk-free rate.

At week $T$. The terminal hedging capital is

$$V_T=(V_{T-1}-\mathscr{C}_{T-1})\frac{B_T}{B_{T-1}}+\theta_T(S_T-F_{T-1,T})-\mathscr{C}_T,$$

where $\mathscr{C}_T$ are clearing fees. Transaction costs are computed following $\mathscr{C}_T=0.004|\theta_T|$ (final clearing costs) and $\mathscr{C}_t=0.004|\theta_{t+1}-\theta_t|$ if $t<T$ with $\theta_{t_0}=0$.

The retailer is at risk of bearing losses when the price it pays to purchase electricity is higher than the price it charges to its customers. To avoid this situation, the hedging algorithm proposed in this paper aims at minimizing risks related to electricity procurement costs incurred by the retailer. Having reliable procurement costs stabilizes the retailer's profitability.[12] Weekly futures, which are used by the retailer to hedge its exposure, allow locking in the payoff of the variable contingent claim $S_T$ to $F_{t_0,T}$ (see the online appendix A.2).[13] However, the retailer has short positions on $S_T^*$ (instead of $S_T$) because it needs to buy electricity at that price. Since $S_T^*=\eta_T S_T$, the electricity procurement target price for each unit of load bought by the retailer during week $T$ is set to $(S_T^*/S_T)F_{t_0,T}=\eta_T F_{t_0,T}$. The retailer's cash flow at time $T$, given by Equation (3), can be separated into an unhedged cash flow $L_T(\Pi_T-\eta_T F_{t_0,T})$, the baseline profit margin, and a more risky component $\mathscr{L}_T(S_T-F_{t_0,T})$, the procurement costs term:

$$\begin{aligned}L_T(\Pi_T-S_t^*)&=L_T(\Pi_T-\eta_T F_{t_0,T})-L_T(S_t^*-\eta_T F_{t_0,T})\\&=L_T(\Pi_T-\eta_T F_{t_0,T})-\mathscr{L}_T(S_T-F_{t_0,T})\end{aligned}$$

where the load-basis $\mathscr{L}_T$ is the product of the load and the basis factor:

$$\mathscr{L}_t=\eta_t L_t. \tag{6}$$

The procurement costs term can cause large losses when the price $S_T$ peaks way above the futures price $F_{t_0,T}$. The hedging strategy aims at offsetting the variation of the quantity

$$\Psi_T=\mathscr{L}_T(S_T-F_{t_0,T}) \tag{7}$$

while the retailer determines the predetermined selling price $\Pi_T$ to extract an expected but uncertain profit. Considering the load-basis $\mathscr{L}$ (instead of the load $L$ and the basis factor $\eta$ separately) is

12. Hedging procurement costs does not remove all risks; profits are still proportional to the load. Adequate hedging of procurement costs will however prevent extreme losses.

13. This is true if transaction fees are disregarded.

convenient since only a single model is required for the former quantity (instead of two models for the latter).

We choose to hedge procurement costs risk, instead of the full cash flow in (3), for several reasons. First, the more critical part of the risk faced by the retailer is found in the procurement costs term. Failures of electricity retailers (e.g. Texas Commercial Energy in 2003) were caused by a large and sudden increase in the spot price. A prime concern in hedging is to avoid extreme losses which could lead to financial hardship or even bankruptcy. Since such losses are likely to be caused by the procurement costs term, the hedging approach developed in the current paper focuses on this term. Second, while this approach leaves some residual risk unhedged since the fluctuation of the load and basis ratio affect the baseline profits, the retailer in the variable price product specification has some control on the retail price $\Pi_T$ over a medium-term time horizon. This can offset a portion of the risk related to the baseline profit margin. Third, since the design of futures contracts allows locking the spot price $S_T$ to the initial futures price $F_{t_0,T}$, it is more natural to hedge the procurement costs term based on the difference in these two quantities than the full cash flow which is driven by the retail selling price $\Pi_T$. Fourth, only considering procurement costs avoids the separate modeling of the load and basis ratio processes. Finally, hedging only procurement costs instead of the full cash flows is consistent with other similar work in the literature (see Rocha and Kuhn (2012) and Kettunen et al. (2010)).

The retailer would like the terminal value of the hedging portfolio $V_T$ to be bigger than the target $\Psi_T$ (or at least as close as possible to it) to offset the procurement costs risk. The global hedging problem that must be solved is thus

$$\min_{(\theta_{t_0+1},\ldots,\theta_T)\in\Theta} \mathbb{E}[G(\Psi_T - V_T) \,|\, \mathscr{G}_{t_0}], \tag{8}$$

where $V_T = V_T(\theta_{t_0+1},\ldots,\theta_T)$, $\mathscr{G} = \{\mathscr{G}_t \,|\, t = t_0,\ldots,T\}$ is the filtration that defines the information available to the retailer,[14] $\Theta$ is the set of all trading strategies available to the retailer[15] and $G$ is a penalty function which weights and sanctions losses. Some integrability and regularity conditions might need to be satisfied to ensure that the solution exists.

There are numerous possibilities for the penalty function $G$. A standard choice in the literature is the quadratic function, $G(x) = x^2$, since it conveniently leads to semi-analytical formulas (Schweizer 1995) and therefore enhances the tractability and the computational speed of the solution. This approach has two principal caveats: (i) the semi-analytical formulas do not take transaction fees into account; (ii) the quadratic penalty is symmetric, such that gains and losses are equally penalized.[16] To remedy the problem of penalized gains, we also consider a semi-quadratic penalty[17]

$$G(x) = x^2\,\mathbb{I}_{\{x>0\}}. \tag{9}$$

14. The retailer is assumed to consider information $\mathscr{G}$ relative to past and contemporaneous load-basis, spot prices and futures prices: $\mathscr{G}_t = \sigma\{\mathscr{L}_u, S_u, F_{u,u+j} \,|\, 0 \le u \le t, j = 1,2,3\}$.

15. In the current paper, this consists of all $\mathscr{G}$-predictable trading strategies, meaning that $\theta_{t+1}$ is $\mathscr{G}_t$−measurable for all $t$.

16. Ni et al. (2012) add a linear term to the quadratic penalty which makes it asymmetric.

17. This penalty is also considered in a hedging problem by François et al. (2012).

A drawback of using a non-quadratic penalty is that it leads to a substantial increase in the numerical burden. The computations are however still feasible for the current framework since the portfolio is only rebalanced three times. The computation of solutions for problem (8) with penalty (9) is discussed in the online appendix A.3. A simulation-based algorithm is proposed to solve the Bellman equation. This algorithm can accommodate a wide class of penalty functions.

Keeping the quadratic order as in penalty (9) has the advantage of giving a lot of weight to large losses, precisely the type of losses that the retailer must avoid. Moreover, smaller losses are still penalized, which is consistent with the hedger's preferences. Other popular risk measures that are used in the risk management literature are the value-at-risk (VaR) and tail value-at-risk (TVaR). The most important pitfall associated with the VaR is that this measure disregards the magnitude of the largest losses. This property means that using the VaR as the objective function in the global risk minimization problem can lead to *gambling for resurrection* behavior, as shown in Godin (2013). As only losses beyond the VaR are penalized, whenever the VaR threshold has been reached, the retailer has the incentive to take very risky positions to maximize the probability of getting back beyond the VaR. This behavior increases the probability of extreme losses and is incompatible with the purpose of hedging. The TVaR measure is preferable to the VaR in a global hedging framework as this risk measure takes into account all worst-case losses by averaging them. However, small losses are not penalized when using the TVaR as the objective function and this results in higher hedging costs. Furthermore, using the TVaR significantly increases the numerical burden: performing the global minimization of the hedging error TVaR with dynamic programming requires to solve a double minimization problem as shown in Godin (2013). This is not easily handled by the numerical algorithm proposed in this paper. The expected semi-quadratic penalty therefore seems the most appropriate choice in the current context.

Penalty (9) removes losses as much as possible, and this without affecting gains. Its use is justified in the case of a highly risk averse retailer wishing to remove downside risk as much as possible, without penalizing the upside risk. However, retailers using weekly electricity futures to hedge incur a cost on average due to the presence of a positive risk premium (contango) on these contracts. An important strand of the literature studies the risk premium on electricity futures contracts, and some of these papers study risk premiums on weekly futures for Nord Pool. Lucia and Torro (2011) observe a risk premium on weekly futures with four or less weeks-to-maturity that is positive on average, but that varies through seasons: it is greatest during winter and approximately null during summer. The risk premium is also shown to be affected by hydropower reservoir levels; an unexpectedly low level inducing a higher premium. The risk premium also increases with increasing time to maturity. Botterud et al. (2010) study the risk premium and convenience yield on weekly futures with one to six weeks to maturity and also observe that futures prices are on average higher than realized spot prices. Because the contango situation leads to substantial costs to retailers using weekly futures to hedge their exposure, it is important to perform the hedging in the most cost-efficient way possible. Solving the optimization problem (8) does the latter.

Retailers with a hedging policy that allows for speculation might decide to accept additional risk in order to increase their expected return.[18] Because of the contango situation of weekly electricity futures in Nord Pool, this would be achieved by under-hedging, i.e. reducing the exposure that is hedged.

18. Sanda et al. (2013) find evidence of speculative trading within the hedging portfolio of electric utilities in Nord Pool which leads to substantial profits.

## 3. MODELS FOR THE STATE VARIABLES

To compute the optimal trading strategy, the dynamics of the state variables $\mathscr{L}_t$ and $F_{t,T}$, the key components in the hedging problem, must be modeled. The proposed models are constructed from historical data.

### 3.1 Load-basis

We assume that the load the retailer must supply is proportional to the entire system load on the Nord Pool spot market.[19] This proportionality assumption, which is justified by a high correlation between firm load and market load, is also found in Coulon et al. (2013) for the Texas electricity market.

Load forecasting has been studied in the literature. Weron (2006) surveys different load forecasting methods and divides them in two classes: artificial intelligence models (neural networks, fuzzy logic, support vector machines) and statistical models (regression models, exponential smoothing, Box-Jenkins type time series models). Load forecasting methods are split into three different segments: short-term load forecasting (STLF), medium-term load forecasting (MTLF) and long-term load forecasting (LTLF). STLF is interested in hourly forecasts up to one week ahead, MTLF considers forecasts from one week to one year ahead and LTLF considers even longer horizons. The vast majority of the load forecasting literature considers STLF (Hahn et al. 2009), but MTLF has attracted more attention recently. Gonzalez et al. (2008) use a combination of neural networks and Fourier series to represent respectively the trend and the cyclical fluctuation of the monthly load in the Spanish market. In their paper, Fourier series outperform neural networks in their predictive ability for the cyclical load fluctuations. Abdel-Aal (2008) compares the use of neural and abductive networks to forecast the monthly load supplied by a power utility based in Seattle. Abdel-Aal and Al-Garni (1997) compare the use of univariate ARIMA process, abductive networks and multivariate regression models incorporating demographic, economic and weather related covariates to forecast the monthly domestic energy consumption in the Eastern province of Saudi Arabia. ARIMA processes outperformed their competitors in the study. Barakat and Al-Qasem (1998) propose a regression model with time and temperature as covariates to forecast the weekly load on the Riyadh system (Saudi Arabia).

A few papers attempt MTLF in Nord Pool. For example, Johnsen (2001) models the electricity demand as a linear function of current and lagged variables including the previous demand, the electricity price, heating degree days (HDD) and other exogenous variables such as the price of alternative fuels, the day-length and the economic activity level. Vehviläinen and Pyykkonen (2005) propose that the monthly electricity demand is the sum of a white noise and a piecewise linear function of the temperature. Fuglerud et al. (2012) use a SARIMA model with HDD as an input to represent the weekly electricity consumption in Nord Pool. To keep the model parsimonious and because the numerical scheme is affected by the dimension of the state variable vector, the current paper proposes a reduced-form parametric statistical model for the weekly load dynamics on Nord Pool which does not include exogenous variables.

19. If the internal load data of the retailer do not support this assumption, the load model should be reworked. For differences between the load consumption patterns across the four countries that are part of the Nord Pool market, see Huovila (2003).

*3.1.1 Load-basis data*

Time series of hourly load (in MegaWatt-hours, MWh) and hourly day-ahead spot price (in Euros, €) on Nord Pool for the January 1, 2007 to July 29, 2012 period are obtained through the Nord Pool FTP server.[20] The hourly load is aggregated as shown in Figure 1 and yields 291 weekly load observations. The resulting load series $L$ and basis ratio series $\eta$ defined by Equation (5) are then combined to obtain the load-basis series $\mathscr{L}$ in (6).

The most salient feature of the load time series is a seasonal pattern, both in the mean and in the variance. Autocorrelation between consecutive load departures from its trend is also present. The model chosen for the dynamics of the load-basis $\mathscr{L}$ (observed in Figure 1) is thus:

$$\mathscr{L}_t - g(t) = \gamma(\mathscr{L}_{t-1} - g(t-1)) + \sqrt{v(t)}\varepsilon_t \tag{10}$$

$$g(t) = \beta_0 + \sum_{j=1}^{P} \beta_j C_t^{(\sin,j)} + \sum_{j=1}^{P} \beta_{j+P} C_t^{(\cos,j)} \tag{11}$$

$$\log v(t) = \alpha_0 + \sum_{j=1}^{Q} \alpha_j C_t^{(\sin,j)} + \sum_{j=1}^{Q} \alpha_{j+Q} C_t^{(\cos,j)} \tag{12}$$

where $\varepsilon$ is a strong standardized Gaussian white noise. The $g$ function represents the seasonal trend of the load-basis level and its fitted value is represented by the dashed line in Figure 1. The $v$ function characterizes the trend in the variance of seasonally corrected load-basis observations and the square root of its fitted value is represented by the dashed line in Figure 2. Terms of a Fourier expansion

$$C_t^{(\sin,j)} = \sin\left(\frac{3\pi}{2} + \frac{2\pi jt}{365.25/7}\right), \quad C_t^{(\cos,j)} = \cos\left(\frac{3\pi}{2} + \frac{2\pi jt}{365.25/7}\right)$$

are used to capture yearly cycles (see also Gonzalez et al. (2008)). The $\gamma$ parameter in Equation (10) represents the autocorrelation in seasonally corrected load-basis observations. To preserve the Markov property, only one lag is considered.[21] Parameters to be estimated are $\gamma, \beta_0, \ldots, \beta_{2P}, \alpha_0, \ldots, \alpha_{2Q}$.

*3.1.2 Estimation of model parameters*

The model estimation is performed in two steps.[22] The first consists in estimating $\gamma$ and $\beta_0, \ldots, \beta_{2P}$ by quasi-maximum likelihood under the assumption that $v(t)$ is constant. The optimal number of Fourier terms in the mean trend ($P = 3$) is chosen using the cross-validation procedure described in the online appendix A.4.1. Table 3 gives estimated parameters and their standard errors for this step. Figure 1 shows the load series $L$, the load-basis series $\mathscr{L}$ and the estimated load-basis seasonality trend $g$. Even if the variance $v$ is presumed constant during the estimation of the trend

20. Nord Pool uses the expression "turnover" to designate the load.

21. Otherwise each additional lag would have to be included as a state variable.

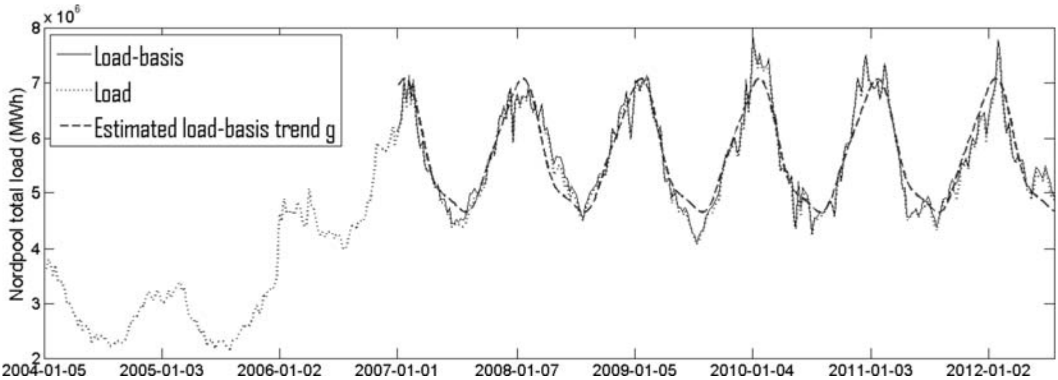
22. Results in the online appendix A.4.2 show that the fitted model is good and a more numerically challenging single-step estimation was thus not attempted.

**Table 3: Load-basis Seasonality Trend Parameters**

| Parameter | Estimated Value | Standard Error |
|---|---|---|
| $\gamma$ | 0.68 | 0.04 |
| $\beta_0 \times 10^6$ | 5.70 | 0.04 |
| $\beta_1 \times 10^6$ | −1.13 | 0.06 |
| $\beta_2 \times 10^6$ | −0.15 | 0.05 |
| $\beta_3 \times 10^6$ | 0.06 | 0.04 |
| $\beta_4 \times 10^6$ | 0.24 | 0.06 |
| $\beta_5 \times 10^6$ | 0.10 | 0.05 |
| $\beta_6 \times 10^6$ | 0.11 | 0.04 |

*Notes*. Estimated parameters and their standard error for the load-basis seasonality trend $g$ defined by Equation (11). Data between January 1, 2007 and July 29, 2012. Estimated parameter variance is obtained through the inverse of the observed Fisher information matrix.

**Figure 1: Load-basis Seasonality Trend Curves**




*Notes*. Observed total weekly load on the Nord Pool market as defined by (1), corresponding load-basis $\mathscr{L}$ described in (6) and fitted seasonality trend $g(t)$ of Equation (11) with parameters of Table 3. Data between January 1, 2007 and July 29, 2012. Load data before 2007 are also included to show the shift in the overall system load level and justify the use of data starting from January 2007. The load-basis model used is given by (10)–(12).

parameters,[23] the overall trend seems reasonably captured. The load-basis is much larger in winter than in summer; this is expected given the winter heating requirements for Scandinavian countries. The autocorrelation parameter $\gamma$ is estimated at 0.68, this large value indicating a high persistence in load deviations from the trend. This persistence can be seen in Figure 1 where the observed load tends to remain away from the trend for a few weeks following a deviation.

Once the trend parameters are estimated, proxy values for $\sqrt{v(t)}\varepsilon_t$, denoted $\sqrt{\hat{v}(t)}\hat{\varepsilon}_t$, can be computed using Equation (10). These proxies serve as input in the second step which consists in estimating $\alpha_0, \ldots, \alpha_{2Q}$ by maximum likelihood (ML). The optimal number of Fourier terms in the variance trend ($Q = 2$) is selected through the cross-validation procedure described in the online appendix A.4.1. Table 4 presents the estimated parameters for the variance model (12). Figure 2 shows the estimated standard deviation trend $\sqrt{\hat{v}_t}$ (dashed curve) and the absolute value of the

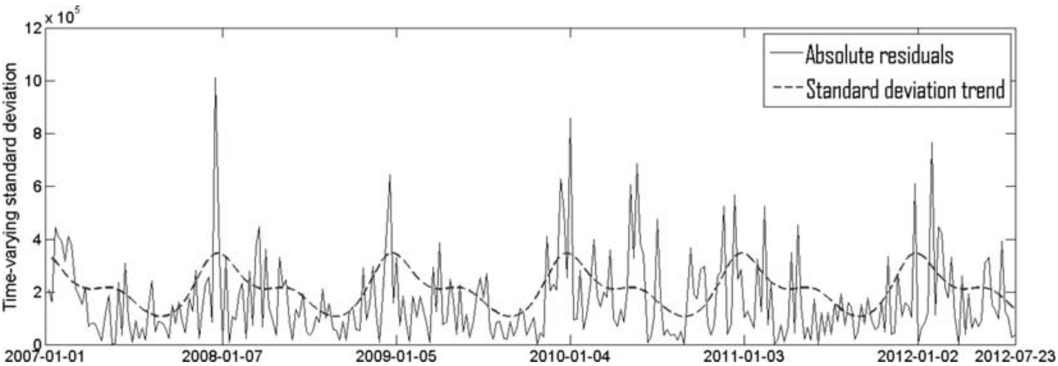
23. At this step, the constant estimated volatility is $\sqrt{\hat{v}} = 2.264 \times 10^5$.

**Table 4: Load-basis variance trend**

| Parameter | $\alpha_0$ | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_4$ |
|---|---|---|---|---|---|
| Estimated Value | 24.40 | −0.72 | −0.38 | 0.49 | −0.32 |
| Standard Error | 0.08 | 0.11 | 0.11 | 0.11 | 0.12 |

*Notes*. Estimated parameters and their standard error for the load-basis variance trend $v$ defined by Equation (12). Data between January 1, 2007 and July 29, 2012. Estimated parameter variance is obtained through the inverse of the observed Fisher information matrix.

**Figure 2: Load-basis Standard Deviation Trend Curves**




*Notes*. Realized absolute load-basis volatility $\sqrt{\hat{v}(t)}|\hat{\varepsilon}_t|$ and fitted standard deviation trend $\sqrt{\hat{v}}$ as defined by Equation (12) with parameters found in Table 4. Data between January 1, 2007 and July 29, 2012.

$\sqrt{\hat{v}(t)}\hat{\varepsilon}_t$ proxies (full curve). The peak in volatility occurs in the beginning of winter, while the lowest volatility is observed during the end of the summer. Note that the high volatility in the winter could generate spikes like those observed in Figure 1. Goodness-of-fit tests that confirm the adequacy of the load model are found in the online appendix A.4.2.

### *3.1.3 Load-basis forecasting from incomplete information*

The selection of $\theta_{t+1}$, the number of futures shares detained in the portfolio from the Friday of week $t$ until the Friday of week $t+1$ (or Sunday if $t+1=T$), is based on $\mathscr{L}_t$, the weekly load-basis on week $t$. However $\mathscr{L}_t$ is only observed at midnight on Sunday of week $t$, and not at the closing time of markets on Friday. What is observed at the latter time is the sum of hourly loads from the beginning of week $t$ to 4:00 p.m. on Friday[24]:

$$\tilde{L}_t = \sum_{d=1}^{4} \sum_{h=1}^{24} L_{t,d,h} + \sum_{h=1}^{16} L_{t,5,h}.$$

24. If the load information is not available in real-time due to delays needed to integrate data in information technology systems, the load data from Monday to Thursday at midnight can be used instead to perform the forecast. The method to compute the forecast in this situation remains the same.

When $\theta_{t+1}$ is selected, the value of $\mathscr{L}_t$ must thus be forecast using $\tilde{L}_t$. The accuracy of several forecasting models were compared through a cross-validation test and the model

$$\hat{\mathscr{L}}_t = \tilde{L}_t \times \left( \ell + c \sum_{j=1}^{q} \frac{\mathscr{L}_{t-j}}{\tilde{L}_{t-j}} \right) \tag{13}$$

produced the lowest out-of-sample forecasting root-mean-square error (RMSE). The out-of-sample mean absolute percentage error (MAPE) is 1.12%.[25] The parameter $\ell$ drives the long-term average of the ratio $\mathscr{L}_t/\tilde{L}_t$, while the autoregressive coefficient $c$ characterizes the dependence of the current ratio on previous ratios. The estimated parameters obtained when re-estimating with the full dataset are $\hat{\ell} = 0.716$, $\hat{c} = 0.171$ and $\hat{q} = 3$. Nested models where one parameter is dropped yield a lower predictive power. The long-term average of the $\mathscr{L}_t/\tilde{L}_t$ ratio is given by $\hat{\ell}/(1-\hat{q}\hat{c}) = 1.47$.[26]

### 3.2 Futures and Spot Price

In this section, time series of futures prices are modeled. Modeling the relation between the spot and futures prices in the context of electricity markets is complicated by the fact that electricity is not storable. This prevents the use of the usual cash-and-carry scheme to price futures contracts.

Solving problem (8) requires a model that completely specifies the stochastic dynamics of futures prices. Several approaches are however proposed in the literature for this purpose and they are now discussed.

Despite the fact that electricity is not storable and not openly traded, some authors follow the risk-neutral approach commonly used in finance. The dynamics of the spot price are modeled and a martingale measure is selected to compute futures prices as an expectation of the discounted cash flows. The literature proposing models for the spot price of electricity is extensive. Two main classes of models exist: reduced-form and structural (or bottom-up) models. Reduced-form models directly specify the stochastic dynamics of the spot price. Many papers follow this approach, e.g. Knittel and Roberts (2005) and Escribano et al. (2011) who test the performance of several models across the different electricity markets. In the structural class of models, the main factors driving the spot price (e.g. weather, alternative fuels price, inflows to hydro-reservoirs, the spot price in adjacent electricity markets) are individually modeled and the interaction of these factors leads to an equilibrium spot price through the intersection of supply and demand. See e.g. Fuglerud et al. (2012) and Vehviläinen and Pyykkonen (2005). Risk-neutral measures are used to compute derivatives prices from the spot price dynamics. E.g. the Girsanov transform is proposed in Lucia and Schwartz (2002), Cartea and Villaplana (2008) and Coulon et al. (2013). Benth et al. (2008) use the Esscher transform to compute futures prices from their spot model which is a linear combination of non-Gaussian Ornstein-Uhlenbeck processes.

25. The out-of-sample MAPE obtained by using the naive benchmark $\hat{\mathscr{L}}_t := \ell\tilde{L}_t$ is 1.21%. Obtaining good load forecasts is crucial to the success of the hedging procedure and the small improvement of model (13) over the naive method justifies its use. Non-Gaussian residuals invalidate the usual t-tests on model parameters. Nested sub-models gave larger out-of-sample prediction RMSE and MAPE.

26. This is consistent with what is expected; since $\mathscr{L}$ and $\tilde{L}$ are respectively approximately the sum of 168 and 112 hourly loads, the long-term average of the ratio should revolve around $168/112 = 1.5$.

Besides the non-storability of electricity, there is another potential pitfall with the risk-neutral approach to price futures. On the Nord Pool market, a principal component analysis applied to weekly futures returns shows that the spot price might be driven by factors different than those driving futures prices Benth et al. (2008). The martingale measure approach discounting the expected spot price to obtain the futures price might thus be inappropriate. This result is consistent with the study of Koekebakker and Ollmar (2005) who use principal component analysis to propose a multi-factor model for forward returns. They find that the number of factors necessary to represent the full forward curve is much larger for electricity futures on the Nord Pool market than for other commodities; the correlation between short-term and long-term electricity forward prices is smaller than in other markets.

Benth et al. (2008) also suggest adapting the Heath-Jarrow-Morton framework to electricity markets. Under such a methodology, the dynamics of forward prices that deliver an infinitesimal volume of electricity are directly specified. However, futures prices, which are really swap prices in the context of electricity markets, suffer from severe intractability issues under this model and we did not retain this approach.

Bessembinder and Lemmon (2002), in their equilibrium scheme, propose to model futures prices as a linear combination of the first three moments of the spot price at maturity. However, empirical work in Redl et al. (2009) shows that the latter model does not perform well for the Nord Pool market.

The third method proposed in Benth et al. (2008) is to find a statistical model that reproduces the dynamics of the observed futures returns. This approach is followed in the current paper since it better suits our need to fully specify the distribution and the stochastic dynamics of futures and spot prices of electricity. Benth et al. (2008) use a deterministic volatility with seasonal and time to maturity regressors. We use a GARCH-type stochastic volatility to accomodate the random (non seasonal) volatility clusters observed in our futures returns series. This approach better reproduces stylized facts.

Daily prices of futures on NASDAQ OMX are provided by Bloomberg. Since futures prices vary during the day, closing prices are used.

*3.2.1 Our model*

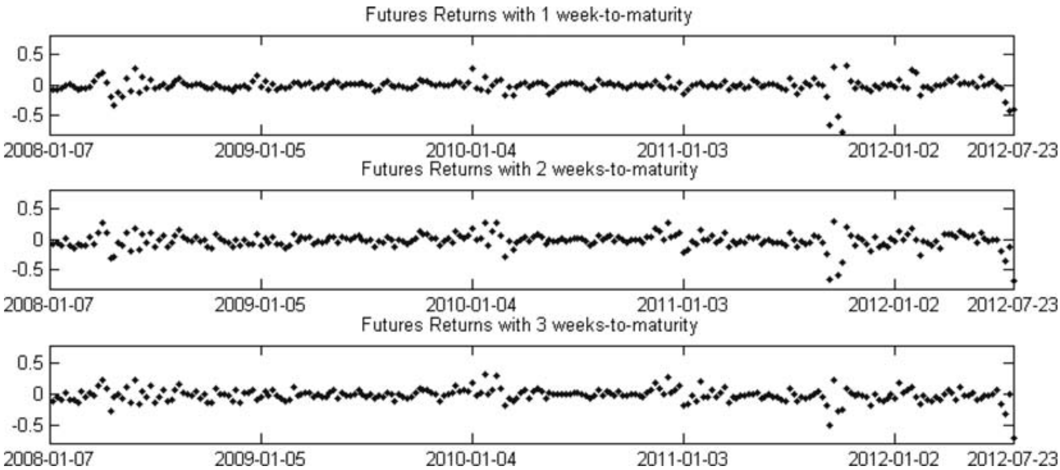
For a market participant hedging the cost of electricity at maturity week $T$, the sequence of observed futures prices that must be modeled is $\{F_{T-j,T} | j = 3,2,1,0\}$. We propose a multivariate time series model for the joint dynamics of the spot and futures prices. As with financial assets, futures price returns are modeled (instead of the futures prices) as they are more likely to be stationary. Futures returns defined by

$$\omega_{t,T} = \log(F_{t,T}/F_{t-1,T}) \tag{14}$$

are shown in Figure 3 for $t = T, T-1, T-2$.

Futures price returns exhibit autocorrelation, volatility clustering and fat tails. These features suggest a multivariate AR-GARCH process with innovations drawn from a fat-tail distribution. For the latter, we choose a Normal Inverse Gaussian (NIG) distribution. This distribution is shown to provide a good fit to Nord Pool's electricity futures returns in Frestad et al. (2010) and Andresen

**Figure 3: Futures Price Returns for Different Times-to-maturity**




*Notes*. NASDAQ OMX electricity weekly futures returns (on Nord Pool day-ahead spot price) as defined by (14) between January 1, 2007 and July 29, 2012. The trivariate time series illustrated contains 290 observations.

et al. (2010). More specifically, for $i = 0,1,2$, the trivariate AR(1)-GARCH(1,1) with NIG innovations is

$$\omega_{t,t+i} = \mu_i + a_i \omega_{t-1,t-1+i} + \sigma_{i,t} z_{i,t} \tag{15}$$

$$\sigma^2_{i,t+1} = \min\{\varsigma^2, \kappa_i + \gamma_i \sigma^2_{i,t} + \xi_i \sigma^2_{i,t} z^2_{i,t}\} \tag{16}$$

where $\mathbf{z}_t = (z_{0,t}, z_{1,t}, z_{2,t})$ has the following properties:

> if $s \neq t$, $\mathbf{z}_t$ and $\mathbf{z}_s$ are independent; $z_{i,t}$ are drawn from a standardized[27] NIG($\alpha_i,\beta_i$) distribution; conditionally upon $\mathcal{G}_{t-1}$, $z_{0,t}$, $z_{1,t}$ and $z_{2,t}$ are linked by the Gaussian copula.

A bound $\varsigma$ is used on the volatility to ensure that futures prices are square-integrable.[28] The $a_i$ parameter represents autocorrelation of futures returns while $\mu_i$ adjusts their long-term expected level. The $\kappa_i$ parameter adjusts the long-term level of futures return volatility, the $\gamma_i$ characterizes the persistence in returns volatility, and $\xi_i$ determines how shocks associated with current returns

27. A standardized NIG is a NIG distribution with mean 0 and variance 1. Such a distribution only has two free parameters: $\alpha$ and $\beta$. Note that these $\alpha$ and $\beta$ should not be confused with those used in the load-basis model in Section 3.1.1.

28. More precisely, the condition

$$\sigma_{i,t} < \frac{\alpha_i - \beta_i}{2} \quad \text{a.s.} \tag{17}$$

is necessary and sufficient to obtain $\mathbb{E}[e^{2\omega_{t,t+i}}] < \infty$. Thus, the volatility bound $\varsigma$ combined with the additional constraints $\alpha_i > \varsigma$, and $\beta_i \in (-\alpha_i, \alpha_i - 2\varsigma]$ assure (17) is satisfied.

**Table 5: Futures Return Parameters**

| Parameter | $i=1$ | $i=2$ | $i=3$ |
|---|---|---|---|
| $\mu_i \times 10^2$ | −0.72 (0.38) | −1.57 (0.51) | −1.27 (0.48) |
| $a_i$ | 0.22 (0.07) | 0.14 (0.07) | 0.07 (0.07) |
| $\kappa_i \times 10^2$ | 0.18 (0.06) | 0.12 (0.05) | 0.11 (0.05) |
| $\gamma_i$ | 0.28 (0.13) | 0.58 (0.09) | 0.60 (0.10) |
| $\xi_i$ | 0.50 (0.16) | 0.37 (0.09) | 0.34 (0.10) |
| $\alpha_i$ | 1.10 (0.10) | 1.27 (0.30) | 1.24 (0.15) |
| $\beta_i$ | −0.11 (0.03) | −0.06 (0.31) | 0.01 (0.08) |

*Notes*. Estimated parameters (standard error) for futures returns model defined in (15)–(16). Data between January 1, 2007 and July 29, 2012 for futures with $i=1$, 2 and 3 weeks to maturity.

**Table 6: Futures Return Copula Parameters**

| Parameter | $\rho_{0,1}$ | $\rho_{0,2}$ | $\rho_{1,2}$ |
|---|---|---|---|
| Estimate (Standard Error) | 0.76 (0.02) | 0.67 (0.03) | 0.88 (0.05) |

*Notes*. Estimated parameters (standard errors) for the Gaussian copula described in Section 3.2.1. $\rho_{i,j}$ links returns on futures with respectively $i+1$ and $j+1$ weeks to maturity. Data between January 1, 2007 and July 29, 2012.

affect the current volatility. The NIG parameter $\alpha_i$ drives the tail thickness in the distribution of the futures return while the $\beta_i$ drives its asymmetry.

*3.2.2 Model estimation*

A two-step procedure is applied. First, the parameters for the three marginal AR(1)-GARCH(1,1) processes ($\omega_{t,t+1}$,$\omega_{t,t+2}$ and $\omega_{t,t+3}$) are estimated by ML.[29] Plugging the estimated parameters in (15)–(16) yields proxy values $\hat{\mathbf{z}}_t$ for $\mathbf{z}_t$. Then, the proxies are used to estimate the parameters of the Gaussian copula. Letting $F_{\text{NIG}}$ denote the cumulative distribution function (cdf) associated with the NIG distribution and applying the Rosenblatt (1952) transform to the proxy $\hat{\mathbf{z}}_t$ yields a series of approximatively independent uniformly distributed observations $\mathbf{U}_t = (F_{\text{NIG}(\hat{\alpha}_0,\hat{\beta}_0)}(\widehat{z_{0,t}}),F_{\text{NIG}(\hat{\alpha}_1,\hat{\beta}_1)}(\widehat{z_{1,t}}),F_{\text{NIG}(\hat{\alpha}_2,\hat{\beta}_2)}(\widehat{z_{2,t}}))$ drawn from the Gaussian copula. ML is used and the closed-form solution is

$$\hat{\rho}_{i,j} = \mathbb{C}orr(\Phi^{(-1)}(U_{i,t}),\Phi^{(-1)}(U_{j,t})),$$

where $\mathbb{C}orr$ is the sample correlation and $\Phi^{(-1)}$ is the inverse cdf of a standard Gaussian variable.

Parameter estimates are shown in Tables 5 and 6. Negative estimates for the $\mu_i$ parameters indicate the futures market is in contango. The GARCH parameters $\gamma_i$ and $\xi_i$ are highly significant, confirming the presence of volatility clustering in futures returns. The autocorrelation parameters $a_i$ are positive, indicating that futures returns are partially predictable. The $\alpha_i$ parameters are all low (smaller than 2) so the kurtosis of futures returns is more pronounced than in a Gaussian

29. The proxy for the initial value for the volatility $\hat{\sigma}_{i,0}$ is its long-term stationary average. The bound is set at $\varsigma = 0.6$ since such a constraint is not numerically binding with the available data.

**Table 7: Correlation Between Load-basis and Futures Return Residuals**

| | Weeks to maturity | | |
|---|---|---|---|
| Quarter | 1 | 2 | 3 |
| 1 | 0.23 | 0.21 | 0.32 |
| 2 | 0.03 | 0.05 | $3 \times 10^{-3}$ |
| 3 | 0.14 | $-3 \times 10^{-3}$ | $-0.01$ |
| 4 | 0.05 | 0.09 | 0.08 |

*Notes*. Sample Spearman correlation across quarters between residuals of the load-basis series and the three futures returns series. Data between January 1, 2007 and July 29, 2012.

distribution (which corresponds to an infinite $\alpha$). The correlation parameters of the Gaussian copula are all higher than 0.65, indicating a somewhat high correlation between futures returns across the time to maturity dimension.

Goodness-of-fit tests that confirm the adequacy of the futures return model are shown in the online appendix A.4.3. None of the main features (autocorrelation, volatility clustering and non-Gaussianity) in the futures returns model can be removed without at least one of the tests in the online appendix A.4.3 indicating an inadequate fit (p-values for nested models not shown).

### 3.3 Dependence Between Load-basis and Futures Innovations

In this section, the dependence between load-basis and futures return innovations is specified. The structure of dependence between the load-basis and futures is known to vary across seasons. The dependence is stronger during winter where the price of electricity is more demand-driven. Table 7 gives estimates of correlation between the load-basis and futures returns residuals across quarters, confirming the time-varying correlation structure.

To reflect this dependence, the Gaussian copula linking futures innovations is extended to include the load-basis innovations. Thus, conditionally upon $\mathscr{G}_{t-1}$, $(z_{0,t}, z_{1,t}, z_{2,t}, \varepsilon_t)$ are assumed to be linked by a Gaussian copula with the following correlation parameters matrix:

$$R = \begin{bmatrix} 1 & \rho_{0,1} & \rho_{0,2} & \rho_{LF,t} \\ \rho_{0,1} & 1 & \rho_{1,2} & \rho_{LF,t} \\ \rho_{0,2} & \rho_{1,2} & 1 & \rho_{LF,t} \\ \rho_{LF,t} & \rho_{LF,t} & \rho_{LF,t} & 1 \end{bmatrix} \tag{18}$$

where $\rho_{LF,t}$ is the parameter linking the load-basis and futures returns innovations. We suppose the correlation between load-basis and futures return innovations to be identical for the three considered maturities. To capture the cyclical behavior of the correlation, a trigonometric function is used:

$$\rho_{LF,t} = \tau_1 0.5\left(1 + \sin\left(\frac{3\pi}{2} + \frac{2\pi \min\left((t-\tau_3)\operatorname{mod}\frac{365.25}{7}, \tau_2\right)}{\tau_2}\right)\right) \tag{19}$$

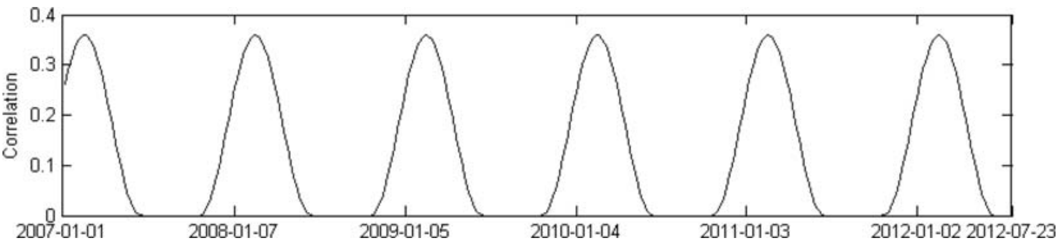
where $\tau_1$ is the maximum correlation during the year, $\tau_2$ determines the span of time across which the correlation is non-null and $\tau_3$ sets the location of the correlation peak. The parameters $\tau_1, \tau_2$ and

**Table 8: Estimated Parameters of Dynamic Correlation Between Load-basis and Futures**

| | $\tau_1$ | $\tau_2$ | $\tau_3$ |
|---|---|---|---|
| Estimated Parameter | 0.36 | 34.1 | −11.1 |
| Standard Error | 0.10 | 10.7 | 5.8 |

*Notes*. Estimated parameters and standard errors for the functional form (19) of the dynamic correlation between load-basis and futures innovations. Data between January 1, 2007 and July 29, 2012.

**Figure 4: Estimated Correlation Function Between Load-basis and Futures Innovations**




*Notes*. Estimated correlation function between load-basis and futures innovations between January 1, 2007 and July 29, 2012, as obtained by (19) with parameters found in Table 8.

$\tau_3$ are estimated by ML, using that the distribution of $(\Phi^{(-1)}(U_{0,t}),\Phi^{(-1)}(U_{1,t}),\ \Phi^{(-1)}(U_{2,t}),\hat{\varepsilon}_t)$ is approximately Gaussian with zero mean and covariance matrix $R$. The estimated parameters are given in Table 8.

The estimated correlation function $\hat{\rho}_{LF,t}$ is given in Figure 4. The resulting correlation parameter is null in the summer and attains its peak value in the middle of winter at the sixth week of the year.

## 4. PERFORMANCE ASSESSMENT

We carry out numerical experiments to assess the performance of the hedging strategy given by solutions of problem (8). We propose two different hedging procedures: (i) the hedging methodology which solves problem (8) with $G(x) = x^2$ is referred to as quadratic dynamic global hedging (QDGH); (ii) the methodology solving that same problem but without penalizing the gains, i.e. using (9), is called semi-quadratic dynamic global hedging (SQDGH). The benchmarks are described in Section 4.1 while the backtests are explained in Section 4.2.

### 4.1 Benchmarks

#### *4.1.1 Delta Hedging*

If the load to be served by the retailer is known with certainty and no transaction fees exist, the delta hedging strategy proposed by Eydeland and Wolyniec (2003) completely eliminates the price risk borne by the retailer by locking in the spot price to $F_{t_0,T}$ (see the online appendix

A.2). This strategy is adapted to the case of a stochastic load by hedging the expected load-basis, i.e. the retailer enters into

$$\theta_{t+1} = \frac{B_{t+1}}{B_T}\mathbb{E}[\mathscr{L}_T | \mathscr{G}_t] \quad (20)$$

long positions in the futures contract at time $t$ to cover its exposure at time $T$. Improved delta hedging (IDH) uses the load-basis model (10)–(12) to compute $\mathbb{E}[\mathscr{L}_T | \mathscr{G}_t]$ in (20).

To quantify the impact of using the (10)–(12) load-basis model in the hedging algorithm, alternative load-basis models are also proposed to compute $\mathbb{E}[\mathscr{L}_T | \mathscr{G}_t]$. For example, one may state that a good prediction of the expected load-basis in a near future is the last observed load-basis. This points to the first alternative, the naive delta hedging (NDH), which uses the naive prediction model

$$\mathbb{E}[\mathscr{L}_T^{(NDH)} | \mathscr{G}_t] = \mathscr{L}_t^{(NDH)}.$$

The second alternative, referred to as delta hedging (DH), uses a load-basis model inspired from Wagner et al. (2003) where the latent variable found in their model is removed for simplicity. Their model specifies the load dynamics, but is applied here to the load-basis. More specifically, the load-basis model for DH is

$$\mathscr{L}_{t+1}^{(DH)} = \mathscr{L}_t^{(DH)} + \gamma^{(DH)}(\bar{\mathscr{L}}_{m_{t+1}} - \mathscr{L}_t^{(DH)}) + \mathscr{E}_{t+1}$$

where $\mathscr{E}$ is a Gaussian white noise, $\bar{\mathscr{L}}_m$ is the historical mean value of the load-basis during the $mth$ month of the year ($m = 1,\ldots,12$) in the estimation set, $m_{t+1}$ is the month associated with week $t+1$ and $\gamma^{(DH)}$ is estimated by ML. We find $\hat{\gamma}^{(DH)} = 0.3477$. Note that any dependence between the load and the futures cannot be easily accounted for under all delta hedging approaches.

#### *4.1.2 Local Minimal Variance Hedging (LMVH)*

The objective of this strategy, which is based on the Ederington (1979) scheme, is to construct a portfolio of futures whose variation mimics the variation of the spot price as closely as possible for the current period. More precisely, for each unit of load to serve, the retailer would detain $\vartheta_{t+1}$ units of futures at time $t$, where $\vartheta_{t+1}$ minimizes $\mathbb{V}ar[(S_{t+1} - S_t) - \vartheta_{t+1}(F_{t+1,T} - F_{t,T}) | \mathscr{G}_t]$. This yields the solution $\vartheta_{t+1} = \mathbb{C}ov[S_{t+1}, F_{t+1,T} | \mathscr{G}_t] / \mathbb{V}ar[F_{t+1,T} | \mathscr{G}_t]$. To adapt this scheme to the case of stochastic load, the retailer hedges its expected load-basis by detaining at time $t$,

$$\theta_{t+1}^{(LMVH)} = \mathbb{E}[\mathscr{L}_T | \mathscr{G}_t]\frac{\mathbb{C}ov[S_{t+1}, F_{t+1,T} | \mathscr{G}_t]}{\mathbb{V}ar[F_{t+1,T} | \mathscr{G}_t]} \quad (21)$$

long positions in the futures contract to cover its exposure at time $T$. As for all delta hedging procedures, the correlation between the load-basis and the futures contract is not accounted for. There are many ways to estimate $\mathbb{E}[\mathscr{L}_T | \mathscr{G}_t]$, our reported results use our model outlined in Section 3.1.1. Many different models are used in the literature to compute the conditional variance and covariance terms in (21). We compute these quantities with the futures model (15)–(16) for consistency and refer to the approach as local minimal variance hedging (LMVH).

### *4.1.3 Static Hedging*

Since many papers are devoted to static hedging procedures, we include them in our study. To apply static hedging (STAH), the retailer identifies the solution to problem (8) under the constraint $\theta_{t_0+1} = \ldots = \theta_T$. We use the semi-quadratic penalty (9) and identify the optimal trading strategy through simulation.

## 4.2 Backtests

In all tests, the initial value of the portfolio $V_{t_0}$ is set to 0 and the weekly continuously compounded risk free rate is $r = 0.0193/52$.[30] The case of a retailer serving 1% of the Nord Pool load is considered.

### *4.2.1 In-sample backtest*

In this experiment, our global hedging and the benchmarks are applied to historical data during the 287 weeks over the January 29, 2007 to July 23, 2012 period. Hedging errors $\Psi_T - V_T$ are recorded at the end of week $T$ and the performance of the various approaches are compared using the root-mean- square error (RMSE), the semi-root-mean-square error (semi-RMSE) and tail value-at-risk (TVaR) metrics:

$$\text{RMSE} = \sqrt{\frac{1}{287}\sum_{T=1}^{287}(\Psi_T - V_T)^2}, \tag{22}$$

$$\text{semi-RMSE} = \sqrt{\frac{1}{287}\sum_{T=1}^{287}((\Psi_T - V_T)\mathbb{I}_{\{\Psi_T > V_T\}})^2}, \tag{23}$$

$$\text{TVaR}_\alpha = \frac{\sum_{T=1}^{287}(\Psi_T - V_T)\mathbb{I}_{\{\Psi_T - V_T \geq q_{(1-\alpha)}\}}}{\sum_{T=1}^{287}\mathbb{I}_{\{\Psi_T - V_T \geq q_{(1-\alpha)}\}}}. \tag{24}$$

where $\text{VaR}_\alpha = q_{(1-\alpha)}$ is the quantile[31] of level $1-\alpha$ of hedging errors $\Psi_T - V_T$. Results are reported in Table 9.

The main result is that the semi-quadratic SQDGH outperforms all other methods in terms of risk reduction. It reduces the semi-RMSE, the $\text{TVaR}_{5\%}$ and the $\text{TVAR}_{1\%}$ by 2,560€, 13,190€ and 33,800€, respectively (i.e. by 9.8%, 13.3% and 20.9% in relative measurement), with respect to IDH, the best non-global benchmark. Those improvements can be attributed to using global hedging procedures instead of delta hedging since both approaches share the same load model.[32] To put

30. The average EURO LIBOR rate over January 1, 2007 to July 29, 2012 period.

31. Let $x^{(1)},\ldots,x^{(n)}$ be the set of ordered hedging errors $\Psi_T - V_T$. Then, $q_{(1-\alpha)} = x^{(\lfloor \alpha n + 0.5 \rfloor)} + (\alpha n + 0.5 - \lfloor \alpha n + 0.5 \rfloor)(x^{(\lfloor \alpha n + 0.5 \rfloor + 1)} - x^{(\lfloor \alpha n + 0.5 \rfloor)})$.

32. The results are qualitatively the same when the correlation model in Section 3.3 is omitted and the correlation is set to zero so the differences cannot be attributed to the consideration of the load/futures dependence.

**Table 9: In-sample Backtest Results**

| Model | SQDGH | QDGH | IDH | DH | NDH | STAH | LMVH | NOH |
|---|---|---|---|---|---|---|---|---|
| Mean | 6.12 | 7.37 | 7.69 | 8.17 | 7.06 | 8.31 | −18.9 | −70.7 |
| RMSE | 26.16 | 26.04 | 27.36 | 31.37 | 35.03 | 27.74 | 274.8 | 474.1 |
| Semi-RMSE | 23.54 | 24.55 | 26.10 | 29.75 | 32.15 | 26.67 | 208.4 | 319.5 |
| $VaR_{5\%}$ | 49.76 | 53.46 | 54.53 | 60.56 | 73.85 | 52.77 | 414.0 | 688.7 |
| $VaR_{1\%}$ | 113.9 | 118.3 | 129.2 | 142.7 | 153.5 | 133.7 | 1206 | 1471 |
| $TVaR_{5\%}$ | 86.12 | 92.79 | 99.31 | 115.3 | 121.0 | 101.7 | 784.5 | 1201 |
| $TVAR_{1\%}$ | 128.1 | 134.5 | 161.9 | 172.9 | 180.9 | 152.2 | 1273 | 1831 |

*Notes*. Hedging error risk metrics for the in-sample backtest (in 1000€). Semi-quadratic dynamic global hedging (SQDGH), quadratic dynamic global hedging (QDGH), delta hedging (DH), improved delta hedging (IDH), naive delta hedging (NDH), static hedging (STAH), local minimal variance hedging (LMVH) and no hedging (NOH), i.e. $\theta_t^{(NOH)} = 0$ for all $t$. Historical data from January 29, 2007 to July 23, 2012.

these numbers in context, the mean weekly procurement costs of electricity (the average of $\mathscr{L}_T S_T$ for the January 2007 to August 2012 period) for the considered retailer is $2.35M$ €. Von der Fehr and Hansen (2010) identify a retail price mark-up over the wholesale price ranging between 4.4% and 10.8% for variable price contracts and between 7.2% and 13% for fixed price contracts in Norway. Using a 10% mark-up for ballpark calculations, this leaves the retailer with an average weekly margin of 235,000€ to cover expenses and profit; average profits will be a fraction of that amount. SQDGH reduces the 1% worst-scenarios average loss with respect to IDH by 33,800€, a substantial fraction of average profits.

Note that IDH benefits from our load-basis model (10)–(12). The added value of the latter model is isolated by comparing IDH with DH and NDH. The $TVAR_{1\%}$ is reduced from 180,900€ for the NDH to 172,900€ for the DH, and further reduced to 161,900€ for the IDH. This illustrates the importance of having an accurate load-basis model and the benefits provided by the model (10)-(12) in terms of risk reduction.

The combined reduction in $TVAR_{1\%}$ due to methodology presented in this paper obtained by comparing SQDGH and NDH is 52,800€, with a combined reduction in semi-RMSE of 8,610€.

It is also interesting that the mean hedging error is lower for SQDGH than for all other models except LVMH and no hedging (NOH). This indicates the risk reduction yielded by the SQDGH method is not obtained at the expense of a lesser profitability. The LMVH and NOH methods are the two most profitable on average, but they yield extremely poor results in terms of risk management. The poor performance of the LMVH method is explained by positions in the futures that are significantly too low. Indeed, since the cash-and-carry relationship of futures price and the spot price does not hold in this market, the correlation between spot price and futures price variations are much lower than in other markets. This reduces the $\theta^{(LMVH)}$ position and produces under-hedging. Because the Nord Pool electricity futures market is in contango,[33] under-hedging produces higher average profits than full hedging.

In terms of semi-RMSE, STAH underperforms IDH, QDGH and SQDGH, showing the benefits of dynamic hedging over a static procedure.

### *4.2.2 Out-of-sample backtest*

In this experiment, we carry out an out-of-sample test that replicates more realistic application conditions where future observations cannot be used to estimate state variable models. The

33. The average 3-weeks futures price is 7.7% higher than the arithmetic average spot price for the January 2007 to July 2012 period.

**Table 10: Out-of-sample Backtest Results**

| Model | SQDGH | QDGH | IDH | DH | NDH | STAH | LMVH | NOH |
|---|---|---|---|---|---|---|---|---|
| Mean | 13.21 | 13.15 | 13.68 | 14.82 | 11.29 | 15.35 | −0.60 | −59.9 |
| RMSE | 33.71 | 36.60 | 35.15 | 42.04 | 42.64 | 37.71 | 347.5 | 585.5 |
| Semi-RMSE | 31.45 | 34.01 | 34.19 | 40.10 | 40.15 | 36.77 | 275.4 | 409.0 |
| $\text{VaR}_{5\%}$ | 79.54 | 87.50 | 92.51 | 104.7 | 97.51 | 105.1 | 678.7 | 907.9 |
| $\text{VaR}_{1\%}$ | 112.5 | 133.4 | 121.5 | 150.7 | 155.5 | 131.5 | 1208 | 1633 |
| $\text{TVaR}_{5\%}$ | 103.9 | 113.6 | 116.3 | 137.0 | 134.9 | 122.6 | 1040 | 1483 |
| $\text{TVAR}_{1\%}$ | 114.1 | 142.5 | 150.0 | 166.1 | 168.8 | 145.1 | 1311 | 2005 |

*Notes*. Hedging error risk metrics for the out-of-sample backtest (in 1000€). Semi-quadratic dynamic global hedging (SQDGH), quadratic dynamic global hedging (QDGH), delta hedging (DH), improved delta hedging (IDH), naive delta hedging (NDH), static hedging (STAH), local minimal variance hedging (LMVH) and no hedging (NOH), i.e. $\theta_t^{(NOH)} = 0$ for all $t$. The three-week hedging procedure is applied weekly between December 28, 2009 and July 23, 2012.

three-week hedging procedure is applied weekly between December 28, 2009 and July 23, 2012. For each iteration of the test (an iteration corresponds to the starting point of the hedging procedure), all load and futures price models are estimated based on historical data from the three previous years (rolling-window estimation set). The hedging algorithm is then applied out-of-sample on the three weeks following the estimation set, and the terminal hedging error is recorded. There are 132 iterations in total. Descriptive statistics and risk metrics (22)–(24) applied to hedging errors are given in Table 10.

Once again, SQDGH outperforms all the benchmarks, reducing the semi-RMSE by 7.5%, the $\text{TVaR}_{5\%}$ by 8.5% and the $\text{TVAR}_{1\%}$ by 19.9% with respect to QDGH, the best competitor. The SQDGH is therefore the best hedging method among all the proposed methods in both the in-sample and the out-of-sample backtests.

### 4.3 Drivers of the SQDGH Hedge Ratio

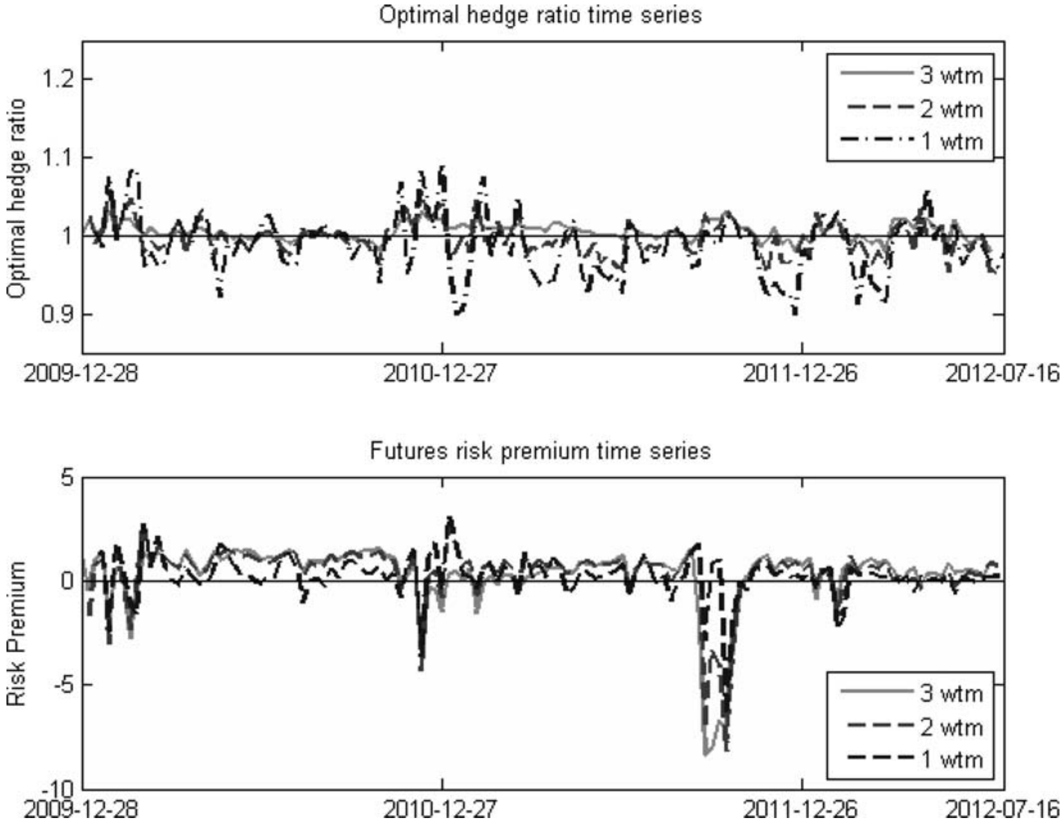
The hedge ratio is the proportion of futures contract shares to hold per unit of expected MWh:

$$\tilde{\theta}_{t+1} = \frac{\theta_{t+1}}{\mathrm{E}[\mathscr{L}_T | \mathscr{G}_t]}$$

Delta-hedging prescribes holding one futures contract per unit of the underlying asset, i.e. the delta-hedging hedge ratio is $e^{-r(T-t)}$, which is approximately one. However, in electricity markets, futures prices, futures volatilities, load-basis, current portfolio value and previous portfolio composition affect the optimal hedge ratio prescribed by our model in a complex manner. First, to gain some intuition on the drivers of the optimal hedge ratio, let us consider a simple one-period version of the hedging problem (instead of the three-period version in Section 2.3). In this case, the loss is

$$\begin{aligned}
\text{Loss}_T &= \Psi_T - V_T \\
&= \mathscr{L}_T(S_T - F_{T-1,T}) - \big((V_{T-1} - \mathscr{C}_{T-1})e^r + \tilde{\theta}_T \mathrm{E}[\mathscr{L}_T | \mathscr{G}_{T-1}](S_T - F_{T-1,T}) + \mathscr{C}_T\big) \\
&= \underbrace{(\mathscr{L}_T - \tilde{\theta}_T \mathrm{E}[\mathscr{L}_T | \mathscr{G}_{T-1}])}_{\text{term 1}}\underbrace{(S_T - F_{T-1,T})}_{\text{term2}} - V_{T-1}e^r + \underbrace{\mathscr{C}_{T-1}e^r + \mathscr{C}_T}_{\text{transaction costs}}.
\end{aligned}$$

**Figure 5: SQDGH Hedge Ratio and Futures Risk Premium Over Time**




*Notes*. Optimal hedge ratios and futures risk premium for the three-week hedging procedure applied weekly between December 28, 2009 and July 23, 2012; wtm = weeks to maturity.

Large losses may occur when terms 1 and 2 are simultaneously positive (type-a loss) or negative (type-b loss). Controlling type-a losses requires increasing the hedge ratio while managing type-b losses suggests decreasing the hedge ratio. Because the risk premium $F_{t,T} - \mathrm{E}[S_T | \mathscr{G}_t]$ is usually positive on the futures market, $F_{T-1,T}$ is usually larger than $\mathrm{E}_{T-1}[S_T | \mathscr{G}_{T-1}]$ (contango) and the term $(S_T - F_{T-1,T})$ tends to be negative, making type-b losses more likely. In this case, choosing $\tilde{\theta}_T < 1$ mitigates the probability of having the two components being simultaneously negative. Analogously, when the risk premium is negative, one would choose $\tilde{\theta}_T > 1$. This has the beneficial by-product that more (less) futures are bought when they are less (more) expensive.

When there is positive correlation between futures prices and load-basis, terms 1 and 2 are more likely to have the same sign and the probability of experiencing large losses increases. Because a higher correlation increases both the likelihood of type-a and type-b losses, it is unclear a priori if such higher correlation should positively or negatively impact the optimal hedge ratio. The optimal hedge ratio will depend on how the other state variables influence the likelihood of losses of type a or b. These influences are complex and non-linear, and a global optimization procedure is required to mitigate the largest losses.

We now go back to our three-period hedging problem and consider the results from our out-of-sample backtest. Figure 5 shows the optimal hedge ratio calculated under SQDGH (panel 1)

**Table 11: Explanatory Power of State Variables on SQDGH Optimal Hedge Ratio**

| | R-square (%) | | | Standardized slope (bsp) | | |
|---|---|---|---|---|---|---|
| | Weeks-to-maturity | | | Weeks-to-maturity | | |
| Explanatory variable | 1 | 2 | 3 | 1 | 2 | 3 |
| Futures premium risk | 35.2 | 34.2 | 24.0 | −192 | −129 | −64 |
| Load-basis Futures Correlation | 9.6 | 1.1 | 0.0 | −4 | 23 | 34 |

*Notes*. Results are for simple linear regression. Optimal hedge ratio is regressed on each (contemporaneous) explanatory variable. Futures risk premium is obtained through our model ex-ante. The standardized slope is the slope multiplied by the sample standard deviation of the explanatory variable and can be interpreted as the average impact, measured in basis points (bsp) on the optimal hedge ratio of a one standard deviation increase in the explanatory variable. Optimal hedge ratios for the three-week hedging procedure applied weekly between December 28, 2009 and July 23, 2012.

and the futures risk premium (panel 2). The first panel shows that the volatility of the optimal hedge ratio is (i) more important during the winter season and (ii) decreases with the time to maturity. Given that there is a certain dependence between panels 1 and 2, it is then a legitimate question to ask whether the futures risk premium is a good predictor of the optimal hedge ratio.

Table 11 shows simple linear regression results where the realized SQDGH optimal hedge ratio is regressed on the futures risk premium. A simple rule-of-thumb hedging decision based only on the futures risk premium, i.e. changing the hedge ratio in a linear way with respect to the futures risk premium, is clearly suboptimal as the latter only explains 24% to 35% of the variation in the optimal hedge ratio. The average impact on the optimal hedge ratio of a one standard deviation increase in the risk premium is $-64$ to $-192$ basis points.

The inadequacy of the load-basis and futures prices correlation as a simple predictor of the optimal hedge ratio is also confirmed by the very small $R^2$ values in Table 11. In summary, the explanatory variables play on the hedging decision, but not in a simple way and there is no easy-to-compute hedging decision rule.

## 5. CONCLUSION

A dynamic global hedging methodology involving futures contracts is developed to allow retailers to cover their exposure to price and load risk. Global hedging procedures have received less attention in the electricity markets literature because they often yield solutions which are computationally more complex than their local counterparts. We show that the approach is not only feasible but allows us to account for load uncertainty, basis risk and transaction costs when seeking the optimal trading strategy.

Statistical models were proposed for the load to be served by the retailer, the electricity spot price and futures contract prices on the Nord Pool market. Those models were built from weekly historical data and reproduce their stylized facts. The load-basis model accounts for seasonality in the mean and the variance, as well as autocorrelation in seasonally corrected shocks. The proposed model for futures price returns, a multivariate AR(1)-GARCH with NIG innovations, exhibits stochastic volatility, partially predictable returns and fat tails. Dependence between load-basis and futures innovations is also modeled. Multiple goodness-of-fit tests validate the adequacy of all models developed.

Backtests using historical market data show the superiority of the semi-quadratic global hedging procedure compared to various benchmarks of the literature in terms of risk reduction.

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