# DISCOUNTING AND EQUITY CONSIDERATIONS IN ENERGY-CLIMATE POLICY MODELING

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## (1) Overview

Most energy-climate policy assessment models including "integrated assessment models" are based on the Ramsey model. Its basic structure formulates the economic dynamics as an optimization model. Its objective function describes social welfare that is defined as a weighted sum of instantaneous utilities of consumption stream. On one hand, the weights are called discount factors, the feature of which is characterized by social time preference,  $\rho$ . On the other hand, a typical instantaneous utility function has the form of CRRA, constant relative risk aversion, with the parameter of the elasticity of marginal utility,  $\eta$ . These two features indicate that it is a pair of parameters ( $\eta$ ,  $\rho$ ) that determines one of the most important parts of energy-climate policy modeling.

Needless to say, there is a long list of studies in the economics literature that contributed to theoretical foundations regarding  $\eta$  and  $\rho$ . Thanks to their contributions, the rationale of utilizing these parameters has been well established as an economic theory. On the other hand, in practices of the policy modeling community, setting actual values for these parameters is not an easy task: It is hard to say that there is a well-established methodology or procedure for it so far. It is because parameter setting must be consistent to the real economic data as well as ethical or philosophical considerations regarding equity among economic agents. In particular, climate change has an exceptionally long time horizon, and thus relates to intergenerational equity issues to which the setting of social time preference is crucial. This fact has led to a lot of controversies among climate change policy analysts. According to Dasgupta (2008), the model of Cline (1992) used the pair of parameters as  $(\eta, \rho) = (1.5, 0\%)$  while Nordhaus (1994) used (1, 3%). In the well-known study of Stern (2006), it was set as (1, 0%), however. These differences in parameter values in fact are considered to be one of primal sources of different calculation results out of their models.

Out of the two parameters, adopting specific numbers for  $\rho$  is much more controversial than for  $\eta$  in policy modeling practices. The role of  $\rho$  is to determine the weights for the stream of instantaneous utilities of generations, and thus is closely related to ethical considerations regarding intergenerational equity. At the same time, it is related to market interest rates (or capital rents) in theory. As a result, when one chooses a specific number for  $\rho$ , (s)he sometimes faces a puzzle that is not easy to solve in light of ethical considerations, consistency with actual data, and other factors to be considered in economic theory. In the recent literature, providing a survey of related studies, Dasgupta (2008), cited above, illustrates that there are various constraints in setting the value of  $\rho$  for a fixed  $\eta$ .

In the context of climate policy modeling, while the role of  $\rho$  in models has drawn many researchers' attention, the role of  $\eta$  does not seem to have been treated in a similar manner so far. As is mentioned above, however,  $\eta$  being together with  $\rho$  determines the representation of social welfare function. Thus, the role might carry the same importance with that of  $\rho$ . Emphasizing this point, Buchholz and Schumacher (2010) examined the interchangeability between  $\eta$ and  $\rho$ . In particular, they indicated that the role of  $\rho$  as a determinant of intergenerational equity can be replaced by the adjustment of  $\eta$  to some extent. The study of Buchholz and Schumacher definitely contributed to a rich understanding of the relationship between intergenerational equity and social time preference. On the other hand, however, it failed to examine the role of  $\eta$  itself in contrast to that of  $\rho$  because it only focused to  $\eta$ 's supplemental role to  $\rho$ .

The purpose of this paper is to examine the interchangeability between  $\eta$  and  $\rho$  to explore insights regarding intra- and intergenerational equities as well as implications to parameter setting techniques in energy-climate policy modeling.

#### (2) Methods

To focus on theoretical aspects of the combination of  $\eta$  and  $\rho$  and its implications to energy-climate policy modeling, we confine our analysis to the Ramsey model structure in which the social welfare function is defined as:

$$\int_{0}^{\infty} u(c_{t}) L_{t} e^{-\rho t} dt \text{ where } L_{t} = e^{nt} \text{ and } u(c) = \frac{1}{1 - \eta} c^{1 - \eta}$$

We consider perturbations of  $\eta$  and  $\rho$  by introducing the following functional:

$$W\left(\left\{c_{t}\right\}_{t=0}^{\infty},\eta,\rho,\Delta\eta,\left\{\delta\rho_{t}\right\}_{t=0}^{\infty}\right)\equiv\int_{0}^{\infty}c_{t}^{1-(\eta+\Delta\eta)}L_{t}e^{-\int_{0}^{t}(\rho+\delta\rho_{\tau})d\tau}dt$$

An optimal consumption path is described as follows:

$$\left\{c_{t}^{*}\left(\eta,\rho,\Delta\eta,\left\{\delta\rho_{t}\right\}_{t=0}^{\infty}\right)\right\}_{t=0}^{\infty}\equiv\arg\max_{\left\{c_{t}\right\}_{t=0}^{\infty}\in\mathcal{F}}W\left(\left\{c_{t}\right\}_{t=0}^{\infty},\eta,\rho,\Delta\eta,\left\{\delta\rho_{t}\right\}_{t=0}^{\infty}\right)$$

where  $\mathcal{F}$  represents a feasible set of consumption path and the superscript \* indicates optimal solutions. Then, interchangeability is defined as follows:

Definition (Interchangeability between  $\eta$  and  $\rho$ ). If there exists a pair of  $\Delta \eta (\neq 0)$  and  $\{\delta \rho_t\}_{t=0}^{\infty} (\neq \{0\}_{t=0}^{\infty})$  that satisfies

$$\arg\max_{\{c_t\}\in\mathcal{F}} W\left(\left\{c_t\right\}_{t=0}^{\infty}, \eta, \rho, 0, \left\{0\right\}_{t=0}^{\infty}\right) = \arg\max_{\{c_t\}\in\mathcal{F}} W\left(\left\{c_t\right\}_{t=0}^{\infty}, \eta, \rho, \Delta\eta, \left\{\delta\rho_t\right\}_{t=0}^{\infty}\right)$$

then it is said that  $\eta$  is interchangeable with time-varying social time preference ( $\rho_t \equiv \rho + \delta \rho_t$ ). Moreover, if  $\{\delta \rho_t\}_{t=0}^{\infty}$  is constant in time, i.e.,  $\delta \rho_t = \delta \rho \,\forall t$ , then it is said that  $\eta$  and  $\rho$  are completely interchangeable with each other.

#### (3) Results

Main propositions obtained are the following:

Proposition 1. If per-capita consumption keeps rising or declining in equilibrium,  $\eta$  is interchangeable with timevarying social time preference. If per-capita consumption stays constant in equilibrium, then  $\eta$  is never interchangeable with any form of social time preference.

Proposition 2.  $\eta$  and  $\rho$  are completely interchangeable with each other if and only if per-capita consumption rises at a constant rate in equilibrium.

*Corollary 1. With a one-sector production function,*  $\eta$  *and*  $\rho$  *are completely interchangeable with each other if and only if the production function is reduced to be the AK model.* 

Corollary 2. Perturbation of  $\eta$  can be cancelled out by adjustments of the degree of exponential discounting if and only if per-capita consumption rises at a constant rate in equilibrium.

Numerical examples to illustrate these results are provided in the paper.

## (4) Conclusions

This paper examined the interchangeability between the coefficient of relative inequality-aversion  $\eta$  and social time preference  $\rho$  to explore insights regarding intra- and intergenerational equities as well as implications to parameter setting techniques in climate policy modeling practices. The pair of parameters  $(\eta, \rho)$  is a key determinant of characterizing objective functions in Ramsey-type models. When we adopt specific values for these two parameters, we must be sure that they are closely related to each other in light of ethical considerations as well as consistency to actual economic data. However, past studies in the economics literature focus on these two parameters separately, and have never treated them together. This perception motivated us to launch our study.

As was shown in Proposition 2, the necessary and sufficient conditions for these two parameters to possess the characteristics of complete interchangeability are that per-capita consumption rises at a constant rate in equilibrium. According to Corollary 1, the condition is equivalent to the one that the production function is the AK model given that the economy is assumed to consist of one-sector production. With such condition, the pair of values for  $(\eta, \rho)$  are not unique: for example, for a given pair of values, we will be able to obtain a new pair such as  $(\eta + \Delta \eta, 0)$  with which equilibrium path of economic growth does not change. It is underlined that given  $r \equiv A - d = 0.08 \sim 0.10$ , parameter pairs used in Nordhous (1994) and Cline (1992) are almost equivalent to each other; in particular, when  $r \equiv A - d = 0.09$ , the relation is exact.

Another important implication of these propositions and corollaries is connected to the properties of discounting factors. If the complete interchangeability holds, even if we make an adjustments of  $(\eta, \rho)$ , we can keep exponential discounting. Otherwise, adjustments require an introduction of general hyperbolic discounting or endogenous discounting so as to have the same equilibrium. Findings obtained here illustrated a new perspective of policy modeling.

#### References

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