APPENDIX

INVESTMENT WITHOUT POLICY UNCERTAINTY

The value of the option to invest in the presence ($\zeta = 1$) or absence ($\zeta = 0$) of a subsidy is described in (A–1).

$$F_{\epsilon,0,0}(E) = \begin{cases} (1 - \rho dt)E \left[ F_{\epsilon,0}^{(j)}(E + dE) \right], & E < \epsilon_{\epsilon,0,0}^{(j)} \\ \frac{E k_{\epsilon,0,0}^{(j)}(1 + y_{1}(\zeta = 1))}{\rho - \mu} - I \left( k_{\epsilon,0,0}^{(j)} \right), & E \geq \epsilon_{\epsilon,0,0}^{(j)} \end{cases} \quad \text{(A–1)}$$

By expanding the first branch on the right–hand side of (A–1) using Itô’s lemma, we obtain the differential equation for $F_{\epsilon,0,0}^{(j)}(E)$, which, together with its solution for $E < \epsilon_{\epsilon,0,0}^{(j)}$, is indicated in (A–2).

$$\frac{1}{2} \sigma^2 E F_{\epsilon,0,0}^{(j)''}(E) + \mu F_{\epsilon,0,0}^{(j)'}(E) - \rho F_{\epsilon,0,0}^{(j)}(E) = 0 \quad \Rightarrow \quad F_{\epsilon,0,0}^{(j)}(E) = A_{\epsilon,0,0}^{(j)} E^{B_1} + C_{\epsilon,0,0}^{(j)} E^{B_2} \quad \text{(A–2)}$$

Note that the value of the project becomes very small as $E \to 0$. Since $\beta_2 < 0$, we have $E \to 0 \Rightarrow C_{\epsilon,0,0}^{(j)} E^{B_2} \to \infty$. Consequently, $C_{\epsilon,0,0}^{(j)} = 0$, and, thus, the expression for $F_{\epsilon,0,0}^{(j)}(E)$ is indicated in (13). By applying value–matching and smooth–pasting conditions between the two branches of (13), we obtain the expression for the endogenous constant, $A_{\epsilon,0,0}^{(j)}$, and the optimal investment threshold, $\epsilon_{\epsilon,0,0}^{(j)}$.

$$A_{\epsilon,0,0}^{(j)} = \frac{1}{\epsilon_{\epsilon,0,0}^{(j)}} \left[ \frac{\epsilon_{\epsilon,0,0}^{(j)} k_{\epsilon,0,0}^{(j)} (1 + y_{1}(\zeta = 1))}{\rho - \mu} - I \left( k_{\epsilon,0,0}^{(j)} \right) \right] \quad \text{(A–3)}$$

$$\epsilon_{\epsilon,0,0}^{(j)} (k_{\epsilon,0,0}^{(j)}) = \frac{I (k_{\epsilon,0,0}^{(j)})}{k_{\epsilon,0,0}^{(j)} (1 + y_{1}(\zeta = 1))} \frac{\beta_1 (\rho - \mu)}{\beta_1 - 1} \quad \text{(A–4)}$$

Finally, by inserting (A–4) into (11) we obtain the expression for the optimal capacity.

$$k_{\epsilon,0,0}^{(j)} = \left[ \frac{a_j}{b_j \gamma_j (\beta_1 - 1) - \beta_1} \right]^{\frac{1}{\gamma_j - 1}}, \quad \gamma_j (\beta_1 - 1) - \beta_1 > 0 \quad \text{(A–5)}$$

Moreover, from (A–5) we see that the existence of an optimal solution to the investment problem requires that the cost function is strictly convex, i.e., $\gamma_j (\beta_1 - 1) - \beta_1 > 0 \Leftrightarrow \gamma_j > \frac{\beta_1}{\beta_1 - 1} > 1$.

**Proposition 1** $\epsilon_{1,0,0}^{(j)} < \epsilon_{0,0,0}^{(i)}$ and $k_{1,0,0}^{(j)} = k_{0,0,0}^{(j)}$.

**Proof:** In the presence of a subsidy, the value of the option to invest is indicated in (13). The value–matching and
smooth–pasting conditions between the two branched of (13) are:

\[
A_{1,0,0}^{(j)} - \frac{\epsilon_{0,0,0}^{(j)} k_{1,0,0}^{(j)} (1 + y)}{\rho - \mu} - l \left( k_{1,0,0}^{(j)} \right) = 0
\]  \hspace{1cm} (A–6)

\[
\beta_1 A_{1,0,0}^{(j)} - \frac{\epsilon_{0,0,0}^{(j)} k_{1,0,0}^{(j)} (1 + y)}{\rho - \mu} + \frac{\epsilon_{0,0,0}^{(j)} k_{1,0,0}^{(j)} (1 + y)}{\rho - \mu} - a_j k_{1,m,n}^{(j)} - \gamma_j b_j k_{1,m,n}^{(j)} k_{1,m,n}^{(j)} = 0
\]  \hspace{1cm} (A–7)

The expression for the endogenous constant, \( A_{1,0,0}^{(j)} \), and the optimal investment threshold, \( \epsilon_{1,0,0}^{(j)} \), is indicated in (A–8).

\[
A_{1,0,0}^{(j)} = \frac{\epsilon_{1,0,0}^{(j)} - \beta_1}{\beta_1 (\rho - \mu)} \frac{k_{1,0,0}^{(j)} (1 + y)}{\beta_1 (\rho - \mu)} \quad \text{and} \quad \epsilon_{1,0,0}^{(j)} = \frac{(\rho - \mu) l \left( k_{1,0,0}^{(j)} \right)}{\beta_1} - 1
\]  \hspace{1cm} (A–8)

Notice that \( \epsilon_{1,0,0}^{(j)} = \epsilon_{0,0,0}^{(j)} / (1 + y) \) and that by inserting the expression for \( \epsilon_{1,0,0}^{(j)} \) into (11) we obtain:

\[
k_{1,0,0}^{(j)} \left( \epsilon_{1,0,0}^{(j)} \right) = \left[ \frac{1}{b_j \gamma_j} \left( \frac{\epsilon_{0,0,0}^{(j)} (1 + y)}{\rho - \mu} - a_j \right) \right]^{\gamma_j} = k_{0,0,0}^{(j)} \left( \epsilon_{0,0,0}^{(j)} \right)
\]  \hspace{1cm} (A–9)

\section*{INVESTMENT UNDER A RETRACTABLE SUBSIDY}

First, we determine the expected value of the active project in the presence of a retractable subsidy. Notice that, within an infinitesimal time interval \( dt \), either the subsidy will be retracted with probability \( \lambda dt \) and the instantaneous revenue will decrease by \( E K_{1,1,0}^{(j)} y \), or no policy intervention will take place with probability \( 1 - \lambda dt \) and the reduction in the instantaneous revenue will be zero. Consequently, the expected reduction in the instantaneous revenue over a small time interval \( dt \) is \( \lambda E K_{1,1,0}^{(j)} y dt \) and the expected present value of this reduction is \( \frac{\lambda E K_{1,1,0}^{(j)} y}{\rho - \mu} \). By subtracting this from the expected revenues under a permanent subsidy, \( \frac{E K_{1,1,0}^{(j)} (1 + y)}{\rho - \mu} \), we obtain the expected value of the revenues under sudden subsidy retraction, i.e., \( \frac{E K_{1,1,0}^{(j)} [1 + (1 - \lambda) y]}{\rho - \mu} \).

\section*{Proposition 2}

\( \lambda \geq 0 \Rightarrow \epsilon_{1,1,0}^{(j)} \leq \epsilon_{0,0,0}^{(j)} \) and \( k_{1,1,0}^{(j)} \leq k_{0,0,0}^{(j)} \) while, for low values of \( \lambda \), \( \epsilon_{1,1,0}^{(j)} \leq \epsilon_{1,0,0}^{(j)} \).

\textbf{Proof:} The value of the option to invest in the presence of a retractable subsidy is indicated in (17). Notice that \( \lambda = 0 \Rightarrow F_{1,1,0}^{(j)} (E) = F_{1,0,0}^{(j)} (E) \), and, therefore, \( \epsilon_{1,1,0}^{(j)} = \epsilon_{1,0,0}^{(j)} \leq \epsilon_{0,0,0}^{(j)} \) and \( k_{1,1,0}^{(j)} = k_{1,0,0}^{(j)} = k_{0,0,0}^{(j)} \). From (15), we know that \( \lambda \uparrow \Rightarrow k_{1,1,0}^{(j)} \downarrow \gamma_j \), which implies that a higher \( \lambda \) lowers the expected value of the project, and, in turn, both the amount of installed capacity and the optimal investment threshold. Hence, for small values of \( \lambda \), \( \epsilon_{1,1,0}^{(j)} \leq \epsilon_{1,0,0}^{(j)} \), whereas \( \lambda \rightarrow 1 \Rightarrow \epsilon_{1,1,0}^{(j)} \rightarrow \epsilon_{1,0,0}^{(j)} \).
Proposition 3 \[ \frac{F^{(j)}_{1,0,0}(E) - F^{(j)}_{1,1,0}(E)}{F^{(j)}_{1,1,0}(E)} \in \left(0, \frac{A^{(j)}_{1,0,0} - A^{(j)}_{1,1,0}}{A^{(j)}_{1,0,0}}\right) \]

Proof: In the presence of a retractable subsidy, the value of the option to invest is:

\[ F^{(j)}_{1,1,0}(E) = A^{(j)}_{1,0,0}E^{B_1} + B^{(j)}_{1,1,0}E^{B_1}, \quad E < \varepsilon^{(j)}_{1,1,0} \]  

(A-10)

If \( \lambda = 0 \), then the subsidy will never be retracted. This implies that \( F^{(j)}_{1,1,0}(E) = F^{(j)}_{1,0,0}(E) \), and, in turn, that the relative loss in option value is zero. By contrast, as \( \lambda \) increases, the likelihood of subsidy retraction increases, and, as a result, \( B^{(j)}_{1,1,0}E^{B_1} \to 0 \), and, in turn, \( F^{(j)}_{1,1,0}(E) \to A^{(j)}_{1,0,0}E^{B_1} \), which implies that the relative loss in option value is \( \frac{A^{(j)}_{1,0,0} - A^{(j)}_{1,1,0}}{A^{(j)}_{1,0,0}} \). 

\[
\text{INVESTMENT UNDER SUDDEN PROVISION OF A PERMANENT SUBSIDY}
\]

The extra instantaneous revenue from subsidy provision is \( EK^{(j)}_{0,1,y} \) and will be realised with probability \( \lambda dt \), whereas with probability \( 1 - \lambda dt \), no subsidy will be provided. Hence, the expected value of the subsidy is \( \lambda EK^{(j)}_{0,1,y} dt \) and its expected present value is \( \frac{\lambda EK^{(j)}_{0,1,y}}{\rho - \mu} \). Consequently, the expected value of the revenues under sudden provision of a permanent subsidy consist of the expected revenues without the subsidy, \( \frac{E K^{(j)}_{0,1}}{\rho - \mu} \), and the extra revenues due to the subsidy, \( \frac{\lambda E K^{(j)}_{0,1,y}}{\rho - \mu} \), i.e., \( \frac{E K^{(j)}_{0,1} + (1 + \lambda y)}{\rho - \mu} \).

Proposition 4 \( \lambda \geq 0 \Rightarrow \varepsilon^{(j)}_{0,0,1} \geq \varepsilon^{(j)}_{1,0,0} \) and \( k^{(j)}_{0,0,1} \geq k^{(j)}_{1,0,0} \), while, for low values of \( \lambda \), \( \varepsilon^{(j)}_{0,0,1} \geq \varepsilon^{(j)}_{0,0,0} \).

Proof: The value of the option to invest under sudden provision of a permanent subsidy is indicated in (21). Notice that \( \lambda = 0 \Rightarrow F^{(j)}_{0,0,1}(E) = F^{(j)}_{0,0,0}(E) \), and, therefore, \( \varepsilon^{(j)}_{0,0,1} = \varepsilon^{(j)}_{0,0,0} > \varepsilon^{(j)}_{1,0,0} \) and \( k^{(j)}_{0,0,1} = k^{(j)}_{0,0,0} = k^{(j)}_{1,0,0} \). As \( \lambda \) increases, the likelihood of subsidy provision increases, thereby raising the expected value of the project, and, in turn, the incentive to install greater capacity. Indeed, \( \lambda \searrow \Rightarrow \varepsilon^{(j)}_{0,0,1} \searrow \), and, therefore, at low values of \( \lambda \) we have \( \varepsilon^{(j)}_{0,0,1} \geq \varepsilon^{(j)}_{0,0,0} \). By contrast, at high values of \( \lambda \) it is very likely that the subsidy will be provided, and, therefore, \( \lambda \rightarrow 1 \Rightarrow \varepsilon^{(j)}_{0,0,1} \rightarrow \varepsilon^{(j)}_{1,0,0} \).

Proposition 5 \( F^{(j)}_{1,0,0}(E) - F^{(j)}_{0,0,1}(E) \in \left[ -\frac{B^{(j)}_{1,0,0}}{A^{(j)}_{1,0,0}}, 0 \right] \).

Proof: Under sudden provision of a permanent subsidy, the value of the option to invest is:

\[ F^{(j)}_{0,0,1}(E) = A^{(j)}_{1,0,0}E^{B_1} + B^{(j)}_{0,0,1}E^{B_1}, \quad E < \varepsilon^{(j)}_{0,0,0} \]  

(A-11)

If \( \lambda = 0 \), then the subsidy will never be provided. This implies that \( F^{(j)}_{0,0,1}(E) = (A^{(j)}_{1,0,0} + B^{(j)}_{0,0,1})E^{B_1} \), and, in turn, that the relative loss in option value is maximised. By contrast, as \( \lambda \) increases, the likelihood of subsidy provision increases, and, as a result, \( B^{(j)}_{0,0,1}E^{B_1} \to 0 \), and, in turn, \( F^{(j)}_{0,0,1}(E) \to A^{(j)}_{1,0,0}E^{B_1} \), which implies that the relative loss in option value is zero.
INVESTMENT UNDER INFINITE PROVISIONS AND RETRACTIONS

The dynamics of the value of the option to invest under infinite provision and retractions for \( \zeta = 0, 1 \), are described in (A–12) and (A–13), respectively.

\[
F^{(i)}_{0,\infty,m}(E) = (1 - \rho dt) \left[ \lambda dt E \left[ F^{(i)}_{1,\infty,m}(E + dE) \right] + (1 - \lambda dt) E \left[ F^{(i)}_{0,\infty,m}(E + dE) \right] \right] \quad (A–12)
\]

\[
F^{(i)}_{1,\infty,m}(E) = (1 - \rho dt) \left[ \lambda dt E \left[ F^{(i)}_{1,\infty,m}(E + dE) \right] + (1 - \lambda dt) E \left[ F^{(i)}_{0,\infty,m}(E + dE) \right] \right] \quad (A–13)
\]

By expanding the right-hand side of (A–12) and (A–13) using Itô’s lemma, we have

\[
\frac{1}{2} \sigma^2 E^2 F^{(i)''}_{0,\infty,m}(E) + \mu E F^{(i)'}_{0,\infty,m}(E) - (\lambda + \rho) F^{(i)}_{0,\infty,m}(E) + \lambda F^{(i)}_{1,\infty,m}(E) = 0 \quad (A–14)
\]

\[
\frac{1}{2} \sigma^2 E^2 F^{(i)''}_{1,\infty,m}(E) + \mu E F^{(i)'}_{1,\infty,m}(E) - (\lambda + \rho) F^{(i)}_{1,\infty,m}(E) + \lambda F^{(i)}_{0,\infty,m}(E) = 0 \quad (A–15)
\]

and by adding and subtracting (A–14) and (A–15) we obtain (A–16) and (A–17), respectively, where \( F_a(E) = F^{(i)}_{0,\infty,m}(E) + F^{(i)}_{1,\infty,m}(E) \) and \( F_b(E) = F^{(i)}_{1,\infty,m}(E) - F^{(i)}_{0,\infty,m}(E) \).

\[
\frac{1}{2} \sigma^2 E^2 F^{(i)'}_a(P) + \mu E F^{(i)}_a(E) - \rho F^{(i)}_a(E) = 0 \quad (A–16)
\]

\[
\frac{1}{2} \sigma^2 E^2 F^{(i)'}_b(P) + \mu E F^{(i)}_b(E) - (\rho + 2\lambda) F^{(i)}_b(E) = 0 \quad (A–17)
\]

The solution to (A–16) and (A–17) can be obtained by setting \( F_a(E) = A^{(i)}_a E^{\beta_1} \) and \( F_b(E) = A^{(i)}_b E^{\beta_2} \). Thus, we obtain:

\[
F^{(i)}_{0,\infty,m}(E) = \frac{1}{2} \left[ A^{(i)}_a E^{\beta_1} + A^{(i)}_b E^{\beta_2} \right] \quad (A–18)
\]

\[
F^{(i)}_{1,\infty,m}(E) = \frac{1}{2} \left[ A^{(i)}_a E^{\beta_1} - A^{(i)}_b E^{\beta_2} \right] \quad (A–19)
\]

where \( \eta \) is the positive root of the quadratic \( \frac{1}{2} \sigma^2 x(x - 1) + \mu x - (\rho + 2\lambda) = 0 \). □