Price Regulation and the Incentives to Pursue Energy Efficiency by Minimising Network Losses

Joisa Dutra[a], *Flavio M. Menezes*[b*], *and Xuemei Zheng*[b]

APPENDIX

Proof of Lemma 3

The associated Lagrangian function for price cap regulation is given by:

$$
\begin{aligned}
L^{pc} &= \frac{(a-P^{pc})^2}{2b} + (\gamma+\lambda^{pc})[\frac{P^{pc}(a-P^{pc})}{b} - \frac{(a-P^{pc})c}{b\Phi(E)} - E] \\
&= \frac{\Phi(a^2-2aP^{pc}+P^{pc2}) + 2(\gamma+\lambda^{pc})[(a\Phi+c)P^{pc} - \Phi P^{pc2} - ac]}{2b\Phi} - (\gamma+\lambda^{pc})E,
\end{aligned}
$$

where $\lambda^{pc} > 0$ is the Lagrangian multiplier. The Kuhn-Tucker conditions are

$$
\frac{\partial L^{pc}}{\partial P^{pc}} = \frac{(P^{pc}-a)\Phi + (\gamma+\lambda)(a\Phi+c-2\Phi P^{pc})}{b\Phi} \leqslant 0, P^{pc} \geqslant 0, \text{ and } P^{pc}\frac{\partial L^{pc}}{\partial P^{pc}} = 0,
$$

$$
\frac{\partial L^{pc}}{\partial \lambda} = \frac{(a-P^{pc})(P^{pc}\Phi-c)}{b\Phi} - E \geqslant 0, \lambda^{pc} \geqslant 0 \text{ and } \lambda^{pc}\frac{\partial L^{pc}}{\partial \lambda^{pc}} = 0,
$$

from which we identify three possible solutions:

$$
\begin{cases}
P_1^{pc*} = \frac{a\Phi+c}{2\Phi} - \frac{\sqrt{(a\Phi-c)^2-4b\Phi^2E}}{2\Phi} \\
\lambda_1^{pc} = \frac{a\Phi-c}{2\sqrt{(a\Phi-c)^2-4b\Phi^2E}} + \frac{1}{2} - \gamma
\end{cases}, \quad \text{(Eq. (A.1))}
$$

$$
\begin{cases}
P_2^{pc*} = \frac{a\Phi+c}{2\Phi} + \frac{\sqrt{(a\Phi-c)^2-4b\Phi^2E}}{2\Phi} \\
\lambda_2^{pc} = -\frac{a\Phi-c}{2\sqrt{(a\Phi-c)^2-4b\Phi^2E}} + \frac{1}{2} - \gamma
\end{cases},
$$

or

$$
\begin{cases}
P_3^{pc*} = \frac{(\gamma-1)a\Phi+\gamma c}{(2\gamma-1)\Phi} \\
\lambda_3^{pc} = 0
\end{cases}
$$

The participation constraint $\overline{\pi}^{pc}(P^{pc}, E) \geqslant 0$ implies that $P_1^{pc*} \leqslant P^{pc} \leqslant P_2^{pc*}$ with $P_1^{pc*} = \frac{a\Phi+c}{2\Phi} - \frac{\sqrt{(a\Phi-c)^2-4b\Phi^2E}}{2\Phi}$ and $P_2^{pc*} = \frac{a\Phi+c}{2\Phi} + \frac{\sqrt{(a\Phi-c)^2-4b\Phi^2E}}{2\Phi}$. We can readily check that P_3^{pc*} does not satisfy the participation constraint as:

$$
\overline{\pi}^{pc}(\overline{P}_3^{pc*}, E) = \frac{\gamma(\gamma-1)(a\Phi-c)^2}{(1-2\gamma)^2 b\Phi^2} - E < 0.
$$

[a]Centre for Regulation, FGV, Rio de Janeiro, Brazil.
[b]School of Economics, The University of Queensland, Brisbane, Australia.
[*]Corresponding author. Professor Flavio Menezes, School of Economics, The University of Queensland, Brisbane, QLD, 4072, Australia. Email: f.menezes@uq.edu.au.

To select between $P_1^{pc*}$and P_2^{pc*}, we compute the expected welfare associated with each price, W_1^{pc} and W_2^{pc}, respectively, as follows:

$$
\begin{aligned}
& W_1^{pc} - W_2^{pc} \\
&= \frac{(a - P_1^{pc*})[(a - P_1^{pc*})\Phi + 2\gamma(P_1^{pc*}\Phi - c)]}{2b\Phi} - \frac{(a - P_2^{pc*})[(a - P_2^{pc*})\Phi + 2\gamma(P_2^{pc*}\Phi - c)]}{2b\Phi} \\
&= \frac{(a\Phi - c)\sqrt{(a\Phi - c)^2 - 4b\Phi^2 E}}{2b\Phi^2} \geqslant 0 \text{ (from the assumption that } a\underline{\Phi} \geqslant \frac{3c}{2})
\end{aligned}
$$

It follows that the optimal price cap is $P^{pc*} = \frac{a\Phi+c}{2\Phi} - \frac{\sqrt{(a\Phi-c)^2-4b\Phi^2 E}}{2\Phi}$.

Proof of Lemma 4

Under revenue-cap regulation, the Lagrangian function of the welfare maximization problem could be written as

$$L^{rc} = \frac{(a - \sqrt{a^2 - 4bR^{rc}})^2}{8b} + (\lambda^{rc} + \gamma)(R^{rc} - \frac{c(a - \sqrt{a^2 - 4bR^{rc}})}{2b\Phi(E)} - E),$$

and the Kuhn-Tucker conditions (KTCs) are

$$\frac{\partial L^{rc}}{\partial R^{rc}} \leqslant 0,\ R^{rc} \geqslant 0,\ \text{and } R^{rc}\frac{\partial L^{rc}}{\partial R^{rc}} = 0,$$

$$\frac{\partial L^{rc}}{\partial \lambda^{rc}} = R^{rc} - \frac{c(a - \sqrt{a^2 - 4bR^{rc}})}{2b\Phi} - E \geqslant 0, \lambda^{rc} \geqslant 0 \text{ and } \lambda^{rc}\ \frac{\partial L^{rc}}{\partial \lambda^{rc}} = 0.$$

From the KTCs we get three sets of solution, i.e.,

$$\begin{cases} R_1^{rc*} = \frac{-c^2+ac\Phi+2b\Phi^2 E-c\sqrt{(a\Phi-c)^2-4b\Phi^2 E}}{2b\Phi^2} \\ \lambda_1^{rc} = \frac{a\Phi-2\gamma c+(2\gamma-1)\Phi\sqrt{a^2-4bR_1^{rc}}}{2(c-\Phi\sqrt{a^2-4bR_1^{rc}})} \end{cases},$$

$$\begin{cases} R_2^{rc*} = \frac{-c^2+ac\Phi+2b\Phi^2 E+c\sqrt{(a\Phi-c)^2-4b\Phi^2 E}}{2b\Phi^2} \\ \lambda_2^{rc} = \frac{a\Phi-2\gamma c+(2\gamma-1)\Phi\sqrt{a^2-4bR_2^{rc}}}{2(c-\Phi\sqrt{a^2-4bR_2^{rc}})} \end{cases},$$

or

$$\begin{cases} R_3^{rc*} = \frac{\gamma(a\Phi-c)(c\gamma+\gamma a\Phi-a\Phi)}{b(1-2\gamma)^2\Phi^2} \\ \lambda_3^{rc} = 0 \end{cases}.$$

Given the above, we obtain that the monopolist's expected profit with R_3^{rc} is equal to $\overline{\pi}_3^{rc} < 0$. Hence, the third potential solution does not satisfy the participation constraint $\pi^{rc} \geqslant 0$ and, as a result, we only need to consider R_1^{rc} and R_2^{rc} when identifying the socially optimal revenue cap. Denote the social welfare associated to R_1^{rc} and R_2^{rc} by W_1^{rc} and W_2^{rc}, respectively. Then

$$W_1^{rc} = \frac{(a\Phi - c)^2 - 2b\Phi^2 E - (a\Phi - c)\sqrt{(a\Phi - c)^2 - 4b\Phi^2 E}}{4b\Phi^2},$$

$$W_2^{rc} = \frac{(a\Phi - c)^2 - 2b\Phi^2 E + (a\Phi - c)\sqrt{(a\Phi - c)^2 - 4b\Phi^2 E}}{4b\Phi^2}.$$

As the expected welfare difference between W_1^{rc} and W_2^{rc} is

$$W_2^{rc} - W_1^{rc} = \frac{(a\Phi - c)\sqrt{(a\Phi - c)^2 - 4b\Phi^2 E]}}{2b\Phi^2} > 0,$$

the optimal revenue cap is $R^{rc*} = R_2^{rc} = \frac{-c^2+ac\Phi+2b\Phi^2E+c\sqrt{(a\Phi-c)^2-4b\Phi^2E}}{2b\Phi^2}$.

Note that the condition $R^{rc*} < R^*$ is satisfied. As

$$(a\Phi - c)^2 - 4b\Phi^2 E > 0$$

$$\begin{aligned}
&\Rightarrow (a\Phi - c)^2 - 4b\Phi^2 E + 2c\sqrt{(a\Phi - c)^2 - 4b\Phi^2 E} > 0\\
&\Rightarrow -2c^2 + 2ac\Phi + 4b\Phi^2 E - 2c\sqrt{(a\Phi - c)^2 - 4b\Phi^2 E} < a^2\Phi^2 - c^2\\
&\Rightarrow \frac{-c^2 + ac\Phi + 2b\Phi^2 E + c\sqrt{(a\Phi - c)^2 - 4b\Phi^2 E}}{2b\Phi^2} < \frac{(a\Phi + c)(a\Phi - c)}{4b\Phi^2}\\
&\Rightarrow R^{rc*} < R^*
\end{aligned}$$

Proof of Proposition 4

A) Consider $\delta \leqslant \widetilde{\delta}$

From Lemma 5, if $\delta \leqslant \widetilde{\delta}$, the threshold values of effort cost under different regulatory circumstances satisfy $\widetilde{e}_1 < \widetilde{e}_4 < \widetilde{e}_2$. In the following analysis, we will consider the four possible situations, i.e., $e \geqslant \widetilde{e}_2$, $\widetilde{e}_4 \leqslant e < \widetilde{e}_2$, $\widetilde{e}_1 \leqslant e < \widetilde{e}_4$ and $0 < e < \widetilde{e}_1$.

- Case A.1: When $e \geqslant \widetilde{e}_2$, the monopolists chooses to exert no effort under all possible scenarios. The resulting expected social welfare under different regulatory circumstances are given by:

$$W_l^* = W^*(E = 0) = \frac{(2\gamma + 1)(a\Phi_l - c)^2}{8b\Phi_l^2},$$

$$W^{ror*} = W^{ror*}(E = 0) = \frac{\nu(a\underline{\Phi} - c)^2}{2b\underline{\Phi}^2} + \frac{(1 - \nu)(a\overline{\Phi} - c)^2}{2b\overline{\Phi}^2},$$

$$W_l^{pc*} = W^{pc*}(E = 0) = \frac{(a\Phi_l - c)^2}{2b\Phi_l^2}, \text{ and}$$

$$W_l^{mt*} = W_l^{mt*}(E = 0) = \frac{(2\gamma + 1)(a\Phi_l - c)^2}{8b\Phi_l^2} + (1 - \gamma)\nu\delta.$$

It follows that:

$$\begin{aligned}
&W_l^{pc*} - W^{ror*}\\
&= \frac{(a\Phi_l - c)^2}{2b\Phi_l^2} - \frac{\nu(a\underline{\Phi} - c)^2}{2b\underline{\Phi}^2} - \frac{(1 - \nu)(a\overline{\Phi} - c)^2}{2b\overline{\Phi}^2}\\
&= \frac{-c^2\nu(1 - \nu)(\overline{\Phi} - \underline{\Phi})^2}{2b\underline{\Phi}^2\overline{\Phi}^2} < 0 \Rightarrow W_l^{pc*} < W^{ror*}.
\end{aligned}$$

Moreover,

$$W_l^{mt*} - W_l^* = (1-\gamma)\nu\delta > 0$$

$$\begin{aligned}
&W_l^{pc*} - W_l^{mt*} \\
&= \frac{(a\Phi_l - c)^2}{2b\Phi_l^2} - \frac{(2\gamma+1)(a\Phi_l - c)^2}{8b\Phi_l^2} - (1-\gamma)\nu\delta \\
&> \frac{(a\Phi_l - c)^2}{2b\Phi_l^2} - \frac{(2\gamma+1)(a\Phi_l - c)^2}{8b\Phi_l^2} - \frac{(1-\gamma)\nu(a\Phi_l - c)^2}{4b\nu\Phi_l^2} \\
&= \frac{(a\Phi_l - c)^2}{8b\Phi_l^2} > 0
\end{aligned}$$

In summary, if the cost of effort is sufficiently high that zero effort is chosen under all different regimes, then the following ranking holds:

$$W^{ror*} > W_l^{pc*} > W_l^{mt*} > W_l^*.$$

- Case A.2: When $\widetilde{e}_4 \leqslant e < \widetilde{e}_2$, the monopolists only exerts positive effort under price cap regulation. In this case, the expected social welfare under each regime is:

$$W_l^* = W^*(E=0) = \frac{(2\gamma+1)(a\Phi_l - c)^2}{8b\Phi_l^2},$$

$$W^{ror*} = W^{ror*}(E=0) = \frac{\nu(a\underline{\Phi} - c)^2}{2b\underline{\Phi}^2} + \frac{(1-\nu)(a\overline{\Phi} - c)^2}{2b\overline{\Phi}^2},$$

$$W_h^{pc*} = W^{pc*}(E=e) = \frac{(a\Phi_h - c)^2 + (a\Phi_h - c)\sqrt{(a\Phi_h - c)^2 - 4b\Phi_h^2 e}}{4b\Phi_h^2} - \frac{e}{2}, \text{ and}$$

$$W_l^{mt*} = W_l^{mt*}(E=0) = \frac{(2\gamma+1)(a\Phi_l - c)^2}{8b\Phi_l^2} + (1-\gamma)\nu\delta.$$

The comparison between W_l^*, W^{ror*} and W_l^{mt*} is the same in case A.1. Therefore, it suffices to compare W_h^{pc*} to W_l^* , W^{ror*} and W_l^{mt*}.

As

$$\begin{aligned}
\frac{\partial W_h^{pc*}}{\partial e} &= \frac{(a\Phi_h - c)}{4b\Phi_h^2}\frac{-4b\Phi_h^2}{2\sqrt{(a\Phi_h - c)^2 - 4b\Phi_h^2 e}} - \frac{1}{2} \\
&= \frac{-(a\Phi_h - c)}{2\sqrt{(a\Phi_h - c)^2 - 4b\Phi_h^2 e}} - \frac{1}{2} < 0,
\end{aligned}$$

we have

$$\begin{aligned} &W_h^{pc*} - W_l^{pc*} \\ &= \frac{(a\Phi_h - c)^2 + (a\Phi_h - c)\sqrt{(a\Phi_h - c)^2 - 4b\Phi_h^2 e}}{4b\Phi_h^2} - \frac{e}{2} - \frac{(a\Phi_l - c)^2}{2b\Phi_l^2} \\ &> \frac{(a\Phi_h - c)^2 + (a\Phi_h - c)\sqrt{(a\Phi_h - c)^2 - 4b\Phi_h^2 \widetilde{e}_2}}{4b\Phi_h^2} - \frac{\widetilde{e}_2}{2} - \frac{(a\Phi_l - c)^2}{2b\Phi_l^2} \\ &\geqslant 0. \end{aligned}$$

It follows that $W_h^{pc*} > W_l^{pc*} > W_l^{mt*} > W_l^*$.

As $\frac{\partial W_h^{pc*}}{\partial e} < 0$, when $\widetilde{e}_4 \leqslant e < \widetilde{e}_2$ we have:

$$W_h^{pc*}(e = \widetilde{e}_2) - W^{ror*} < W_h^{pc*} - W^{ror*} < W_h^{pc*}(e = \widetilde{e}_4) - W^{ror*},$$

and

$$\begin{aligned} W_h^{pc*} - W^{ror*} = &\frac{(a\Phi_h - c)^2 + (a\Phi_h - c)\sqrt{(a\Phi_h - c)^2 - 4b\Phi_h^2 e}}{4b\Phi_h^2} - \frac{e}{2} \\ &- \frac{\nu(a\underline{\Phi} - c)^2}{2b\underline{\Phi}^2} - \frac{(1-\nu)(a\overline{\Phi} - c)^2}{2b\overline{\Phi}^2} \\ &\begin{cases} > 0, \text{ if } \widetilde{e}_4 \leqslant e < \widetilde{e}_3 \\ \leqslant 0, \text{ if } \widetilde{e}_3 \leqslant e < \widetilde{e}_2 \end{cases}, \end{aligned}$$

where $W_h^{pc*} = W^{ror*}$ for $e = \widetilde{e}_3$. It follows that:

e	W
$\widetilde{e}_4 \leqslant e < \widetilde{e}_3$	$W_h^{pc*} > W^{ror*} > W_l^{mt*} > W_l^*$
$\widetilde{e}_3 \leqslant e < \widetilde{e}_2$	$W^{ror*} \geqslant W_h^{pc*} > W_l^{mt*} > W_l^*$

.

- Case A.3: When $\widetilde{e}_1 \leqslant e < \widetilde{e}_4$, the monopolist undertakes positive effort under both price cap and mandated-target regulation. In this case:

$$W_l^* = W^*(E = 0) = \frac{(2\gamma + 1)(a\Phi_l - c)^2}{8b\Phi_l^2},$$

$$W^{ror*} = W^{ror*}(E = 0) = \frac{\nu(a\underline{\Phi} - c)^2}{2b\underline{\Phi}^2} + \frac{(1-\nu)(a\overline{\Phi} - c)^2}{2b\overline{\Phi}^2},$$

$$W_h^{pc*} = W^{pc*}(E = e) = \frac{(a\Phi_h - c)^2 + (a\Phi_h - c)\sqrt{(a\Phi_h - c)^2 - 4b\Phi_h^2 e}}{4b\Phi_h^2} - \frac{e}{2}, \text{ and}$$

$$W_h^{mt*} = W^{mt*}(E = e) = \frac{(2\gamma + 1)(a\Phi_h - c)^2}{8b\Phi_h^2} - \gamma e + (1-\gamma)(1-\nu)\delta.$$

As $e < \widetilde{e}_4 = \frac{(a\Phi_h - c)^2}{4b\Phi_h^2} - \frac{(a\Phi_l - c)^2}{4b\Phi_l^2} + (2\nu - 1)\delta$, it is straightforward to show that $W_h^{mt*} > W_l^{mt*}$. From the previous analysis, we know that $W_l^{mt*} > W_l^*$. Therefore, $W_h^{mt*} > W_l^*$.

As

$$
\begin{aligned}
&W^{ror*} - W_h^{mt*} \\
&= \frac{\nu(a\underline{\Phi}-c)^2}{2b\underline{\Phi}^2} + \frac{(1-\nu)(a\overline{\Phi}-c)^2}{2b\overline{\Phi}^2} - \frac{(2\gamma+1)(a\Phi_h-c)^2}{8b\Phi_h^2} + \gamma e - (1-\gamma)(1-\nu)\delta \\
&\geqslant \frac{\nu(a\underline{\Phi}-c)^2}{2b\underline{\Phi}^2} + \frac{(1-\nu)(a\overline{\Phi}-c)^2}{2b\overline{\Phi}^2} - \frac{(2\gamma+1)(a\Phi_h-c)^2}{8b\Phi_h^2} + \gamma \widetilde{e}_1 - (1-\gamma)(1-\nu)\,\widetilde{\delta} \\
&\geqslant \frac{\nu(a\underline{\Phi}-c)^2}{2b\underline{\Phi}^2} + \frac{(1-\nu)(a\overline{\Phi}-c)^2}{2b\overline{\Phi}^2} - \frac{(a\Phi_h-c)^2}{8b\Phi_h^2} - \frac{\gamma\nu(a\Phi_l-c)^2}{4b\nu\Phi_l^2} - \frac{(1-\gamma)(1-\nu)(a\Phi_l-c)^2}{4b\nu\Phi_l^2} \\
&= \frac{\nu(a\underline{\Phi}-c)^2}{2b\underline{\Phi}^2} + \frac{(1-\nu)(a\overline{\Phi}-c)^2}{2b\overline{\Phi}^2} - \frac{(a\Phi_h-c)^2}{8b\Phi_h^2} - \frac{[1-\nu+(2\nu-1)\gamma](a\Phi_l-c)^2}{4b\nu\Phi_l^2} \\
&\geqslant \frac{\nu(a\underline{\Phi}-c)^2}{2b\underline{\Phi}^2} + \frac{(1-\nu)(a\overline{\Phi}-c)^2}{2b\overline{\Phi}^2} - \frac{(a\Phi_h-c)^2}{8b\Phi_h^2} - \frac{(a\Phi_l-c)^2}{4b\Phi_l^2} \\
&> \frac{\nu(a\underline{\Phi}-c)^2}{2b\underline{\Phi}^2} + \frac{(a\overline{\Phi}-c)^2}{8b\overline{\Phi}^2} - \frac{\nu(a\overline{\Phi}-c)^2}{2b\overline{\Phi}^2} > \frac{(a\overline{\Phi}-c)^2}{8b\overline{\Phi}^2} - \frac{\nu(a\overline{\Phi}-c)^2}{9b\overline{\Phi}^2} > 0, \qquad \text{(Ineq. (A.3))}
\end{aligned}
$$

we have that $W^{ror*} > W_h^{mt*} > W_l^*$.

By definition, $W_h^{pc*} = W^{ror*}$ at $e = \widetilde{e}_3 > \widetilde{e}_4$. Thus, it follows that when $e < \widetilde{e}_4$, we also have $e < \widetilde{e}_3$. As W_h^{pc*} is decreasing in e, we can conclude that $W_h^{pc*} > W^{ror*}$ for this range of the cost of effort. To sum up, in this case we have $W_h^{pc*} > W^{ror*} > W_h^{mt*} > W_l^*$.

- Case A.4: When $0 < e < \widetilde{e}_1$, the monopolist undertakes positive effort in all cases except under ROR regulation. This implies that:

$$W_h^* = W^*(E=e) = \frac{(2\gamma+1)(a\Phi_h-c)^2}{8b\Phi_h^2} - \gamma e,$$

$$W^{ror*} = W^{ror*}(E=0) = \frac{\nu(a\underline{\Phi}-c)^2}{2b\underline{\Phi}^2} + \frac{(1-\nu)(a\overline{\Phi}-c)^2}{2b\overline{\Phi}^2},$$

$$W_h^{pc*} = W^{pc*}(E=e) = \frac{(a\Phi_h-c)^2 + (a\Phi_h-c)\sqrt{(a\Phi_h-c)^2 - 4b\Phi_h^2 e}}{4b\Phi_h^2} - \frac{e}{2}, \text{ and}$$

$$W_h^{mt*} = W^{mt*}(E=e) = \frac{(2\gamma+1)(a\Phi_h-c)^2}{8b\Phi_h^2} - \gamma e + (1-\gamma)(1-\nu)\delta$$

From the analysis above, we know that $W_h^{mt*} > W_h^*$, and therefore, we have the following ranking: $W_h^{pc*} > W^{ror*} > W_h^{mt*} > W_h^*$.

B) Consider now the case where $\widetilde{\delta} < \delta$

As in the analysis of the case for $\widetilde{\delta} < \delta$, the threshold values of effort cost under different regulatory circumstances satisfy $\widetilde{e}_1 < \widetilde{e}_2 < \widetilde{e}_4$. Thus, we need to consider four distinct circumstances: $e \geqslant \widetilde{e}_4$, $\widetilde{e}_2 \leqslant e < \widetilde{e}_4$, $\widetilde{e}_1 \leqslant e < \widetilde{e}_2$ and $0 < e < \widetilde{e}_1$. However, the analysis for the two extreme cases where $e \geqslant \widetilde{e}_4$ and $0 < e < \widetilde{e}_1$ is the same regardless of the value of δ. Hence, it suffices to consider the cases when $\widetilde{e}_2 \leqslant e < \widetilde{e}_4$ and $\widetilde{e}_1 \leqslant e < \widetilde{e}_2$.

- Case B.1. When $\widetilde{e}_2 \leqslant e < \widetilde{e}_4$, the monopolist only undertakes positive effort under price cap regulation, yielding the following expected social welfare values:

$$W_l^* = W^*(E=0) = \frac{(2\gamma+1)(a\Phi_l - c)^2}{8b\Phi_l^2},$$

$$W^{ror*} = W^{ror*}(E=0) = \frac{\nu(a\underline{\Phi} - c)^2}{2b\underline{\Phi}^2} + \frac{(1-\nu)(a\overline{\Phi} - c)^2}{2b\overline{\Phi}^2},$$

$$W_l^{pc*} = W^{pc*}(E=0) = \frac{(a\Phi_l - c)^2}{2b\Phi_l^2}, \text{ and}$$

$$W_h^{mt*} = W^{mt*}(E=e) = \frac{(2\gamma+1)(a\Phi_h - c)^2}{8b\Phi_h^2} - \gamma e + (1-\gamma)(1-\nu)\delta$$

Therefore, $W^{ror*} > W_l^{pc*} > W_l^*$.

From the comparison between W^{ror*} and W_h^{mt*} in (Ineq. (A.3)), we know that $W^{ror*} > W_h^{mt*}$. Thus, it suffices to compare W_l^{pc*} and W_h^{mt*}.

As

$$\begin{aligned}
W_l^{pc*} - W_h^{mt*} &= \frac{(a\Phi_l - c)^2}{2b\Phi_l^2} - \frac{(2\gamma+1)(a\Phi_h - c)^2}{8b\Phi_h^2} + \gamma e - (1-\gamma)(1-\nu)\delta \\
&> \frac{(a\Phi_l - c)^2}{2b\Phi_l^2} - \frac{(2\gamma+1)(a\Phi_h - c)^2}{8b\Phi_h^2} + \gamma \widetilde{e}_2 - (1-\gamma)(1-\nu)\delta \\
&\geqslant \frac{(a\Phi_l - c)^2}{2b\Phi_l^2} - \frac{(2\gamma+1)(a\Phi_h - c)^2}{8b\Phi_h^2} + \frac{\gamma c(a\Phi_l - c)(\Phi_h - \Phi_l)}{b\Phi_h\Phi_l^2} - \frac{(1-\gamma)(1-\nu)(a\Phi_l - c)^2}{4b\nu\Phi_l^2} \\
&> \frac{(a\Phi_l - c)^2}{2b\Phi_l^2} - \frac{(2\gamma+1)(a\Phi_h - c)^2}{8b\Phi_h^2} + \frac{\gamma c(a\Phi_l - c)(\Phi_h - \Phi_l)}{b\Phi_h\Phi_l^2} - \frac{(1-\gamma)(a\Phi_l - c)^2}{4b\Phi_l^2} \\
&> \frac{(a\Phi_l - c)^2}{8b\Phi_l^2} + \frac{\gamma c(a\Phi_l - c)(\Phi_h - \Phi_l)}{b\Phi_h\Phi_l^2} > 0,
\end{aligned}$$

we obtain the following ranking: $W^{ror*} > W_l^{pc*} > W_h^{mt*} > W_l^*$.

- Case B.2: When $\widetilde{e}_1 \leqslant e < \widetilde{e}_2$, the monopolist undertakes positive effort under both price cap regulation and mandated-target regulation. This yields the same result as in the case where $\widetilde{e}_1 \leqslant e < \widetilde{e}_4$ and $\delta \leqslant \widetilde{\delta}_2$, namely $W_h^{pc*} > W^{ror*} > W_h^{mt*} > W_l^*$.