Appendices: Climate Policy and Strategic Operations in a Hydro-Thermal Power System

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Appendix A Nomenclature

Indices and Sets

$e \in \mathcal{E}_{i,n}$ Variable renewable energy (VRE) unit of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$.

$i \in \mathcal{I}$ Firms.

$\ell \in \mathcal{L}$ Transmission lines.

$\ell^{\text{AC}} \in \mathcal{L}^{\text{AC}} \subset \mathcal{L}$ AC transmission lines.

$\ell^{\text{DC}} \in \mathcal{L}^{\text{DC}} \subset \mathcal{L}$ DC transmission lines.

$\mathcal{L}_n^+, \mathcal{L}_n^-$ Transmission line starting/ending at node n.

$n \in \mathcal{N}$ Nodes.

$\mathcal{N}_{i,w} \subset \mathcal{N}$ Nodes containing hydro unit w belonging to firm i.

$n^{\text{AC}} \in \mathcal{N}^{\text{AC}} \subset \mathcal{N}$ AC nodes.

$n^{\text{DC}} \in \mathcal{N}^{\text{DC}} \subset \mathcal{N}$ DC nodes.

n_ℓ^+, n_ℓ^- Node index for starting/ending node of transmission line ℓ.

$t \in \mathcal{T}$ Time periods.

$u \in \mathcal{U}_{i,n}$ Thermal generation units of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$.

$w \in \mathcal{W}_{i,n}$ Hydro unit of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$.

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Parameters

$A^{\mathrm{e}}_{n,t}$	Availability factor for VRE unit $e \in \mathcal{E}_{i,n}$ at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (–).[1]
$B_{\ell^{AC}}$	Susceptance of AC transmission line $\ell^{AC} \in \mathcal{L}^{AC}$ (S).
$C_{i,n,t,u}$	Cost of generation for generation unit $u \in \mathcal{U}_{i,n}$ at node $n \in \mathcal{N}$ for firm $i \in \mathcal{I}$ at time $t \in \mathcal{T}$ (€/MWh).
$D^{\mathrm{int}}_{n,t}$	Intercept of linear inverse-demand curve at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (€/MWh).
$D^{\mathrm{slp}}_{n,t}$	Slope of inverse-demand curve at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (€/MWh^2).
$E^{\mathrm{sto}}_{i,n,w}$	Storage efficiency of hydro unit $w \in \mathcal{W}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ (m^3/m^{3}h).
$F_{i,n,w}$	Pumped-hydro efficiency of hydro unit $w \in \mathcal{W}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ (MWh/m^3).
$\overline{G}_{i,n,u}$	Maximum generation capacity of generation unit $u \in \mathcal{U}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ (MW).
$\overline{G}^{e}_{i,n}$	Maximum generation capacity of VRE unit $e \in \mathcal{E}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ (MW).
$I_{i,n,t,w}$	Natural inflow to hydro unit $w \in \mathcal{W}_{i,n}$ belonging to firm i at node n in period t (m^3).
$\overline{K}_{\ell}$	Capacity of the transmission line $\ell \in \mathcal{L}$ in positive direction (MW).
$\underline{K}_{\ell}$	Capacity of the transmission line $\ell \in \mathcal{L}$ in negative direction (MW).
$P_{i,n,u}$	CO_2 emission rate of generation unit $u \in \mathcal{U}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ (t/MWh).
$Q_{i,n,w}$	Generation efficiency of hydro unit $w \in \mathcal{W}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ (MWh/m^3).
$\overline{R}_{i,n,w}$	Maximum reservoir volume of hydro unit $w \in \mathcal{W}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ (m^3).
$\underline{R}_{i,n,w}$	Minimum reservoir volume of hydro unit $w \in \mathcal{W}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ (m^3).
$R^{\mathrm{in}}_{i,n,w}$	Maximum charging rate for hydro unit $w \in \mathcal{W}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ (m^3/m^{3}h).
R^{up}_{u}	Ramp-up rate for generation unit $u \in \mathcal{U}_{i,n}$ (–).
R^{down}_{u}	Ramp-down rate for generation unit $u \in \mathcal{U}_{i,n}$ (–).
S	Price of CO_2 emissions (€/t).
T_t	Duration of period t (h).
V	Scaling factor for power flow (–).
$Y_{i,n,w}$	Maximum generation capacity of hydro unit $w \in \mathcal{W}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ (MW).
$Z_{i,n}$	Regulation of net-hydro reservoir generation by firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ (MWh).

[1] "(–)" refers to a unitless item.

Variables

Primal variables

$f_{\ell,t}$ — Power flow on transmission line $\ell \in \mathcal{L}$ at time $t \in \mathcal{T}$ (MW).

$g_{i,n,t,u}$ — Generation of thermal unit $u \in \mathcal{U}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (MWh).

$g^e_{i,n,t}$ — Generation of VRE unit $e \in \mathcal{E}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (MWh).

$q_{n,t}$ — Consumption at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (MWh).

$r^{\text{in}}_{i,n,t,w}$ — Volume of water pumped into hydro unit $w \in \mathcal{W}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (m^3).

$r^{\text{sto}}_{i,n,t,w}$ — Volume of water stored in hydro unit $w \in \mathcal{W}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (m^3).

$v_{n,t}$ — Voltage angle of node $n^{AC} \in \mathcal{N}^{\mathcal{AC}}$ at time $t \in \mathcal{T}$ (rad).

$y_{i,n,t,w}$ — Volume of water turbined from hydro unit $w \in \mathcal{W}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (m^3).

$z_{i,n,t,w}$ — Volume of water spilled from hydro unit $w \in \mathcal{W}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (m^3).

Dual variables

$\beta_{i,n,t,u}$ — Shadow price of generation capacity of thermal unit $u \in \mathcal{U}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (€/MWh).

$\beta^e_{i,n,t}$ — Shadow price of generation capacity of VRE unit $e \in \mathcal{E}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (€/MWh).

$\beta^{\text{up}}_{i,n,t,u}$ — Shadow price of ramp-up rate of thermal unit $u \in \mathcal{U}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (€/MWh).

$\beta^{\text{down}}_{i,n,t,u}$ — Shadow price of ramp-down rate of thermal unit $u \in \mathcal{U}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (€/MWh).

$\gamma_{i,n}$ — Shadow price of hydro regulation for firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ (€/MWh).

$\eta_{\ell^{AC},t}$ — Shadow price of energy flow on AC line $\ell^{AC} \in \mathcal{L}^{AC}$ at time $t \in \mathcal{T}$ (€/MWh).

$\theta_{n,t}$ — Shadow price of market-clearing condition at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (€/MWh).

$\overline{\kappa}_{n^{AC},t}$ — Shadow price of maximum voltage angle at node $n^{AC} \in \mathcal{N}^{AC}$ at time $t \in \mathcal{T}$ (€/rad).

$\underline{\kappa}_{n^{AC},t}$ — Shadow price of minimum voltage angle at node $n^{AC} \in \mathcal{N}^{AC}$ at time $t \in \mathcal{T}$ (€/rad).

$\lambda^{\text{bal}}_{i,n,t,w}$ — Shadow price of water stored in reservoir of hydro unit $w \in \mathcal{W}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (€/m^3).

$\lambda^{\text{in}}_{i,n,t,w}$ — Shadow price of charging rate of hydro unit $w \in \mathcal{W}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (€/m^3).

$\lambda^{\mathrm{h}}_{i,n,t,w}$ Shadow price of turbine capacity of hydro unit $w \in \mathcal{W}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (€/MWh).

$\lambda^{\mathrm{ub}}_{i,n,t,w}$ Shadow price of maximum reservoir capacity of hydro unit $w \in \mathcal{W}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (€/m^3).

$\lambda^{\mathrm{lb}}_{i,n,t,w}$ Shadow price of minimum reservoir capacity of hydro unit $w \in \mathcal{W}_{i,n}$ of firm $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ (€/m^3).

$\overline{\mu}_{\ell,t}$ Shadow price of positive transmission capacity of line $\ell \in \mathcal{L}$ at time $t \in \mathcal{T}$ (€/MWh).

$\underline{\mu}_{\ell,t}$ Shadow price of negative transmission capacity of line $\ell \in \mathcal{L}$ at time $t \in \mathcal{T}$ (€/MWh).

Appendix B Mathematical Formulation

B.1 Mathematical Formulation for the ISO

$$\underset{\Gamma^{\mathrm{ISO}}}{\text{Maximise}} \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\left(D^{\mathrm{int}}_{n,t}q_{n,t} - \frac{1}{2}D^{\mathrm{slp}}_{n,t}q^2_{n,t}\right) \tag{B-1}$$

$$\text{s.t. } q_{n,t} = \sum_{i\in\mathcal{I}}\sum_{u\in\mathcal{U}_{i,n}} g_{i,n,t,u} + \sum_{i\in\mathcal{I}}\sum_{e\in\mathcal{E}_{i,n}} g^{e}_{i,n,t} + \sum_{i\in\mathcal{I}}\sum_{w\in\mathcal{W}_{i,n}}\left(Q_{i,n,w}y_{i,n,t,w} - F_{i,n,w}r^{\mathrm{in}}_{i,n,t,w}\right)$$

$$- \sum_{\ell\in\mathcal{L}^{+}_{n}} V\,T_t f_{\ell,t} + \sum_{\ell\in\mathcal{L}^{-}_{n}} V\,T_t f_{\ell,t} : \ \theta_{n,t}, \forall n, t \tag{B-2}$$

$$T_t f_{\ell^{\mathrm{AC}},t} = T_t B_{\ell^{\mathrm{AC}}}\left(v_{n^{+}_{\ell},t} - v_{n^{-}_{\ell},t}\right) : \ \eta_{\ell^{\mathrm{AC}},t}, \forall \ell^{\mathrm{AC}} \in \mathcal{L}^{\mathrm{AC}}, t \tag{B-3}$$

$$\underline{\mu}_{\ell,t} : \ -T_t\underline{K}_{\ell} \le V T_t f_{\ell,t} \le T_t\overline{K}_{\ell} : \ \overline{\mu}_{\ell,t}, \forall \ell, t \tag{B-4}$$

$$\underline{\kappa}_{n^{\mathrm{AC}},t} : \ -\pi \le v_{n^{\mathrm{AC}},t} \le \pi : \ \overline{\kappa}_{n^{\mathrm{AC}},t}, \forall n^{\mathrm{AC}} \in \mathcal{N}^{\mathrm{AC}}, t \tag{B-5}$$

Here, $\Gamma^{\mathrm{ISO}} \equiv \{q_{n,t} \ge 0, f_{\ell,t} \text{ u.r.s.}, v_{n^{AC},t} \text{ u.r.s.}\}$ and "u.r.s." refers to "unrestricted in sign." Lower-case Greek letters next to the constraints indicate the associated dual variables. Dual variables are simply the Lagrange multipliers associated with the constraints. At optimality, they indicate shadow prices of the constraints. For example, the dual variable, $\overline{\mu}_{\ell,t}$, associated with (B-4) reflects the marginal impact on the maximised gross surplus from an infinitesimal increase in the effective transmission limit, $T_t\overline{K}_{\ell}$. Some of them have non-negative signs, e.g., if the associated constraints are inequalities, whereas others are u.r.s., i.e., they are free to assume any sign, because their corresponding constraints are equalities.

B.2 Mathematical Formulation for Firm i

The associated KKT conditions in Appendix C for the following profit-maximisation problem of firm $i \in \mathcal{I}$ are written based on the exercise of market power in all generation.[2]

[2] Price-taking behaviour in both thermal/VRE generation and reservoirs is modelled by treating the price in (B-6) as exogenous, which means that KKT conditions (C-10)–(C-11) and (C-14)–(C-15) omit terms such as $D^{\mathrm{slp}}_{n,t}\left(\sum_{u'\in\mathcal{U}_{i,n}} g_{i,n,t,u'} + \sum_{e'\in\mathcal{E}_{i,n}} g^{e'}_{i,n,t} + \sum_{w'\in\mathcal{W}_{i,n}} Q_{i,n,w'}y_{i,n,t,w'} - \sum_{w'\in\mathcal{W}_{i,n}} F_{i,n,w'}r^{\mathrm{in}}_{i,n,t,w'}\right)$. It is also possible to account for market power that is exercised only in thermal/VRE generation and not by specific reservoirs (Ekholm and Virasjoki, 2020). In that case, only the impact of thermal/VRE generation on the price is reflected in (B-6) by treating $q_{n,t}$ as a constant when multiplying it by $\sum_{w'\in\mathcal{W}_{i,n}} Q_{i,n,w'}y_{i,n,t,w'} - \sum_{w'\in\mathcal{W}_{i,n}} F_{i,n,w'}r^{\mathrm{in}}_{i,n,t,w'}$. Consequently, the KKT conditions (C-10)–(C-11) omit $\sum_{w'\in\mathcal{W}_{i,n}} Q_{i,n,w'}y_{i,n,t,w'} - \sum_{w'\in\mathcal{W}_{i,n}} F_{i,n,w'}r^{\mathrm{in}}_{i,n,t,w'}$

$$\begin{aligned}\underset{\Gamma^{\mathrm{i}}}{\text{Maximise}} \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}} \Bigg[& \left(D^{\mathrm{int}}_{n,t} - D^{\mathrm{slp}}_{n,t} q_{n,t}\right) \Bigg(\sum_{u\in\mathcal{U}_{i,n}} g_{i,n,t,u} + \sum_{e\in\mathcal{E}_{i,n}} g^e_{i,n,t} + \sum_{w\in\mathcal{W}_{i,n}} Q_{i,n,w} y_{i,n,t,w} \\ & - \sum_{w\in\mathcal{W}_{i,n}} F_{i,n,w} r^{\mathrm{in}}_{i,n,t,w} \Bigg) - \sum_{u\in\mathcal{U}_{i,n}} \left(C_{i,n,t,u} + SP_{i,n,u}\right) g_{i,n,t,u} \Bigg] \end{aligned} \quad \text{(B-6)}$$

$$\text{s.t. } g_{i,n,t,u} \le T_t \overline{G}_{i,n,u} : \ \beta_{i,n,t,u}, \forall n,t,u \in \mathcal{U}_{i,n} \quad \text{(B-7)}$$

$$g^e_{i,n,t} = T_t A^e_{n,t} \overline{G}^e_{i,n} : \ \beta^e_{i,n,t}, \forall e \in \mathcal{E}_{i,n}, n, t \quad \text{(B-8)}$$

$$\beta^{\mathrm{down}}_{i,n,t,u} : \ -T_t R^{\mathrm{down}}_u \overline{G}_{i,n,u} \le g_{i,n,t,u} - g_{i,n,t-1,u} \le T_t R^{\mathrm{up}}_u \overline{G}_{i,n,u} : \ \beta^{\mathrm{up}}_{i,n,t,u}, \forall n,t,u \in \ \mathcal{U}_{i,n} \quad \text{(B-9)}$$

$$r^{\mathrm{sto}}_{i,n,t,w} = (1 - E^{\mathrm{sto}}_{i,n,w})^{T_t} r^{\mathrm{sto}}_{i,n,t-1,w} + r^{\mathrm{in}}_{i,n,t,w} - y_{i,n,t,w} - z_{i,n,t,w} + I_{i,n,t,w} : \ \lambda^{\mathrm{bal}}_{i,n,t,w}, \forall n,t,w \in \mathcal{W}_{i,n} \quad \text{(B-10)}$$

$$\lambda^{\mathrm{lb}}_{i,n,t,w} : \ \underline{R}_{i,n,w} \le r^{\mathrm{sto}}_{i,n,t,w} \le \overline{R}_{i,n,w} : \ \lambda^{\mathrm{ub}}_{i,n,t,w}, \forall n,t,w \in \mathcal{W}_{i,n} \quad \text{(B-11)}$$

$$r^{\mathrm{in}}_{i,n,t,w} \le T_t R^{\mathrm{in}}_{i,n,w} \overline{R}_{i,n,w} : \ \lambda^{\mathrm{in}}_{i,n,t,w}, \forall n,t,w \in \mathcal{W}_{i,n} \quad \text{(B-12)}$$

$$Q_{i,n,w} y_{i,n,t,w} \le T_t Y_{i,n,w} : \ \lambda^{\mathrm{h}}_{i,n,t,w}, \forall n,t,w \in \mathcal{W}_{i,n} \quad \text{(B-13)}$$

$$\sum_{t\in\mathcal{T}} \sum_{w\in\mathcal{W}_{i,n}} \left(Q_{i,n,w} y_{i,n,t,w} - F_{i,n,w} r^{\mathrm{in}}_{i,n,t,w}\right) \ge Z_{i,n} : \ \gamma_{i,n}, \forall n \quad \text{(B-14)}$$

Here, $\Gamma^{\mathrm{i}} \equiv \{g_{i,n,t,u} \ge 0, g^e_{i,n,t} \ge 0, r^{\mathrm{sto}}_{i,n,t,w} \ge 0, r^{\mathrm{in}}_{i,n,t,w} \ge 0, y_{i,n,t,w} \ge 0, z_{i,n,t,w} \ge 0\}$.

B.3 Equilibrium Problem and Single Equivalent Optimisation Problem

Since each optimisation problem, (B-1)–(B-5) and (B-6)–(B-14), $\forall i \in \mathcal{I}$, is convex, it may be replaced by its KKT conditions, thereby rendering a mixed-complementarity problem (MCP), (C-1)–(C-9) and (C-10)–(C-25), $\forall i \in \mathcal{I}$ (Gabriel et al., 2013). Subsequently, the MCP can be recast as a single optimisation problem. The optimisation problem maximises a quadratic objective function and incorporates Cournot behaviour via the extended-cost term, $-\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} \frac{D^{\mathrm{slp}}_{n,t}}{2} \left(\sum_{u\in\mathcal{U}_{i,n}} g_{i,n,t,u} + \sum_{e\in\mathcal{E}_{i,n}} g^e_{i,n,t} + \sum_{w\in\mathcal{W}_{i,n}} \left(Q_{i,n,w} y_{i,n,t,w} - F_{i,n,w} r^{\mathrm{in}}_{i,n,t,w}\right)\right)^2$. This transformation is due to the fact that the inverse-demand curve in the model is linear and transportation costs are proportional to the distances travelled (Hashimoto, 1985; Hobbs, 2001). The constraints of the resulting quadratic programming (QP) problem are simply those from the underlying optimisation problems, (B-2)–(B-5) and (B-7)–(B-14), $\forall i \in \mathcal{I}$.

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and KKT conditions (C-14)–(C-15) omit $D^{\mathrm{slp}}_{n,t}\left(\sum_{u'\in\mathcal{U}_{i,n}} g_{i,n,t,u'} + \sum_{e'\in\mathcal{E}_{i,n}} g^{e'}_{i,n,t} + \sum_{w'\in\mathcal{W}_{i,n}} Q_{i,n,w'} y_{i,n,t,w'} - \sum_{w'\in\mathcal{W}_{i,n}} F_{i,n,w'} r^{\mathrm{in}}_{i,n,t,w'}\right)$ when allowing for Cournot behaviour in thermal/VRE generation but price-taking behaviour in reservoirs. Market power by reservoirs only is handled analogously. Likewise, the equivalent quadratic programming (QP) reformulation in (B-15) can capture either perfect competition by dropping the "extended cost" term altogether, perfect competition in reservoirs by dropping the relevant $\sum_{w'\in\mathcal{W}_{i,n}} Q_{i,n,w'} y_{i,n,t,w'} - \sum_{w'\in\mathcal{W}_{i,n}} F_{i,n,w'} r^{\mathrm{in}}_{i,n,t,w'}$ terms from the extended cost, or perfect competition in thermal/VRE generation by dropping the relevant $\sum_{u'\in\mathcal{U}_{i,n}} g_{i,n,t,u'} + \sum_{e'\in\mathcal{E}_{i,n}} g^{e'}_{i,n,t}$ terms from the extended cost.

$$\underset{\Gamma}{\text{Maximise}} \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\left[\left(D^{\text{int}}_{n,t}q_{n,t}-\frac{1}{2}D^{\text{slp}}_{n,t}q^2_{n,t}\right)-\sum_{i\in\mathcal{I}}\left\{\frac{D^{\text{slp}}_{n,t}}{2}\left(\sum_{u\in\mathcal{U}_{i,n}}g_{i,n,t,u}+\sum_{e\in\mathcal{E}_{i,n}}g^e_{i,n,t}\right.\right.\right.$$
$$\left.\left.\left.+\sum_{w\in\mathcal{W}_{i,n}}\left(Q_{i,n,w}y_{i,n,t,w}-F_{i,n,w}r^{\text{in}}_{i,n,t,w}\right)\right)^2+\sum_{u\in\mathcal{U}_{i,n}}\left(C_{i,n,t,u}+SP_{i,n,u}\right)g_{i,n,t,u}\right\}\right] \tag{B-15}$$

s.t. (B-2) – (B-5)
(B-7) – (B-14), $\forall i\in\mathcal{I}$

where Γ comprises the ISO's decisions, Γ^{ISO}, and all of the firms' decisions, Γ^{i}, $\forall i\in\mathcal{I}$.[3]

Appendix C KKT Conditions

C.1 KKT Conditions for the ISO

$$0\le q_{n,t}\perp-\left(D^{\text{int}}_{n,t}-D^{\text{slp}}_{n,t}q_{n,t}\right)+\theta_{n,t}\ge 0,\forall n,t \tag{C-1}$$

$$f_{\ell,t}\text{ u.r.s., }T_t\eta_{\ell^{\text{AC}},t}+V\,T_t\overline{\mu}_{\ell,t}-V\,T_t\underline{\mu}_{\ell,t}+V\,T_t\theta_{n^+_\ell,t}-V\,T_t\theta_{n^-_\ell,t}=0,\forall\ell,t \tag{C-2}$$

$$v_{n^{\text{AC}},t}\text{ u.r.s., }-\sum_{\ell\in\mathcal{L}^+_n}T_tB_{\ell^{\text{AC}}}\eta_{\ell^{\text{AC}},t}+\sum_{\ell\in\mathcal{L}^-_n}T_tB_{\ell^{\text{AC}}}\eta_{\ell^{\text{AC}},t}+\overline{\kappa}_{n^{\text{AC}},t}-\underline{\kappa}_{n^{\text{AC}},t}=0,$$
$$\forall n^{\text{AC}}\in\mathcal{N}^{\text{AC}},t \tag{C-3}$$

$$\theta_{n,t}\text{ u.r.s., }q_{n,t}-\sum_{i\in\mathcal{I}}\sum_{u\in\mathcal{U}_{i,n}}g_{i,n,t,u}-\sum_{i\in\mathcal{I}}\sum_{e\in\mathcal{E}_{i,n}}g^e_{i,n,t}-\sum_{i\in\mathcal{I}}\sum_{w\in\mathcal{W}_{i,n}}\left(Q_{i,n,w}y_{i,n,t,w}-F_{i,n,w}r^{\text{in}}_{i,n,t,w}\right)$$
$$+\sum_{\ell\in\mathcal{L}^+_n}V\,T_tf_{\ell,t}-\sum_{\ell\in\mathcal{L}^-_n}V\,T_tf_{\ell,t}=0,\forall n,t \tag{C-4}$$

$$\eta_{\ell^{\text{AC}},t}\text{ u.r.s., }T_tB_{\ell^{\text{AC}}}\left(v_{n^+_\ell,t}-v_{n^+_\ell,t}\right)-T_tf_{\ell^{\text{AC}},t}=0,\forall\ell^{\text{AC}}\in\mathcal{L}^{\text{AC}},t \tag{C-5}$$

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[3]For sake of clarity, social welfare (SW) is different from (B-15) because it equals the sum of consumer surplus (CS), producer surplus (PS), merchandising surplus (MS), and government revenue (GR), where:

- $CS=\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\left(D^{\text{int}}_{n,t}q_{n,t}-\frac{1}{2}D^{\text{slp}}_{n,t}q^2_{n,t}\right)-\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\theta_{n,t}q_{n,t}$, i.e., gross consumer surplus minus the cost of electricity purchases.
- $PS=\sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\theta_{n,t}\left(\sum_{u\in\mathcal{U}_{i,n}}g_{i,n,t,u}+\sum_{e\in\mathcal{E}_{i,n}}g^e_{i,n,t}+\sum_{w\in\mathcal{W}_{i,n}}\left(Q_{i,n,w}y_{i,n,t,w}-F_{i,n,w}r^{\text{in}}_{i,n,t,w}\right)\right)-\sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\sum_{u\in\mathcal{U}_{i,n}}\left(C_{i,n,t,u}+SP_{i,n,u}\right)g_{i,n,t,u}$, i.e., revenues from electricity sales minus the costs of generation and CO_2 permits, excluding capital costs.
- $MS=\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\theta_{n,t}\left(\sum_{\ell\in\mathcal{L}^-_n}V\,T_tf_{\ell,t}-\sum_{\ell\in\mathcal{L}^+_n}V\,T_tf_{\ell,t}\right)$, i.e., the revenues from net imports at each node.
- $GR=\sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\sum_{u\in\mathcal{U}_{i,n}}SP_{i,n,u}g_{i,n,t,u}$, i.e., the CO_2 permit price multiplied by nodal CO_2 emissions.

The payment term in CS plus the revenue term in PS plus MS equal zero via energy balance (B-2), and the cost of CO_2 permits in PS cancels with GR. Thus, $SW=\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\left(D^{\text{int}}_{n,t}q_{n,t}-\frac{1}{2}D^{\text{slp}}_{n,t}q^2_{n,t}\right)-\sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\sum_{u\in\mathcal{U}_{i,n}}C_{i,n,t,u}g_{i,n,t,u}$. With exogenous net imports to the Nordic region, $X_{n,t}$ (in MWh), we modify the numerical implementation as follows:

- In nodal energy balance (B-2) and (C-4), subtract $X_{n,t}$ from $q_{n,t}$.
- Calculate the cost of exogenous net imports to the Nordic region, $IC=\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\theta_{n,t}X_{n,t}$.
- Subtract IC from the sum of CS, PS, MS, and GR to yield $SW=\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\left(D^{\text{int}}_{n,t}q_{n,t}-\frac{1}{2}D^{\text{slp}}_{n,t}q^2_{n,t}\right)-\sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\sum_{u\in\mathcal{U}_{i,n}}C_{i,n,t,u}g_{i,n,t,u}-\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\theta_{n,t}X_{n,t}$.

$$0 \leq \underline{\mu}_{\ell,t} \perp T_t \underline{K}_\ell + VT_t f_{\ell,t} \geq 0, \forall \ell, t \tag{C-6}$$

$$0 \leq \overline{\mu}_{\ell,t} \perp T_t \overline{K}_\ell - VT_t f_{\ell,t} \geq 0, \forall \ell, t \tag{C-7}$$

$$0 \leq \underline{\kappa}_{n^{\mathrm{AC}},t} \perp \pi + v_{n^{\mathrm{AC}},t} \geq 0, \forall n^{\mathrm{AC}} \in \mathcal{N}^{\mathrm{AC}}, t \tag{C-8}$$

$$0 \leq \overline{\kappa}_{n^{\mathrm{AC}},t} \perp \pi - v_{n^{\mathrm{AC}},t} \geq 0, \forall n^{\mathrm{AC}} \in \mathcal{N}^{\mathrm{AC}}, t \tag{C-9}$$

The KKT conditions lend themselves to straightforward economic interpretations. For example, (C-1) states that if consumption is strictly positive, then the marginal utility of electricity consumption is equal to its marginal generation value. However, if consumption is zero, then the marginal generation value exceeds the marginal utility of electricity consumption.

C.2 KKT Conditions for Firm i

$$\begin{aligned} 0 \leq g_{i,n,t,u} \perp \Bigg[& C_{i,n,t,u} + SP_{i,n,u} - \left(D^{\mathrm{int}}_{n,t} - D^{\mathrm{slp}}_{n,t} q_{n,t}\right) \\ & + D^{\mathrm{slp}}_{n,t} \left(\sum_{u' \in \mathcal{U}_{i,n}} g_{i,n,t,u'} + \sum_{e \in \mathcal{E}_{i,n}} g^e_{i,n,t} + \sum_{w \in \mathcal{W}_{i,n}} Q_{i,n,w} y_{i,n,t,w} - \sum_{w \in \mathcal{W}_{i,n}} F_{i,n,w} r^{\mathrm{in}}_{i,n,t,w} \right) \Bigg] \\ & + \beta_{i,n,t,u} + \beta^{\mathrm{up}}_{i,n,t,u} - \beta^{\mathrm{up}}_{i,n,t+1,u} + \beta^{\mathrm{down}}_{i,n,t+1,u} - \beta^{\mathrm{down}}_{i,n,t,u} \geq 0, \\ & \forall n, t, u \in \mathcal{U}_{i,n} \end{aligned} \tag{C-10}$$

$$\begin{aligned} 0 \leq g^e_{i,n,t} \perp \Bigg[& -\left(D^{\mathrm{int}}_{n,t} - D^{\mathrm{slp}}_{n,t} q_{n,t}\right) \\ & + D^{\mathrm{slp}}_{n,t} \left(\sum_{u \in \mathcal{U}_{i,n}} g_{i,n,t,u} + \sum_{e' \in \mathcal{E}_{i,n}} g^{e'}_{i,n,t} + \sum_{w \in \mathcal{W}_{i,n}} Q_{i,n,w} y_{i,n,t,w} - \sum_{w \in \mathcal{W}_{i,n}} F_{i,n,w} r^{\mathrm{in}}_{i,n,t,w} \right) \Bigg] \\ & + \beta^e_{i,n,t} \geq 0, \;\; \forall e \in \mathcal{E}_{i,n}, n, t \end{aligned} \tag{C-11}$$

$$0 \leq r^{\mathrm{sto}}_{i,n,t,w} \perp \lambda^{\mathrm{bal}}_{i,n,t,w} - (1 - E^{\mathrm{sto}}_{i,n,w})^{T_t} \lambda^{\mathrm{bal}}_{i,n,t+1,w} + \lambda^{\mathrm{ub}}_{i,n,t,w} - \lambda^{\mathrm{lb}}_{i,n,t,w} \geq 0, \forall n, t, w \in \mathcal{W}_{i,n} \tag{C-12}$$

$$0 \leq z_{i,n,t,w} \perp \lambda^{\mathrm{bal}}_{i,n,t,w} \geq 0, \forall n, t, w \in \mathcal{W}_{i,n} \tag{C-13}$$

$$\begin{aligned} 0 \leq r^{\mathrm{in}}_{i,n,t,w} \perp \Bigg[& F_{i,n,w} \left(D^{\mathrm{int}}_{n,t} - D^{\mathrm{slp}}_{n,t} q_{n,t}\right) \\ & - F_{i,n,w} D^{\mathrm{slp}}_{n,t} \left(\sum_{u \in \mathcal{U}_{i,n}} g_{i,n,t,u} + \sum_{e \in \mathcal{E}_{i,n}} g^e_{i,n,t} + \sum_{w' \in \mathcal{W}_{i,n}} Q_{i,n,w'} y_{i,n,t,w'} - \sum_{w' \in \mathcal{W}_{i,n}} F_{i,n,w'} r^{\mathrm{in}}_{i,n,t,w'} \right) \Bigg] \\ & - \lambda^{\mathrm{bal}}_{i,n,t,w} + \lambda^{\mathrm{in}}_{i,n,t,w} + F_{i,n,w} \gamma_{i,n} \geq 0, \forall n, t, w \in \mathcal{W}_{i,n} \end{aligned} \tag{C-14}$$

$$\begin{aligned} 0 \leq y_{i,n,t,w} \perp \Bigg[& -Q_{i,n,w} \left(D^{\mathrm{int}}_{n,t} - D^{\mathrm{slp}}_{n,t} q_{n,t}\right) \\ & + Q_{i,n,w} D^{\mathrm{slp}}_{n,t} \left(\sum_{u \in \mathcal{U}_{i,n}} g_{i,n,t,u} + \sum_{e \in \mathcal{E}_{i,n}} g^e_{i,n,t} + \sum_{w' \in \mathcal{W}_{i,n}} Q_{i,n,w'} y_{i,n,t,w'} - \sum_{w' \in \mathcal{W}_{i,n}} F_{i,n,w'} r^{\mathrm{in}}_{i,n,t,w'} \right) \Bigg] \\ & + \lambda^{\mathrm{bal}}_{i,n,t,w} + Q_{i,n,w} \lambda^{\mathrm{h}}_{i,n,t,w} - Q_{i,n,w} \gamma_{i,n} \geq 0, \forall n, t, w \in \mathcal{W}_{i,n} \end{aligned} \tag{C-15}$$

$$\begin{aligned} \lambda^{\mathrm{bal}}_{i,n,t,w} \text{ u.r.s., } & r^{\mathrm{sto}}_{i,n,t,w} - (1 - E^{\mathrm{sto}}_{i,n,w})^{T_t} r^{\mathrm{sto}}_{i,n,t-1,w} - r^{\mathrm{in}}_{i,n,t,w} + y_{i,n,t,w} + z_{i,n,t,w} - I_{i,n,t,w} = 0, \\ & \forall n, t, w \in \mathcal{W}_{i,n} \end{aligned} \tag{C-16}$$

$$\beta^e_{i,n,t} \text{ u.r.s., } g^e_{i,n,t} - T_t A^e_{n,t} \overline{G}^e_{i,n} = 0, \forall e \in \mathcal{E}_{i,n}, n, t \tag{C-17}$$

$$0 \leq \beta_{i,n,t,u} \perp T_t \overline{G}_{i,n,u} - g_{i,n,t,u} \geq 0, \forall n, t, u \in \mathcal{U}_{i,n} \tag{C-18}$$

$$0 \leq \beta^{\mathrm{up}}_{i,n,t,u} \perp T_t R^{\mathrm{up}}_u \overline{G}_{i,n,u} + g_{i,n,t-1,u} - g_{i,n,t,u} \geq 0, \forall n, t, u \in \mathcal{U}_{i,n} \tag{C-19}$$

$$0 \leq \beta^{\mathrm{down}}_{i,n,t,u} \perp T_t R^{\mathrm{down}}_u \overline{G}_{i,n,u} + g_{i,n,t,u} - g_{i,n,t-1,u} \geq 0, \forall n, t, u \in \mathcal{U}_{i,n} \tag{C-20}$$

$$0 \leq \lambda^{\mathrm{in}}_{i,n,t,w} \perp T_t R^{\mathrm{in}}_{i,n,w} \overline{R}_{i,n,w} - r^{\mathrm{in}}_{i,n,t,w} \geq 0, \forall n, t, w \in \mathcal{W}_{i,n} \tag{C-21}$$

$$0 \leq \lambda^{\mathrm{h}}_{i,n,t,w} \perp T_t Y_{i,n,w} - Q_{i,n,w} y_{i,n,t,w} \geq 0, \forall n, t, w \in \mathcal{W}_{i,n} \tag{C-22}$$

$$0 \le \lambda^{\text{ub}}_{i,n,t,w} \perp \overline{R}_{i,n,w} - r^{\text{sto}}_{i,n,t,w} \ge 0, \forall n, t, w \in \mathcal{W}_{i,n} \tag{C-23}$$

$$0 \le \lambda^{\text{lb}}_{i,n,t,w} \perp r^{\text{sto}}_{i,n,t,w} - \underline{R}_{i,n,w} \ge 0, \forall n, t, w \in \mathcal{W}_{i,n} \tag{C-24}$$

$$0 \le \gamma_{i,n} \perp \sum_{t \in \mathcal{T}} \sum_{w \in \mathcal{W}_{i,n}} \left(Q_{i,n,w} y_{i,n,t,w} - F_{i,n,w} r^{\text{in}}_{i,n,t,w} \right) - Z_{i,n} \ge 0, \forall n \tag{C-25}$$

Firm $i \in \mathcal{I}$'s KKT conditions also have explicit economic interpretations. For example, consider (C-10) in case of a price-taker, i.e., ignoring the derivative of the extended-cost term, $D^{\text{slp}}_{n,t}$ $\left(\sum_{u' \in \mathcal{U}_{i,n}} g_{i,n,t,u'} + \sum_{e \in \mathcal{E}_{i,n}} g^{e}_{i,n,t} + \sum_{w \in \mathcal{W}_{i,n}} Q_{i,n,w} y_{i,n,t,w} - \sum_{w \in \mathcal{W}_{i,n}} F_{i,n,w} r^{\text{in}}_{i,n,t,w}\right)$. It states that if thermal generation is strictly positive, then the price of electricity equals the marginal cost of generation plus the cost of CO_2 permits plus any capacity rents. On the contrary, if thermal generation is zero, then the marginal cost of generation plus the cost of CO_2 permits plus any capacity rents exceeds the electricity price (again ignoring the derivative of the extended-cost term). In case of a Cournot firm, (C-10) states that if thermal generation is zero, then the marginal cost of generation plus the cost of CO_2 permits plus any capacity rents exceeds the marginal revenue, i.e., the electricity price minus the derivative of the extended-cost term that internalises the price impact of a marginal increase in thermal output.

Appendix D 2030C Scenario

The 2030C scenario, where "C" refers to a high CO_2 price, is the same as the base 2018 scenario but with a CO_2 price of €100/t. By analysing the 2030C scenario, we can decompose the results in a plausible 2030CV scenario envisaged by future climate packages. Table D-1 reveals that a high CO_2 price alone lowers CO_2 emissions by over 80% vis-à-vis the base 2018 scenario under PC, cf. Table 6. Furthermore, because the cost of damage from CO_2 emissions is not included in SW, there is a decrease in overall welfare with a net transfer from consumers to producers. Intuitively, the high CO_2 price makes fossil-fuelled plants like coal and natural gas less profitable. Without a countervailing increase in VRE capacity, this policy intervention increases the average price under PC to €64.17/MWh, which benefits especially VRE, nuclear, and hydro producers. For example, Vattenfall's PS under PC increases to €3.77 billion from €2.01 billion in the base 2018 scenario. However, if Vattenfall exercises market power in thermal generation under COG in the 2030C scenario, then the increase in PS to €4.43 billion from €3.77 billion, i.e., an increase of 17.5%, is relatively less than in the base 2018 scenario, i.e., an increase of 30.8%. This is because the CO_2 price also chokes off consumption, which gives nuclear withholding less leverage in raising prices by forcing fossil-fuelled plants to generate at capacity. Thus, although Vattenfall could still withhold nuclear generation, its reduction is now from 43.27 TWh to 13.11 TWh per annum, cf. from 43.27 TWh to 7.86 TWh per annum in the base 2018 scenario.

Table D-1: Numerical Results for 2030C Scenario (in Billion € Unless Indicated).

Metric \ Case	PC	COG	COR
SW	141.42	139.71	141.33
CS	118.05	103.99	117.22
PS	21.37	33.31	21.90
MS	1.44	0.99	1.67
GR	0.55	1.43	0.55
EM (Mt)	5.52	14.26	5.47
Vattenfall PS	3.77	4.43	3.83

Likewise, Vattenfall's potential temporal arbitrage under COR is less successful than in the base 2018 scenario as its PS changes from €3.77 billion to €3.83 billion, i.e., an increase of

1.59% as opposed to 1.99% in the base 2018 scenario. In effect, the high CO_2 price means less fossil-fuelled generation, which results in a more even seasonal distribution of hydro production under PC (see Figure D-1) and increased exports even in the spring (see Figures D-2–D-3). The higher average prices amplify the opportunity cost for Vattenfall from shifting its hydro production to the spring season (see Figure D-4). Indeed, not only is forgone revenue higher due to shifting production from the winter and fall, but also the loss in revenue in the spring season greater due to the depression of the market-clearing price on a higher production level. Hence, from the perspective of Vattenfall, although the high CO_2 price in the 2030C scenario increases its PS, its payoff from exerting market power (whether under COG or COR) is blunted.

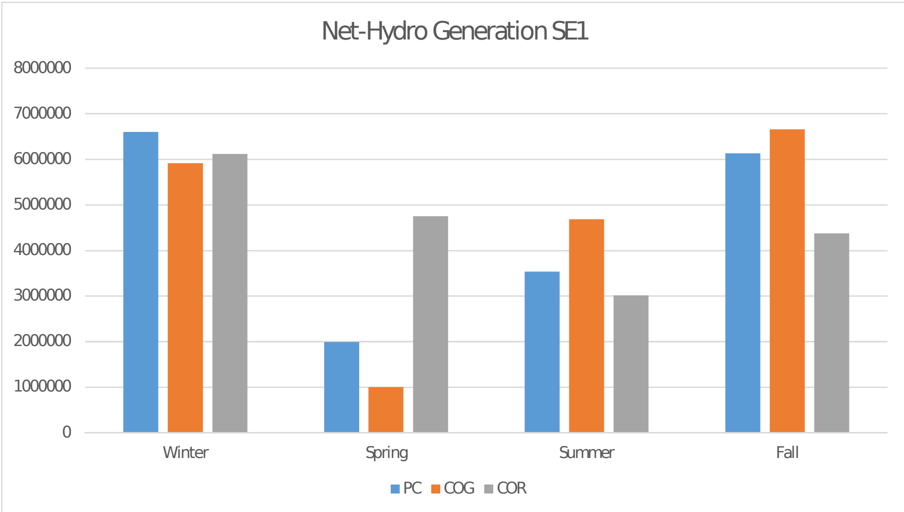

Figure D-1: Net-Hydro Operations at $SE1$ in the 2030C Scenario [in MWh].

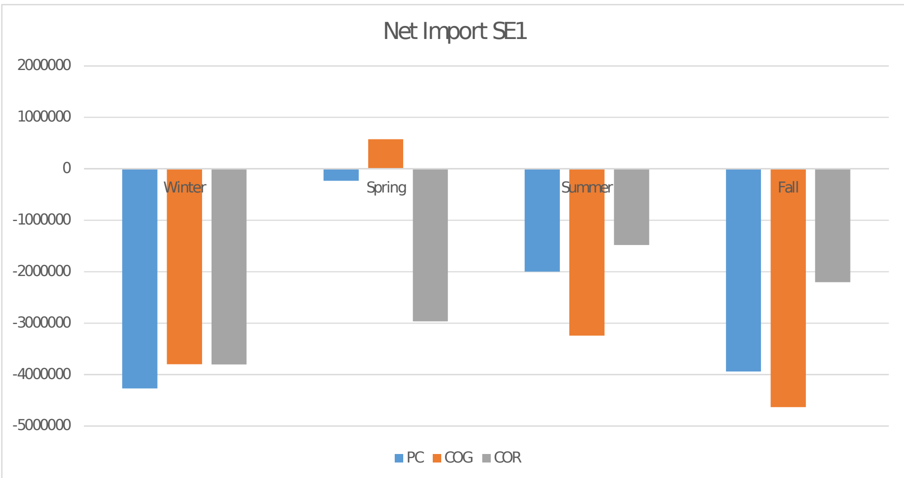

Figure D-2: Net Imports at $SE1$ in the 2030C Scenario [in MWh].

Appendix E Numerical Results with High Price Elasticity of Demand

We conduct a sensitivity analysis with a high price elasticity of demand, i.e., -0.25, in order to check the robustness of our results. In spite of greater price response by consumers, the main qualitative insight, viz., that a future climate package in the 2030CV will exacerbate the potential for temporal arbitrage from strategic reservoirs, still holds. Specifically, Vattenfall's PS in the

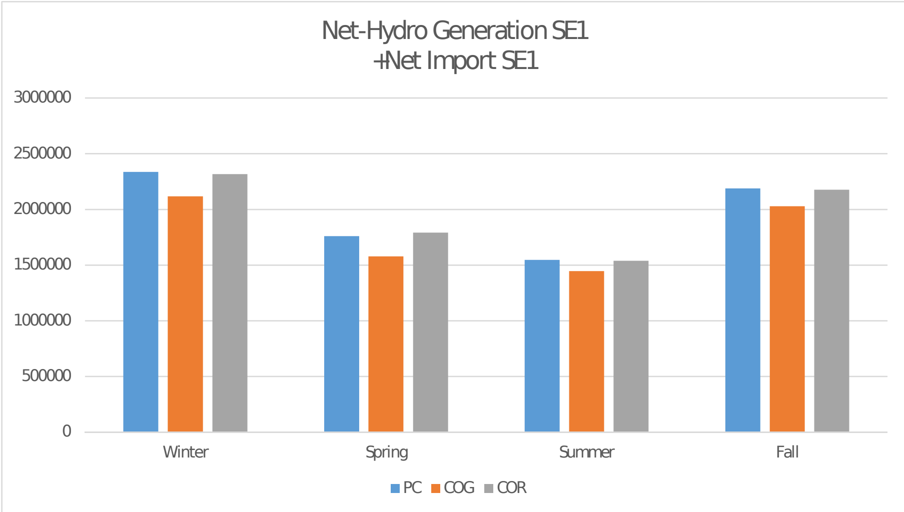

Figure D-3: Net-Hydro Operations Plus Net Imports at $SE1$ in the 2030C Scenario [in MWh].

Figure D-4: Seasonal Average Prices at $SE1$ in the 2030C Scenario [in €/MWh].

base 2018 scenario increases by 2.83% when going from PC to COR (see Table E-1). However, with a CO_2 price of €100/t and doubled VRE capacity in the 2030CV scenario, its PS increases by 3.36% when going from PC to COR (see Table E-3). Finally, a CO_2 price of €100/t alone as in the 2030C scenario leads to a 0.79% increase in Vattenfall's PS when going from PC to COR (see Table E-2).

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Table E-1: Numerical Results for Base Scenario with a -0.25 Price Elasticity of Demand (in Billion € Unless Indicated).

Metric \ Case	PC	COG	COR
SW	47.15	46.46	47.07
CS	34.35	30.38	33.87
PS	12.03	15.12	12.36
MS	0.30	0.39	0.39
GR	0.46	0.56	0.45
EM (Mt)	30.98	37.57	29.72
Vattenfall PS	2.02	2.06	2.07

Table E-2: Numerical Results for 2030C Scenario with a -0.25 Price Elasticity of Demand (in Billion € Unless Indicated).

Metric \ Case	PC	COG	COR
SW	46.22	45.53	46.16
CS	29.66	25.88	29.37
PS	15.44	18.58	15.60
MS	1.08	1.01	1.14
GR	0.05	0.05	0.05
EM (Mt)	0.50	0.54	0.49
Vattenfall PS	2.54	2.65	2.56

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Table E-3: Numerical Results for 2030CV Scenario with a -0.25 Price Elasticity of Demand (in Billion € Unless Indicated).

Metric \ Case	PC	COG	COR
SW	47.68	47.16	46.64
CS	34.73	29.55	34.35
PS	11.66	16.42	11.91
MS	1.27	1.17	1.36
GR	0.02	0.03	0.02
EM (Mt)	0.21	0.26	0.21
Vattenfall PS	1.56	2.00	1.61