Oil price shocks and current account imbalances within a currency union - Online Appendix

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1. MODEL

In this section of this online appendix, we explain the model we developed to analyze the impact of oil-price shocks on the current account and key macroeconomic variables like tradable and non-tradable production, inflation, and employment in more detail. The model is an open-economy two-country two-sector DSGE model with search and matching labor markets.

1.1 The representative household

Our economy is inhabited by a large number of infinitely living identical households consuming aggregates of domestic and imported monopolistic goods (Dixit and Stiglitz [1977]). Any household is either employed or unemployed due to labor market search frictions. In general, labor is supplied inelastically. As a second source of income, households own shares in domestic firms and receive dividends of $D_t$ from them. We assume that households in the domestic economy and the foreign country have the same preferences and factor endowments, defined over a composite consumption good $C_t$ and real money holdings $M_t/P_t$. In composite consumption good contains four goods, an oil good $O_t$, a domestic tradable good $Y_{H,t}$, a foreign tradable good $Y_{F,t}$ and a domestic non-tradable good $Y_{N,t}$ combined to several composite goods on different nests of the consumption function. As described by Merz [1995], we assume a perfect insurance system where households can insure themselves against variations in income. This assumption removes heterogeneity among households within a given country and enables us to consider the optimization problem of a representative household maximizing expected lifetime utility. During each period $t = 0, 1, 2, \ldots$, the expected lifetime utility function is given by

$$E \sum_{t=0}^{\infty} \beta_t \left[ \ln C_t + \kappa_m \ln \left( \frac{M_t}{P_t} \right) \right],$$

where $\beta_t = \frac{1}{1+\phi (\ln c_t-\bar{c})} \beta_{t-1}$ for $t \geq 0$, $\beta_0 = 1$ represents the endogenous discount factor, with the parameter $\phi$ that is assumed to be small and the shock term $c_t$, and $\kappa_m$ that denotes a scaling parameter for utility from real money holdings with $\kappa_m > 0$. The consumption index $C_t$ is defined as

$$C_t \equiv \left( \chi CO_t^{1-\rho} + (1-\chi) O_t^{1-\rho} \right)^{\frac{1}{1-\rho}},$$

with $CO_t$ as non-oil tradable and non-tradable composite and $O_t$ as oil-related products demanded by consumers. We added an oil consumption good as the empirical literature suggests that

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**Department of Economics, University of Duisburg-Essen, Universitätsstraße 2, D-45117 Essen, Germany and Institute for the Study of Labor (IZA), Schaumburg-Lippe-Strasse 5-9, D-53113 Bonn, Germany
We assume a continuum of monopolistic competitive second-tier retailers on the unit interval indexed by \(i\) that purchases goods from intermediate goods-producing firms. Each retailer transforms the intermediate good \(Y_{j,t}\) into a differentiated retail good using a linear production technology. During each period \(t = 0, 1, 2, \ldots\) a retailer \(j\) of sector \(j = H, F, N\) sells \(Y_{j,t}(i)\) units of the retail goods at the nominal price \(P_{j,t}(i)\). Let \(Y_{j,t}\) denote the composite of individual retails goods which is described by the CES aggregator of Dixit and Stiglitz (1977):

\[
C_{O,t} \equiv C_{H,t}^{1-\frac{1}{\eta}} C_{N,t}^{\frac{1}{\eta}}. \tag{3}
\]

 Tradable goods \(C_{T,t}\) can be obtained from the domestic \(C_{H,t}\) or from the foreign economy \(C_{F,t}\) while non-tradables \(C_{N,t}\) are produced at home only. Following Ferrero et al. (2008), we employ a Cobb-Douglas specification with \(\eta\) as the proportion of total expenditure devoted to tradable goods.

\[
C_{T,t} = \left[ \alpha \frac{1}{\eta} C_{H,t}^{\frac{\eta-1}{\eta}} + (1 - \alpha) \frac{1}{\eta} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^\frac{1}{\eta}. \tag{4}
\]

A household chooses consumption, nominal money, and bond holdings subject to a budget constraint of the form

\[
P_{G,t} C_t + B_t / R_t + M_t = B_{t-1} + P_{Y,t} Y_t + D_t + \varsigma_t + M_{t-1}, \tag{5}
\]

for \(t = 0, 1, 2, \ldots\). At the beginning of period \(t\), the household receives a lump-sum transfer \(\varsigma_t\) from the central bank and dividends \(D_t\) from the representative intermediate-goods-producing firm. Income amounts to \(Y_t\). The household enters period \(t\) with bonds \(B_{t-1}\) and \(M_{t-1}\) units of money. Furthermore, the mature bonds are providing additional \(B_{t-1}\) units which are all sold at the beginning of the period and might be used to purchase \(B_t\) new bonds at the nominal cost \(B_t / R_t\) with \(R_t\) as the nominal interest rate between \(t\) and \(t+1\). Solving the intertemporal optimization problem, we derive the following first-order conditions:

\[
\Lambda_t = C_t^{\frac{1}{\eta}} \tag{6}
\]

\[
E_t \beta_{t,t+1} = E_t \pi_{t+1} \frac{\eta+1}{\eta} \pi_{t+1} \tag{7}
\]

\[
\kappa_m = \Lambda_t - \beta_t E_t \frac{\Lambda_t}{\pi_{t+1}}, \tag{8}
\]

where \(\Lambda_t\) is the shadow price and \(\beta_{t,t+1} = \beta_t E_t \Lambda_t / \pi_{t+1}\) is the stochastic discount factor. Real money holdings are defined as \(m_t = M_t / P_{G,t}\). Combining the first-order conditions with respect to \(C_t\) and \(B_t\), equation (6) and equation (8), yields the standard consumption Euler equation:

\[
\beta_t E_t \left( \frac{C_{t+1}}{C_t} \right)^{-1} = E_t \frac{P_{G,t+1}}{R_t P_{G,t}}. \tag{9}
\]

### 1.2 Firms

We assume a continuum of monopolistic competitive second-tier retailers on the unit interval indexed by \(i\) that purchases goods from intermediate goods-producing firms. Each retailer transforms the intermediate good \(Y_{j,t}\) into a differentiated retail good using a linear production technology. During each period \(t = 0, 1, 2, \ldots\) a retailer \(j\) of sector \(j = H, F, N\) sells \(Y_{j,t}(i)\) units of the retail goods at the nominal price \(P_{j,t}(i)\). Let \(Y_{j,t}\) denote the composite of individual retails goods which is described by the CES aggregator of Dixit and Stiglitz (1977):

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1 We assume a unit elasticity between non-traded and traded goods, which is typical but not undisputed in the literature. Based on the simulations of Obstfeld and Rogoff (2005) with a unit elasticity, an elasticity of two and one of 100, our prior is not to find a strong impact of the elasticity on our simulation results.

2 The Euler equation is a difference or differential equation. It is an intertemporal first-order condition for a dynamic choice problem and describes the evolution of economic variables along an optimal path.
where $\varepsilon$ with $\varepsilon > 1$ is the elasticity of substitution across the differentiated retail goods. As in [Calvo (1983)], only a randomly and independently chosen fraction $1 - \nu$ of the firms in the retail sector are allowed to set their prices optimally, whereas the remaining fraction $\nu$ sets their prices by charging the previous period’s price that was inflated by steady-state inflation. Hence, a retail firm $i$, which can choose its price in period $t$, chooses the price $\hat{p}_{j,t}(i)$ to maximize

$$E_t \sum_{s=0}^{\infty} (\nu \beta)^s \beta_{t+s} \left[ \frac{\hat{p}_{j,t}(i)}{P_{j,t+s}} \right]^{-\varepsilon} Y_{j,t+s} \left( \frac{\hat{p}_{j,t}(i)}{P_{j,t+s}} - mc_{j,t+s} \right),$$

where $\beta_{t+s}$ is the stochastic discount factor used by the firms and $mc_{j,t}$ stands for the real marginal costs. The first-order condition for this problem is

$$\hat{p}_{j,t}(i) = \frac{\varepsilon}{(\varepsilon - 1)} \sum_{s=0}^{\infty} (\nu \beta)^s E_t \left( \Lambda_{j,t+s} P_{j,t+s}^{\gamma} Y_{j,t+s} mc_{j,t+s} \right),$$

Finally, using a CES-production technique, first-tier polypolistic retailer combines the composite good and an oil product $O_{\gamma, j, t}$ to create the final good $YG_{j,t}$,

$$YG_{j,t} = \left( \varphi_{\gamma, j} Y_{\gamma, j, t}^{-\rho_{\gamma}} + (1 - \varphi_{\gamma, j}) O_{\gamma, j, t}^{-\rho_{\gamma}} \right)^{\frac{1}{\rho_{\gamma}}}$$

with $\varphi_{\gamma, j}$ as shift parameter explaining oil use in production and the inverse elasticity of substitution $\rho_{\gamma}$.

### 1.3 The labor market

We distinguish three different statuses of employment of the representative household: let $U_{t}$, $W_{j,t}^{N}$ and $W_{j,t}(a_{t})$ denote respectively the present discounted value of an unemployed, newly employed and continuously employed worker, with $j$ being an index for the two sectors of each economy. In case of unemployment, the worker enjoys a real return $b$ and expects to move into employment with probability $p_{j}(\theta_{j,t})$, becoming employed either in the tradable or the non-tradable sector. Therefore, the present discounted income stream of an unemployed worker is

$$U_{j,t} = b + E_t \beta_{t,t+1} \left[ p_{j}(\theta_{j,t}) W_{j,t+1}^{N} + (1 - p_{j}(\theta_{j,t})) U_{j,t+1} \right].$$

Following [Pissarides (2000)], the flow value of being unemployed, $b = h + \rho_{w} w$, consists of the value of home production or leisure $h$ and unemployment benefits $\rho_{w} w$, where $\rho_{w}$ represents the replacement ratio with $0 < \rho_{w} < 1$ and $w$ the steady-state average wage. The second part of Equation (15) describes the expected capital gain from a change of state. As an equilibrium condition, the value of unemployment has to be identical in the both sectors $(U_{t} = U_{H,t} = U_{N,t})$.

The worker’s value from holding a job with idiosyncratic match productivity $a_{j,t}$, that is assumed to be log-normal distributed with the cumulative distribution function $F(\cdot)$, is given by
which ensures that their productivity is always above the productivity threshold 

\[ \rho_{j,t+1} = \frac{\rho_j(1 - \rho_j)}{\rho_j + (1 - \rho_j)} \]

or she survives exogenous and endogenous job destruction, which happens with a total probability of \( \rho_{j,t+1} \), the match will start to produce goods.

The present discounted value of a new match is

\[ W_{j,t}^{N} = W_{j,t}^{N} + \mathbb{E}_t \beta_{t+1} \left( 1 - \rho_s \right) \int_{\hat{\rho}_{j,t+1}}^{\infty} W_{j,t+1}(a_{j,t+1}) dF(a_{j,t+1}) + \rho_{j,t+1} U_{j,t+1} \]  

Equation (14) tells us that an employed worker is paid a sector-specific wage \( w_{j,t}(a_{j,t}) \), and that if he or she survives exogenous and endogenous job destruction, which happens with a total probability of \( \rho_{j,t+1} \), the match will start to produce goods.

\[ W_{j,t}(a_{j,t}) = w_{j,t}(a_{j,t}) \]

Equation (14) tells us that an employed worker is paid a sector-specific wage \( w_{j,t}(a_{j,t}) \), and that if he or she survives exogenous and endogenous job destruction, which happens with a total probability of \( \rho_{j,t+1} \), the match will start to produce goods.

\[ \frac{\partial W_{j,t}(a_{j,t})}{\partial a_{j,t}} = \frac{\partial w_{j,t}(a_{j,t})}{\partial a_{j,t}} \]

\[ m_{j,t}(u_{j,t}, v_{j,t}) = \chi u_{j,t}^\xi v_{j,t}^{1-\xi} \]

Please note, that equation (15) differs from equation (14) in the wages of new workers, only. The wages of new workers, \( w_{j,t}^{N} \), will be different from those of continuing workers, \( w_{j,t}(a_{j,t}) \) owing to the presence of firing costs that a firm must bear if it decides to fire a worker. As in the first period, no endogenous job destruction takes place, and firing costs in this period do not influence the wages of new workers.

\[ m_{j,t}(u_{j,t}, v_{j,t}) = \chi u_{j,t}^\xi v_{j,t}^{1-\xi} \]

During each period \( t = 0, 1, 2, \ldots \), an intermediate goods-producing firm posts a vacancy or continues the match from the previous period. Every single vacancy has the status filled or vacant. Because of matching frictions, it is assumed that the job search and hiring process is time-consuming and costly for both the worker and the firm. If a firm finds a suitable worker, both form a match. The number of job matches depends on the matching function \( m_{j,t}(u_{j,t}, v_{j,t}) \), where \( v_{j,t} \) denotes the number of vacancies in both sectors of the economy, and \( u_{j,t} \) is the number of unemployed workers searching in each sector \( j \). We assume a Cobb-Douglas matching function, where \( \xi \) denotes the partial elasticities

\[ m_{j,t}(u_{j,t}, v_{j,t}) = \chi u_{j,t}^\xi v_{j,t}^{1-\xi} \]

0 < \( \xi \) < 1 and \( \chi \) is a scale parameter reflecting the efficiency of the matching process. The tighter the labor market, the easier it is for unemployed workers to find a job. At the beginning of any period \( t \), job separations take place as a result of an exogenous negative shock with probability \( \rho_j^x \). Firm and worker may decide to dissolve a match endogenously if the realization of the worker’s idiosyncratic productivity of \( a_{j,t} \) is below a certain threshold productivity \( \hat{a}_{j,t} \). The probability of endogenous job destruction is given by \( \rho_j^x = P(a_{j,t} < \hat{a}_{j,t}) = F(\hat{a}_{j,t}) \). The total job separation rate, therefore, is \( \rho_{j,t} = \rho_j^x + (1 - \rho_j^x) \rho_j^p \). As in den Haan, Wouter J. et al. (2000), the idiosyncratic productivity \( a_{j,t} \) is drawn from a log-normal distribution with mean \( \mu_{jn} \) and standard deviation \( \sigma_{jn} \).

Following Mortensen and Pissarides (1994), new matches have a productivity of \( a_{j,t}^{N} \), which ensures that their productivity is always above the productivity threshold \( \hat{a}_{j,t} \), and that all jobs produce before being destroyed. New matches in \( t, m_{j,t}, \) become productive for the first time in \( t + 1 \). Consequently, the employment in each sector evolves according to \( n_{j,t} = (1 - \rho_{j,t}) n_{j,t-1} + m_{j,t-1}(u_{j,t-1}, v_{j,t-1}) \). As we normalize total employment to unity, the sum of unemployed persons becomes \( u_t = (1 - n_{H,t} - n_{N,t}) \).
The representative intermediate goods-producing firm

If an intermediate goods-producing firm posts a vacancy, it bears costs $c_j$. Labor is the only variable input in the production function. At the beginning of each period, old and new matches draw a idiosyncratic, job-specific productivity $a_{j,t}$. Production in each sector is subject to a productivity shock, common to all firms. If the realization of a worker’s idiosyncratic productivity is above the critical threshold $\hat{a}_{j,t+1}$, the firms will produce output using labor. The total factor productivity $A_{j,t}$ follows an AR(1) process, $\ln(A_{j,t}) = \rho_{Aj} \ln(A_{j,t-1}) + \epsilon_{Aj}$, where $\rho_{Aj}$ is the serial correlation coefficient with $0 < \rho_{Aj} < 1$ and $\epsilon_{Aj}$ follows a white noise process with a standard deviation $\sigma_{Aj}$.

We define the present discounted value of expected profits from a vacant job as follows:

$$V_{j,t} = -c_j + E_t \beta_{t+1} \left[ q_j(\theta_{j,t}) J_{j,t+1}^N + (1 - q_j(\theta_{j,t})) V_{j,t+1} \right].$$ (17)

With a probability of $q_j(\theta_{j,t})$, the firms matches with a worker and the match yields a return of $J_{j,t+1}^N$. With a probability of $1 - q_j(\theta_{j,t})$, the job remains vacant with a return of $V_{j,t+1}$. As long as the value of a vacancy is greater than zero, a firm will post new vacancies. In equilibrium, free market entry drives the profit from opening a vacancy to zero, which implies $V_{j,t} = 0$ for any $t$. This yields the vacancy posting condition

$$\frac{c_j}{q_j(\theta_{j,t})} = E_t \beta_{t+1} J_{j,t+1}^N,$$ (18)

which states that the expected cost of hiring a worker, $c_j/q_j(\theta_{j,t})$, is equal to the expected profit generated by a new match.

The value of a newly hired worker enjoyed by a firm, therefore, is given by

$$J_{j,t}^N = mc_{j,t} \frac{P_{j,t}}{P_{j,t}} A_{j,t} a_{j,t}^N - w_{j,t}^N$$

$$+ E_t \beta_{t+1} (1 - \rho^2_{Aj}) \left[ \int_{\hat{a}_{j,t+1}}^{\infty} J_{j,t+1}^N(a_{j,t+1}) dF_j(a_{j,t+1}) - F_j(\hat{a}_{j,t+1}) T_j \right],$$ (19)

where $mc_{j,t}$ denotes the sector-specific real marginal costs of providing one additional unit of output. We distinguish between endogenous and exogenous separations. With probability $1 - \rho^2_{Aj}$, the worker survives exogenous job destruction. For a surviving match, a realization of the idiosyncratic productivity below the critical threshold $\hat{a}_{j,t+1}$ leads to endogenous separation and the firm incurs firing costs $T_j$.

Similarly, the present discount value of a continuing job with productivity $a_{j,t}$ to the employer is

$$J_{j,t}(a_{j,t}) = mc_{j,t} \frac{P_{j,t}}{P_{j,t}} A_{j,t} a_{j,t} - w_{j,t}(a_{j,t})$$

$$+ E_t \beta_{t+1} (1 - \rho^2_{Aj}) \left[ \int_{\hat{a}_{j,t+1}}^{\infty} J_{j,t+1}(a_{j,t+1}) dF_j(a_{j,t+1}) - F_j(\hat{a}_{j,t+1}) T_j \right].$$ (20)

In equations (19) and (20) the term $mc_{j,t} \frac{P_{j,t}}{P_{j,t}} A_{j,t} a_{j,t} - w_{j,t}(a_{j,t})$ represents the net return of a match, and $J_{j,t+1}^N - F_j(\hat{a}_{j,t+1}) T_j$ represents the present discounted firm surplus, if the match is not destroyed.

Wage bargaining

In each period, firms and workers bargain over the real wage for that period, regardless of whether they form a continuous or a new match. The wage is set according to Nash bargaining. The worker and the firm share the joint surplus, and the worker receives the fraction $\eta \in [0, 1]$. Since the wage
where workers differ from those continuing workers as they do not include firing costs related to endogenous separations in the initial period. Whereas new workers are not subject to firing costs because in the period they are hired their idiosyncratic productivity is assumed to be above the critical threshold $d_j$. The wage of new workers is equal to

$$\eta(J_j, (a_{j,t}) + T_j) = (1 - \eta)(W_j, (a_{j,t}) - U_t), \quad (21)$$

and

$$\eta_j^N (a_{j,t}) = (1 - \eta)(W_j^N - U_t), \quad (22)$$

respectively. The bargaining rule for continuing workers, represented by equation (21), internalizes firing costs $T_j$, whereas new workers are not subject to firing costs because in the period they are hired their idiosyncratic productivity $a_j^N$ is assumed to be above the critical threshold $d_j$.

We can now derive the wage for continuing workers using the Bellman equations (13), (14) and (15) as well as the bargaining rules for continuing and new matches, equation (21) and (22)-

$$w_{j,t}(a_{j,t}) = \eta \left[ mc_{j,t} \frac{P_j}{P_t} A_{j,t} a_{j,t} + c_j \theta_j + (1 - \xi_j) T_j \right] + (1 - \eta) b. \quad (23)$$

The agreed wage for new workers is equal to

$$w_j^N = \eta \left[ mc_{j,t} \frac{P_j}{P_t} A_{j,t} a_j^N + c_j \theta_j - \xi_j T_j \right] + (1 - \eta) b, \quad (24)$$

where $\xi_j, t = E_{t+1} (1 - \rho_j^t)$.

The wages that new and continuing workers receive consist of two elements. First, if firms have complete bargaining power, the bargained wage will equal the benefits from unemployment $b$, which includes unemployment insurance payments and welfare captured by the replacement rate as well as the utility derived from not working. Second, if workers have complete market power, the wage will be the match revenue $mc_{j,t} \frac{P_j}{P_t} A_{j,t} a_{j,t}$, plus the saved hiring costs, $c_j \theta_j$, minus the present discounted firing costs, $\xi_j, T_j$, and the savings on firing cost $\xi_j T_j$ in the case of continuing workers. In cases where the bargaining power of firms and workers is between these two extremes, the bargaining power of workers $\eta$ attaches weight to the two elements. It follows from equation (24) that the wage of new workers differs from those continuing workers as they do not include firing costs related to endogenous job separations in the initial period.

### 1.4 The central bank

The central bank conducts monetary policy according to a modified Taylor (1993) rule:

$$\ln \left( R_t / \bar{R} \right) = \rho_r \ln \left( R_{t-1} / \bar{R} \right) + \rho_y \left( \delta \ln (Y_t / \bar{Y}) + (1 - \delta) \ln (Y^*_t / \bar{Y}^*) \right) + \rho_{\pi} \left( \delta \ln (\pi_{H,t} / \bar{\pi}_H) + (1 - \delta) \ln (\pi^*_F, / \bar{\pi}^*_F) \right) + m \rho_r, \quad (25)$$

where $\bar{R}, \bar{Y}$ and $\bar{\pi}_H, \bar{\pi}^*_F$ are the steady-state values of the gross nominal interest rate, output and CPI inflation rate for domestically and foreign-produced goods, and $m_{\rho_r}$ is a shock to monetary policy. The coefficient of the degree of interest rate smoothing $\rho_r$ and the reaction coefficients to inflation and output, $\rho_{\pi}$ and $\rho_y$, are positive. The parameter $\delta$ denotes the relative steady-state size of the home country vice-versa the foreign country.

3 Firing costs are assumed to affect endogenous separations, only. They do not occur for new workers in the first period, as the idiosyncratic productivity for those is per assumption above the threshold level.

4 The Taylor Rule is an interest rate forecasting model invented by Taylor (1993). It suggests how central banks should change interest rates to account for inflation and other economic conditions. Gerlach and Schnabel (2000) discuss the properties of Taylor rules within a European Monetary Union. They conclude that a Taylor rule should be similar to pre-EMU ones. In this paper, our modified Taylor rule for the EMU-area follows this assumption.
1.5 Trade

The real value of net exports is defined using the weighted difference between home production and tradable consumption \( N X_t \equiv \frac{P_{HT,t}Q_{HT,t}^{t} - P_{P,t}Q_{P,t}^{t}}{P_t} \). Using this definition, we specify total nominal bond holdings \( B_t \) according to

\[
\frac{B_t}{P_t} = \frac{R_{t-1}B_{t-1}}{P_t} + NX_t. \tag{26}
\]

We apply the standard incomplete markets model and assume that international financial markets clear \((B_t + B_t^* = 0)\), with \( B_t^* \) as nominal holdings of the domestic bond by foreign households, so that the net change of real bond holding reflects the current account \( CA_t \equiv \frac{B_t - B_t^*}{P_t} \).

Given two sectors in each economy and the rest of the world, it is convenient to define a set of relative prices. The relative price of non-tradables to tradables is defined as 
\[
\frac{P}{\bar{P}} \equiv \frac{P_{N,t}}{P_{T,t}},
\]

where \( P_{N,t} \) and \( P_{T,t} \) are the prices of non-tradables and tradables, respectively. The relative price of non-tradables to tradables is set as 
\[
\frac{P_{N,t}}{P_{T,t}} = \frac{\sum_{i=1}^{n} \mu_i P_{i,t}}{\sum_{i=1}^{n} \mu_i P_{i,t}}.
\]

The terms of trade are defined as 
\[
\frac{P_{N,t}}{P_{T,t}} = \frac{\sum_{i=1}^{n} \mu_i P_{i,t}}{\sum_{i=1}^{n} \mu_i P_{i,t}}.
\]

1.6 Domestic equilibrium conditions

In equilibrium, the value of an open vacancy is zero in both sectors. Making use of the vacancy posting condition \( (18) \), combined with equations \( (19) \) and \( (24) \), yields the job creation condition.

\[
\begin{aligned}
\frac{c_j}{q_j(\theta_{j,t})} &= \left(1 - \eta\right)E_t \beta_{j,t+1} \left[ mc_{j,t+1}A_{j,t+1}(a_{j,t+1} - \hat{a}_{j,t+1}) - T_j \right].
\end{aligned}
\tag{28}
\]

Equation \( (28) \) states that the expected hiring cost that a firm has to pay must equal the expected gain from a filled job. Jobs are destroyed by the firm when the realization of the worker’s productivity is below the reservation productivity. The reservation productivity is defined as the value of \( a_{j,t} \), which makes the firm’s surplus received from a job equal to zero,

\[
J_{j,t}(\hat{a}_{j,t}) + T_j = 0. \tag{29}
\]

The job destruction condition is derived using equations \( (20) \), \( (23) \) and \( (29) \) and is given by

\[
\begin{aligned}
mc_{j,t}A_{j,t}\hat{a}_{j,t} - b_j - \frac{n}{(1 - \theta_j)} c_{j,t} + (1 - \zeta_{j,t})T_j &= 0,
\end{aligned}
\tag{30}
\]

with \( c_{j,t} \) representing the average hiring costs of all firms in either of the two sectors of the economy.

As in \( \text{Zanetti} \) \( (2011) \), the equilibrium average real wage is a weighted average of continuing workers with weight \( \omega_{j,t}^C = (1 - \rho_j)\frac{\pi_{j,t-1}}{\pi_{j,t}} \) while that for new workers is \( 1 - \omega_{j,t}^C \). Therefore, the average real wage is

\[
\omega_t \equiv \frac{\sum_{j=1}^{n} \omega_{j,t}^C \pi_{j,t-1}}{\sum_{j=1}^{n} \pi_{j,t}}.
\]

5There is a wide discussion about the impact of imperfect financial market assumptions in open-economy models. For instance \( \text{Devereux and Sutherland} \) \( (2011) \) discuss the impact of this assumption on monetary policy while \( \text{Bodenstein} \) \( (2011) \) compares different imperfect market assumptions for open economies.
\[ w_{j,t} = \eta \left[ mc_{j,t}A_{j,t}\bar{a}_{j,t} + c\theta_t + (\omega_c^c - \xi_{j,t})T_{j} \right] + (1 - \eta_j)b, \tag{31} \]

where \( \bar{a}_{j,t} = \omega_c^c H(\hat{a}_{j,t}) + (1 - \omega_c^c)a_{j,t}^N \) is the average idiosyncratic productivity across jobs and \( H(\hat{a}_{j,t}) = E(\hat{a}_{j,t}|\hat{a}_{j,t} > \hat{a}_{j,t}) \) represents the average productivity for continuing workers. The aggregate output, net of vacancy costs, amounts to

\[ y_{j,t} = n_{j,t}A_{j,t}\bar{a}_{j,t} - c_{j,t}v_{j,t}, \tag{32} \]

where \( n_{j,t} \) as the number of workers employed in sector \( j \). Non-tradable production must equal demand

\[ Y_{N,t} = C_{N,t}Y_{N,t}^* = C_{N,t}^*, \]

as must tradable production within the union

\[ Y_{HU,t} = C_{H,t} + C_{H,t}^*, \]

and with the rest of the world

\[ Y_{H,t} = Y_{HU,t} + C_{H,t}^W, \]

with \( C_{H,t}^* \) as the demand for home tradable goods from abroad. Combining this relation with equation (26) reveals that the foreign trade balance in units of home consumption \( Q_tNX_t^* \) equal the negative home trade balance \( NX_t \).

Finally, oil prices follow an auto-regressive (AR) process

\[ \ln P_{Oi,t} = \rho_{Oi} \ln (P_{Oi,t-1}) + \epsilon_{Oi,t} \sim N(0, \sigma_r^2). \]

They enter the price index through maximization of equation (3) and the producer price index through equation (15).

2. CALIBRATION

In this section of the online appendix, we discuss the calibration of the model using data from Eurozone countries. Household preferences are characterized by six parameters: the steady-state discount factor, the partial elasticity for tradables and non-tradables, the elasticity of substitution between home and foreign-produced tradables, the home bias and the two elasticities of substitution for varieties of a tradable or non-tradable good. The periods of the model are calibrated to quarters and we assume both countries to be symmetrical except of the share of the tradable sector and, therefore, oil demand. Parameters are the same in both countries if not indicated otherwise. We set the steady-state discount factor to \( \beta = .995 \), which is in line with the most recent DSGE models of the Eurozone (Poutineau and Vermandel, 2015), and implies an annual steady-state interest rate of 2 percent. The Euler equation (9) implies a relative risk aversion parameter with the standard value of 2 (Benchimol and Fourcans, 2012). However, Smets and Wouters (2003) suggest a smaller value of 1 and Rabanal and Rubio-Ramirez (2005) estimate a posterior mean that implies a significantly higher risk aversion of above 9.

In the literature, we find a variety of definitions distinguishing tradables from non-tradables. We follow Schmilling (2013) who extend a study by Jensen and Kletzer (2012) for the service sectors to assign tradability to NACE sectors. Given this definition, the size of the tradable sector for France is slightly larger than 53 percent of GDP; for Italy, the share is somewhat higher than 57 percent, and Germany has the highest tradable share at 62 percent. However, some southern EMU countries like Greece have much lower tradable shares. We set the tradable share to 55 percent for the home country.

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\(^6\)We tested those values in a sensitivity analysis but the impact on current account imbalances and foreign debt was neglectable.

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which in 2012 was the average for EMU countries, and use this value to calculate the partial elasticities for the Cobb-Douglas function. We follow Obstfeld and Rogoff (2006) in setting the preference share parameter to $\alpha = 0.7$ and the elasticity of substitution between home and foreign tradables to $\gamma = 2.0$. The first value reflects the fact that Europeans and Americans attach a consumption weight of 70 percent to their domestic products. The elasticity of substitution between home and foreign tradables is set according to Obstfeld and Rogoff (1995).

We calibrate the labor market of the model to reproduce the structural characteristics of a typical EMU country. The unemployment rate is set to $u = 9.5$ percent, the long-term average among EMU countries. According to Hobijn and Sahin (2007), the quarterly separation rates are 6 percent for Spain and between 3 and 4 percent for France and Germany. Given that the data reflects the Great Moderation period and that separations seem to have increased during the crisis, we set the total separation rate to $\rho = 0.05$, which is in the upper range of estimates. Unfortunately, the data does not contain information on the endogenous and exogenous separation share in the total separation rate, which must be calibrated using the job creation and destruction function. The reservation productivity threshold of $\delta = 1.8$ is calculated at the steady-state intersection of the job destruction and job creation curve. We follow den Haan, Wouter J. et al. (2000) in assuming the idiosyncratic productivity to be log-normally distributed. As Germany is the biggest country in the Eurozone, we mimic the wage distribution of this country, which we have calculated using SOEP data. The mean of productivity to be log-normally distributed. As Germany is the biggest country in the Eurozone, we mimic the wage distribution of this country, which we have calculated using SOEP data. The mean of which is significantly lower than the matching efficiency we set for the Eurozone. Countries like France, Spain and Italy had a high matching efficiency in the past, where estimates range between $\chi = 0.6$ and $\chi = 0.8$ (Bourk, 2004; Destefanis and Fonseca, 2007; Ahamdanech-Zarco et al., 2009). Germany is perceived to have a low efficiency, calibrated between $\chi = 0.2$ and $\chi = 0.3$ (Jung and Kuhn, 2014; Krause and Uhlig, 2012). Recently, efficiency has tended to increase in Germany (Fahr and Sunde, 2009; Hillmann, 2009) but shrank in the other countries mentioned (Arpaia et al., 2014). We, therefore, follow Lubik and Krause (2014) and set the matching efficiency to $\chi = 0.5$, which is in line with the long-term unemployment level of the Eurozone.

The elasticity of a match w.r.t. unemployment is calibrated to $\xi = .7$, which reflects estimates by Burda and Wyplosz (1994) for Germany and France, Kohlbrecher et al. (2013) for Germany and Broersma (1997) for the Netherlands and is in line with the studies surveyed in Petrongolo and Pissarides (2001). As is standard in the literature, the Nash bargaining coefficient used in the wage-setting equation is set to $\eta = 0.5$, so workers and firms have the same bargaining power. The vacancy posting costs in the baseline scenario $c = 5.2$ and the unemployment benefits $b$ are inferred from the steady-state job destruction and job creation conditions. The parameter measuring leisure is calibrated to $h = 0.3$, so that the income from not working ($b$ and $h$) is worth 77 percent of $w$. Firing costs $T$ are set to 67 percent, which is calculated as the EMU average using the World Development

[Obstfeld and Rogoff, 2000] and [Obstfeld and Rogoff, 2006] discuss the issue of an estimation bias using aggregate trade data, which results in a lower than unity elasticity of substitution.

The value for Germany is close to $\rho = 0.03$, the separation rate calculated by Kohlbrecher et al. (2013) using German administrative data.

The matching efficiency in the Eurozone is perceived to be lower than that of the United States (Jung and Kuhn, 2014). Lubik (2013) estimated the Beveridge curve for the US using data from 2000 to 2008. The point estimate for the matching efficiency is $m = 0.8$ which is significantly lower than the matching efficiency we set for the Eurozone. Most studies like Jung and Kuhn calibrate the US matching efficiency lower between 0.5 and 0.6.

We also run the model with a significantly lower matching efficiency of 0.23 following Jung and Kuhn (2014). This specification’s volatility of total vacancies and unemployment is too low, so we returned to the standard specification. We could improve the business cycle statistics by setting the bargaining power according to Hagedorn and Manovskii (2008). If we, however, run the model with the standard matching efficiency and the Hagedorn-Manovskii specification, the business cycle statistics better matched the data (Business cycle properties for this calibration are available in an online supplement). We did not use this specification as it was inconsistent with the long-term unemployment rate of EU-countries and the distribution of wages.

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Indicators (WDI) database, while the replacement rate is 60 percent of the mean wage. This is in line with the study by van Vliet et al. (2012) which calculates a replacement rate of between 50 and 60 percent for most EU countries. The core countries of the Eurozone have values above 60 percent, while Malta and members of the Eastern enlargement round have lower values (30 to 40 percent).

As is common in the literature, the parameter measuring the market power of retailers is set to $\varepsilon = 11$. This implies a mark-up over marginal costs of 10 percent and reflects empirical findings. The Calvo parameter that governs the frequency of price adjustments is set in accordance with Taylor and Woodford (1999) to $\nu = 0.75$, such that the average binding of prices is 4 quarters. As is common, we normalize steady-state inflation to unity. The Taylor rule is calibrated following Taylor and Woodford (1999), and implies a monetary policy response to inflation equal to $\rho_{\pi} = 1.5$, a response to a change in output of $\rho_y = .5$ and a degree of interest rate smoothing of $\rho_r = .32$.

Finally, we specify the shock processes. In line with most of the literature, we calibrate the productivity shock such that the baseline model replicates the standard deviation of output in the Eurozone, which is 1.64. The standard deviation of the shock in either of the two sectors consequently amounts to $\sigma_a = 0.0087$, while the shock persistence parameter is $\rho_a = 0.94$. From Crespo-Cuaresma and Fernandez-Amador (2013) it follows that the standard deviation of time preference shocks should be roughly similar to that of supply shocks from 1990 onward, supply shocks had twice the standard deviation of time preference shocks in the 1960s. We set the standard deviation of the time preference shock to $\sigma_a = 0.013$ and the shock persistence parameter to $\rho_a = 0.94$ reflecting the importance of time preference shock for the Eurozone (Wyplosz, 2013). We follow the findings of Uhlig (2005) that monetary policy shocks contribute to less than 10 percent of the volatility of output in setting the standard deviation of the monetary policy shock to $\sigma_a = 0.0016$ with persistence of $\rho_a = 0.25$. The matching efficiency shocks are assumed to have a standard deviation of $\sigma_a = 0.0016$ and persistence of $\rho_a = 0.25$. These values align with the estimated DSGE models of the Eurozone, (Smets and Wouters 2003; Ratto et al. 2009).

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11 Time preference shocks affects the intertemporal marginal rate of substitution on consumption, they are also referred to as demand shocks.

12 Hence, we also account for asymmetric time preference shocks but, in difference to Wyplosz, assume the same standard deviation of shocks.
3. FIGURE SUPPLEMENTS: INTERNAL AND EXTERNAL MIGRATION

In this section, we show the sensitivity of our results to different migration scenarios. We did not allow for internal or external migration in the benchmark scenario. In the internal migration scenario, people can move within the Eurozone; in the external migration scenario, people can move between the Eurozone and abroad. The contribution of migration to oil-price shock absorption is small as migration does not fluctuate much during the business cycle. This effect could be because, compared to the US, intra-EMU labor mobility is still tiny, and one-time effects like the opening-up of labor markets for eastern-European countries in 2011 and 2014 and the acceptance of Syrian refugees after 2015 dominate intra-EMU and extra-EMU migration.
Figure 1: Positive oil price shock and migration

Impulse response functions
Notes: Each panel shows the response of the model variables to an oil price shock of one standard deviation. The horizontal axes measure time, expressed in quarters.
Figure 2: Positive oil price shock and migration

Positive oil supply shock
Impulse response functions
Notes: Each panel shows the response of the model variables to an oil price shock of one standard deviation. The horizontal axes measure time, expressed in quarters.
REFERENCES


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