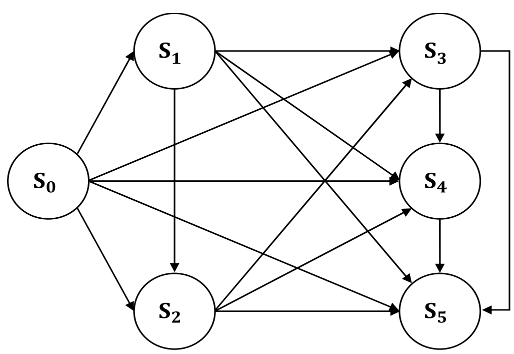
APENDIX 1: METHODOLOGY

We assume a rational household interested in reducing the electricity bill cost through the implementation of an RPV-Storage system. The household can invest multiple times in a limited period of time, always upscaling to states with higher solar power production and/or larger battery capacity. During the investment horizon (T_{inv}), the household can make multiple investments, while during the remaining valuation time ($T - T_{inv}$) the household cannot invest, and she/he will remain with the RPV-Storage system she/he had at the end of T_{inv}.

As an example, Figure A.1 shows five investment possibilities (named "states") that result from some combinations of different levels of power capacity from solar PV modules and different levels of storage capacity from batteries. S_0 represents the base case, where no PV modules or batteries are installed. For every pair of investment states i, j ($j > i$), S_j has larger or equal RPV-Storage capacity than S_i . The household has the option to invest once, moving directly to any state and remain in that state for the rest of the valuation time, or she/he can make multiple investments during the investment horizon (e.g., moving to S_1 in the first period, moving to S_2 in the third period and finally, moving to S_5 in the ninth period).

Figure A.1: Possible States and Transitions.



That is, during the investment time, the household has multiple possible investment paths, composed of one or more transitions. Single Transition Paths are those with only one transition (e.g., moving from S_0 to S_1 in the third period, and staying there for the remaining valuation horizon), while Multi Transitions Paths are those with two or more transitions (e.g., moving from S_0 to S_1 in the fifth period, then moving from S_1 to S_4 in the sixth period, and finally moving to S_5 in the ninth period). Each path has a terminal state (not a steady state), which is the state with the highest capacity of PV modules and batteries of the path. There are 27 possible investment paths in Figure A.1.

We simulate multiple future scenarios to take into account the large uncertainties regarding the profitability of the RPV-Storage system. The price of electricity and the unitary cost of solar modules and batteries are modeled as independent Geometric Brownian Motion (GBM) processes:[1]

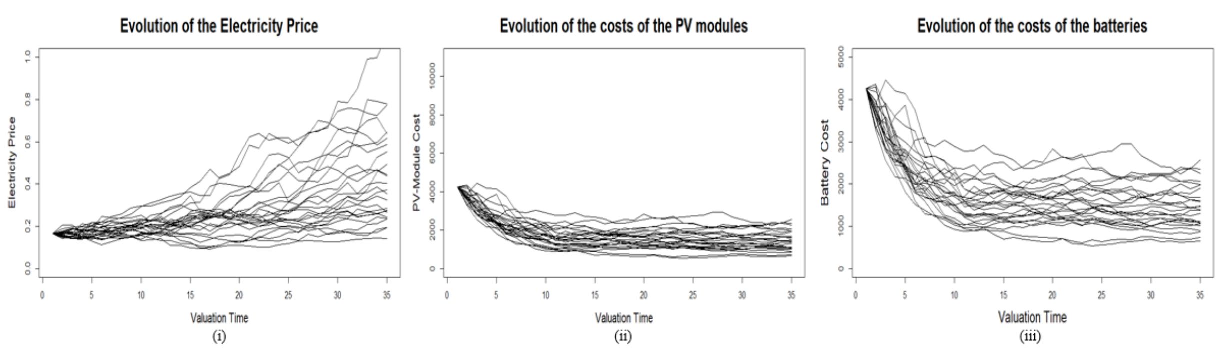
$$dE(t) = E(t) \cdot \alpha_e \cdot dt + E(t) \cdot \sigma_e \cdot dZ_e, \quad \text{(A.1)}$$

$$dM(t) = M(t) \cdot \alpha_m \cdot dt + M(t) \cdot \sigma_m \cdot dZ_m, \quad \text{(A.2)}$$

$$dB(t) = B(t) \cdot \alpha_b \cdot dt + B(t) \cdot \sigma_b \cdot dZ_b, \quad \text{(A.3)}$$

where $E(t)$, $M(t)$ and $B(t)$ are the price of electricity, PV module cost and battery cost at time t, respectively. Additionally, α, σ, and dZ represent the drift, volatility and the increment of a Wiener process, respectively (Hull, 2006). In every period, the price and costs can increase or decrease depending on the drift and the volatility of each GBM. For example, Figure A.2 shows some possible evolutions over time of the electricity price and the costs of PV modules and batteries.

Figure A.2: (i) Electricity price GBM, (ii) PV modules costs GBM, (iii) batteries costs GBM.[2]



To describe the methodology in a logic sequence, the rest of this section is outlined as follows: Section A1.1 explains the benefits and costs of moving between two states of RPV-Storage systems. Section A1.2 first presents the conventional valuation of discounted cash flows for a rigid project (Section A1.2.1); and then, it shows how the

[1] Others studies that have used GBM to model uncertainty in real option valuations are: Moon & Baran (2018); Pindyck (1999); and Tang et al. (2014).

[2] For illustrative purposes, we only show 25 future scenarios in Figure A.2.

traditional LSM method accounts for the single flexibility to postpone the investment (Section A1.2.2); and finally, it shows how the CLSM method accounts for the compound flexibility to postpone the initial investment and to expand the project (Section A1.2.3).

A1.1 Rigid Benefits and Costs of Moving between States.

For a given future scenario, moving from any state, S_i, to another state with higher capacity, S_j, has rigid incremental benefits and costs. Rigid benefits are computed as the difference in the electricity bill cost paid by the household at each state. Rigid costs are all the costs necessary to move from state S_i to state S_j.

A1.1.1 Rigid Benefits

As mentioned above, moving to a higher state in a certain future scenario generates an incremental benefit to the household. In particular, moving from S_i to S_j in period $\hat{t}$ generates benefits equivalent to the difference between the electricity bill cost of states S_i and S_j for the remaining valuation horizon ($T - \hat{t}$). The total benefit of a given transition is called rigid benefit ($RB^{\hat{t}}_{S_i \to S_j}$) and is computed as the net present value of the incremental annual cash flows of moving from S_i to S_j for the remaining years of the valuation horizon. Assuming a discount rate of r, rigid benefits are computed as follows:

$$RB^{\hat{t}}_{S_i \to S_j} = \sum_{t=\hat{t}}^{T} (BS^{t}_{S_j} - BS^{t}_{S_i}) \cdot e^{-r(t-\hat{t}\,)}, \tag{A.4}$$

where, for each period t, benefits per state ($BS^{t}_{S_i}$) are computed as the multiplication of the house demand for electricity (Q^t), the price of electricity (E^t), and the percentage of bill cost savings that a certain state generates ($BCS^{t}_{S_i}$) when compared to S_0 (e.g., the RPV-Storage combination in S_i could decrease the bill cost by 60% with respect to not having any PV module and battery).[3, 4] Therefore, the benefit of being in state i in period t is expressed as follows:

$$BS^{t}_{S_i} = Q^t \cdot E^t \cdot BCS^{t}_{S_i} \tag{A.5}$$

[3] The proposed methodology does not explicitly consider the impact of the uncertainty in the outcomes coming from climatic factors on the computation of the bill cost savings (BCS) parameters. However, we do perform sensitivity analyzes on the variation of total BCS in the main manuscript.
[4] Residential PV-modules' capacity factors are implicitly considered inside the BCS parameters, which in turn affect the computation of the benefits of a given state, as shown in (A.5). In agreement with this, the sensitivity analyses for the BCS parameters presented in the main manuscript can be also used to understand the implications of varying the residential PV-modules' capacity factors in the results.

A1.1.2 Rigid Costs

Investment costs of moving from S_i to S_j in period $\hat{t}$ are divided in three components: (i) initial setup cost, (ii) renovation cost, and (iii) salvage value. Initial setup costs ($SC^{\hat{t}}_{S_i \to S_j}$) are incurred at period $\hat{t}$ ($\hat{t} \in [0, T_{inv}]$) when the household invests to move to a higher state. These setup costs are computed using the GBMs' value of the module and battery costs for a certain future scenario and period.

Since the lifespan of any component of the RPV-Storage system could be shorter than the remaining valuation time after the initial setup ($T - \hat{t}$), the household has to reinvest in the components of the RPV-Storage system at one or multiple times during the valuation time. Thus, renovation costs ($REC^{t}_{S_i \to S_j}$) are incurred between $\hat{t}$ and T when the household has to replace the PV modules and/or batteries for new ones after their lifespans.

Finally, at the end of the valuation horizon, the household recovers the salvage value ($SV^{\hat{t}}_{S_i \to S_j}$) of the RPV-Storage system. The $SV^{\hat{t}}_{S_i \to S_j}$ is computed as the multiplication between the $SC^{t}_{S_i \to S_j}$ in period T and the remaining fraction of the lifespan of the RPV-Storage system. Therefore, the rigid cost ($RC^{\hat{t}}_{S_i \to S_j}$) from moving from S_i to a higher state S_j at time $\hat{t}$ is computed as follows:

$$RC^{\hat{t}}_{S_i \to S_j} = SC^{\hat{t}}_{S_i \to S_j} + \sum_{t=\hat{t}}^{T} \left(REC^{t}_{S_i \to S_j} \right) \cdot e^{-r(t-\hat{t})} - SV^{\hat{t}}_{S_i \to S_j} \cdot e^{-r(T-\hat{t})}, \quad \text{(A.6)}$$

where the renovation cost is zero for the years in which no PV modules or batteries are replaced and it is equal to $SC^{t}_{S_i \to S_j}$ in the years that PV modules or batteries are replaced.

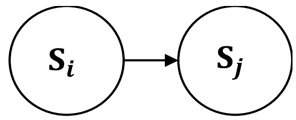
A1.2 RPV-Storage System Valuation

The best way of understanding the proposed CLSM methodology is comparing the traditional valuation methods with our compound approach. In this section, we first review the conventional valuation of a rigid project; and then, we recall how the traditional LSM method accounts for the single flexibility of postponing an investment. Finally, we show how the CLSM method accounts for the compound flexibility of postponing the initial investment and then expanding a project.

A1.2.1 Rigid Valuation

Let us take a subset of two states of Figure A.1, as presented in Figure A.3. For each future scenario, in this case, the household invests immediately (i.e., in $t = 0$) moving from S_i to S_j.

Figure A.3: Representation of a transition in the system represented in Figure A.1 (*j* > *i*).



Rigid valuation calculates the rigid benefits, costs, and NPV of investing immediately, moving from S_i to S_j. We compute a matrix of rigid benefits ($RB_{S_i \to S_j}$) and costs ($RC_{S_i \to S_j}$). Each of these matrices has N future scenarios and only one column since the household only has one period to invest. Therefore, the element $(n, 1)$ of matrix $RB_{S_i \to S_j}$ has the present value of the rigid benefit of upscaling from S_i to S_j in scenario n in time $t = 0$ and is computed as explained in (A.4) with $t = 0$. Analogously, the element $(n, 1)$ of matrix $RC_{S_i \to S_j}$ has the present value of the cost of upscaling from S_i to S_j computed as explained in (A.6) with $t = 0$. Then, the matrix $RNPV_{S_i \to S_j}$ is computed as the difference between the present values of the rigid benefits ($RB_{S_i \to S_j}$) and costs ($RC_{S_i \to S_j}$), and it contains the rigid NPV of investing in $t = 0$ for each of the N future scenarios. Thus:

$$RNPV_{S_i \to S_j} = RB_{S_i \to S_j} - RC_{S_i \to S_j} \tag{A.7}$$

The expected rigid NPV of moving from S_i to S_j is the average $RNPV_{S_i \to S_j}$ of all future scenarios. Therefore, the household decides to invest if the expected NPV is positive.

A1.2.2 Single Flexibility Valuation

In this case, for each future scenario, the household has the flexibility of choosing to postpone the investment. Thus, she/he is not forced to invest in $t = 0$, but has the flexibility of investing at any time between 0 and T_{inv}.

Single flexibility valuation first calculates the rigid benefits, costs, and NPV of investing and moving from S_i to S_j (see Figure A.3) at any time between 0 and T_{inv}. Therefore, we compute a matrix of rigid benefits ($RB_{S_i \to S_j}$) and costs ($RC_{S_i \to S_j}$) of NxT_{inv}, where N is the number of future scenarios, and T_{inv} is the number of periods in which the household can invest. The values in these matrices present the value of the projected benefits and costs during the remaining valuation time after investing in a certain period. For example, element (n, t) in matrix $RB_{S_i \rightarrow S_j}$ has the present value of the incremental benefit of moving from S_i to S_j in scenario n in period t. Analogously, element (n, t) of matrix $RC_{S_i \rightarrow S_j}$ has the present value of the cost of upscaling from S_i to S_j in scenario n in period t. Then, the matrix $RNPV_{S_i \rightarrow S_j}$ is computed as the difference between the present values of the rigid benefits ($RB_{S_i \rightarrow S_j}$) and costs ($RC_{S_i \rightarrow S_j}$), and it shows the NPV of investing at any time between 0 and T_{inv} for each of the N future scenarios. Accordingly:

$$RNPV_{S_i \rightarrow S_j} = RB_{S_i \rightarrow S_j} - RC_{S_i \rightarrow S_j} \tag{A.8}$$

For each future scenario n, the LSM algorithm computes the optimal investment time of moving from S_i to S_j (instead of investing immediately as in the rigid valuation presented in Section A1.2.1). Intuitively, LSM computes the optimal investment time t^* comparing at every period t (between 0 and T_{inv}) the NPV of investing in period t with the value of having the option and not the obligation to invest in the future, which is called the continuation value ($CV_{S_j \rightarrow S_k}$) in the LSM method (Longstaff and Schwartz, 2001). Then, the flexible NPV ($FNPV_{S_i \rightarrow S_j}$) in the future scenario n is simply the discounted value of the n-th element of the rigid NPV ($RNPV_{S_i \rightarrow S_j}$) in the optimal investment time:

$$FNPV_{S_i \rightarrow S_j}(n, 1) = RNPV_{S_i \rightarrow S_j}(n, t^*) \cdot \mathrm{e}^{(-t^* \cdot r)} \tag{A.9}$$

Finally, the expected value of having the option and not the obligation to move from S_i to S_j at any time between 0 and T_{inv} is the average $FNPV_{S_i \rightarrow S_j}$ of all future scenarios.

A1.2.3 Compound Flexibility Valuation

The CLSM method enables the household to valuate multiple investments in a compounded way. For example, let us take a subset of three states of the system represented in Figure A.1, as presented in Figure A.4. Table A.1 shows all three possible paths and their transitions in this selected subset of states. In this case, paths I and II have a single transition, while path III is compound and has two transitions. The valuation of paths I and II is the same used in the single flexibility valuation explained in Section A1.2.2. On the other hand, the compound valuation of path III is only possible with the CLSM algorithm, as explained next.

Figure A.4: Possible transitions in a subset of three states of the system represented in Figure A.1 (*k* > *j* > *i*).

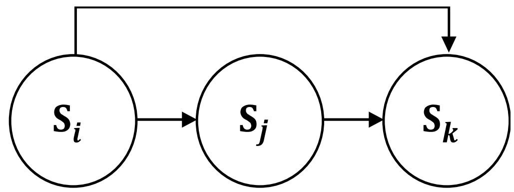


Table A.1: Possible paths of the system represented in Figure A.4.

Path (1)	1st Transition (2)	2nd Transition (3)	Terminal State (4)
I	$S_i \rightarrow S_j$	-	S_j
II	$S_i \rightarrow S_k$	-	S_k
III	$S_i \rightarrow S_j$	$S_j \rightarrow S_k$	S_k

For path III, the CLSM calculates first the rigid benefits, costs, and NPV of investing and moving from S_i to S_j and from S_j to S_k at any time between 0 and T_{inv}. The following matrices of rigid benefits, costs, and NPV are calculated in the same way as calculating the single flexibility valuation for each future scenario and transition.

From S_i to S_j:

$$RNPV_{S_i \rightarrow S_j} = RB_{S_i \rightarrow S_j} - RC_{S_i \rightarrow S_j} \quad \text{(A.10)}$$

From S_j to S_k :

$$RNPV_{S_j \rightarrow S_k} = RB_{S_j \rightarrow S_k} - RC_{S_j \rightarrow S_k} \quad \text{(A.11)}$$

Then, the CLSM method computes the optimal investment times and the NPV of the path considering two steps.

Step I:

In this step, we calculate the optimal investment time of the multiple transitions in path III, for each future scenario, using a compound NPV ($CNPV_{S_i \rightarrow S_j}$) matrix. For a certain future scenario n and period t, the $CNPV_{S_i \rightarrow S_j}$ represents the value of investing and moving from S_i to S_j in path III, and it is computed as the sum of the rigid NPV ($RNPV_{S_i \rightarrow S_j}$) –as explained in section A1.2.2– and the continuation value ($CV_{S_j \rightarrow S_k}$). Thus,

$$CNPV_{S_i \to S_j}(n,t) = RNPV_{S_i \to S_j}(n,t) + CV_{S_j \to S_k}(n,t) \qquad \text{(A.12)}$$

The continuation value ($CV_{S_j \to S_k}$) represents the flexibility to expand afterwards and is quantified as the expected net present value generated by moving from a given state to any higher state in the same path during the future. In other words, moving to a particular RPV-Storage system confers the household the right and not the obligation to continue investing afterwards to move to higher capacity levels of PV modules and batteries under favorable future conditions. As explained before, the continuation value ($CV_{S_j \to S_k}$) is computed using the LSM method.[5]

It is also important to notice that the $CNPV_{S_i \to S_j}$ matrix depends on the path, because the $CV_{S_j \to S_k}$ values all possible future expansions in this path. For instance, for a path that has two transitions, as path III in Table A.1 ($S_i \to S_j$ and $S_j \to S_k$), the continuation value from moving from S_i to S_j (first transition), only considers the option to expand from S_j to S_k (second transition). However, if a path has three transitions (e.g., $S_i \to S_j, S_j \to S_k$, and $S_k \to S_l$), the continuation value of moving from S_i to S_j (first transition), will consider the value of the option to expand from S_j to S_k (second transition), and also the value of the option to continue expanding from S_k to S_l (third transition). When the transition is to the terminal state of the path (e.g., S_k for path III), there are no future expansion possibilities and, therefore, the continuation value is zero.

Step II:

The CLSM calculates the optimal investment time for every transition applying the LSM method with the $CNPV$ matrix (as the expected payoff from immediate exercise of the option) instead of applying the method with the $RNPV$ matrix, as in the case that there is no flexibility to expand. If a path is composed by multiple transitions (e.g., path III: $S_i \to S_j$ and $S_j \to S_k$), there will be one optimal investment time per transition.[6] Then, the flexible NPV vector of an investment path is simply the sum of the discounted values of the elements of the rigid NPV matrices in the optimal investment times of each of the transitions within this path. Thus, for example, the flexible NPV of path III ($FNPV_{S_i \to S_j \to S_k}$) in a certain future scenario n where the optimal investment times are $t^*_{S_i \to S_j}$ and $t^*_{S_j \to S_k}$ is:

[5] In financial terms, the CLSM method fills the null values of the CV matrix used in LSM for the out-of-money options, using the same conditional expectation function used for the CV of the in-the-money options. Thus, out-of-the-money options are not used in the determination of this conditional expectation function.

[6] For example, for path III that has two transitions ($S_i \to S_j$ and $S_j \to S_k$), the CLSM method computes the optimal investment time of the second transition on the condition that it has to be done after the optimal investment time of the first investment.

$$FNPV_{S_i \to S_j \to S_k}(n) = RNPV_{S_i \to S_j}(n,\ t^*_{S_i \to S_j}) \cdot e^{\left(-r \cdot t^*_{S_i \to S_j}\right)} + RNPV_{S_j \to S_k}(n,\ t^*_{S_j \to S_k}) \cdot e^{(-r \cdot t^*_{S_j \to S_k})} \qquad \text{(A.13)}$$

Finally, after computing the optimal investment times and NPVs of all possible paths, the CLSM selects, in each future scenario, the best path by maximizing the NPV. Therefore, the CLSM selects the optimal path in each future scenario n as:

$$OPNPV(n, 1) = Max\left[FNPV(n, 1)_{S_i \to S_j \to S_k};\ FNPV(n, 1)_{S_i \to S_j};\ FNPV(n, 1)_{S_i \to S_k}\right] \qquad \text{(A.14)}$$

where the element $(n, 1)$ of the optimal path NPV matrix $(OPNPV)$ is the value of the maximum flexible NPV of all paths in scenario n. The expected value of having the option and not the obligation to move from S_i to S_j or from S_i to S_k, in a direct or compound way, at any time between 0 and T_{inv} is the average $OPNPV$ of all future scenarios.