APPENDIX

A. OPTIMA AND EQUILIBRIA IN REGIME 2

It follows from the discussion of Regime 2 in Section 3 of the text that expected charge at the end of a daytime period is given by

$$E(s) = \int_{\underline{\theta}}^{1} \left\{ \begin{array}{c} F_D(\theta R + G - S)S \\ + \int_{\theta R + G - S}^{\theta R + G} (\theta R + G - L) f_D(L) dL \end{array} \right\} h(\theta) d\theta. \tag{A.1a}$$

Increases in either *R* or *G* increase the expected value of *Z*:

$$\partial E(s) / \partial R = \int_{\underline{\theta}}^{1} \theta \left[\begin{array}{c} F_D(\theta R + G) \\ -F_D(\theta R + G - S) \end{array} \right] h(\theta) d\theta < \int_{\underline{\theta}}^{1} \theta h(\theta) d\theta, \tag{A.1b}$$

$$\partial E(s) / \partial G = \int_{\underline{\theta}}^{1} \left[F_D(\theta R + G) - F_D(\theta R + G - S) \right] h(\theta) d\theta < 1, \tag{A.1c}$$

$$\partial E(s) / \partial S = \int_{\underline{\theta}}^{1} F_D(\theta R + G - S) h(\theta) d\theta < 1. \tag{A.1d}$$

Expected *daytime* operating cost follows from competitive storage behavior in Regime 2:

$$\Lambda\left(R,G,S\right)\equiv\int_{\underline{\theta}}^{1}\left\{\begin{aligned}&c\int_{\theta R-S}^{\theta R+G-S}\left[L-(\theta R-S)\right]f_D(L)dL\\&\quad+cG\left[1-F_D\left(\theta R+G-S\right)\right]\\&+v\int_{\theta R+G}^{\infty}\left[L-(\theta R+G)\right]f_D(L)dL.\end{aligned}\right\}h(\theta)d\theta. \tag{A.2a}$$

In contrast to Regime 1, daytime operating cost here depends on S because demand from storage may cause gas to turn on. The derivatives of the daytime cost function are the following:

$$\partial\Lambda/\partial R=-\int_{\underline{\theta}}^{1}\theta\left\{\begin{aligned}&c[F_D(\theta R+G-S)-F_D(\theta R-S)]\\&\quad+v[1-F_D(\theta R+G)]\end{aligned}\right\}h(\theta)d\theta<0, \tag{A.2b}$$

$$\partial\Lambda/\partial G=\int_{\underline{\theta}}^{1}\left\{\begin{aligned}&c[1-F_D(\theta R+G-S)]\\&-v[1-F_D(\theta R+G)]\end{aligned}\right\}h(\theta)d\theta, \tag{A.2c}$$

$$\partial\Lambda/\partial S=\int_{\underline{\theta}}^{1}\left\{c\left[F(\theta R+G-S)-F(\theta R-S)\right]\right\}h(\theta)d\theta>0. \tag{A.2d}$$

If ηs MWh are sold from storage, expected *nighttime* operating cost is given by equation (4.3a) in the text:

$$\begin{aligned}\omega\left(G,s\right)\equiv c\int_{\eta s}^{\eta s+G}[L-\eta s]f_N\left(L\right)dL+cG\left[1-F_N\left(G+\eta s\right)\right]\\+v\int_{\eta s+G}^{\infty}[L-(\eta s+G)]f_N\left(L\right)dL.\end{aligned} \tag{A.3}$$

Equations (4.3b) and (4.3c) are thus also valid under Regime 2.

Taking the expectation over s from the description of competitive behavior under Regime 2 in Section 3 yields unconditional expected nighttime operating cost:

$$\Omega(R,G,S) \equiv \int_{\underline{\theta}}^{1} \left\{ \begin{array}{c} F_D(\theta R + G - S)\,\omega(G,S) \\ + \int_{\theta R+G-S}^{\theta R+G} \omega(G,\theta R + G - L) f_D(L)\,dL \\ + \left[1 - F_D(\theta R + G)\right]\omega(G,0) \end{array} \right\} h(\theta)\,d\theta. \tag{A.4a}$$

The derivatives of this function parallel equations (4.4b) – (4.4d) in the text:

$$\begin{aligned} \partial\Omega / \partial R &= \int_{\underline{\theta}}^{1} \left[\int_{\theta R+G-S}^{\theta R+G} \frac{\partial\omega(G,\theta R + G - L)}{\partial R} f_D(L)\,dL \right] h(\theta)\,d\theta \\ &= -\int_{\underline{\theta}}^{1} \theta \left\{ \int_{\theta R+G-S}^{\theta R+G} \eta \overline{P}_N(G,\theta R + G - L)] f_D(L)\,dL \right\} h(\theta)\,d\theta, \end{aligned} \tag{A.4b}$$

(A.4c)

$$\partial\Omega / \partial G = \int_{\underline{\theta}}^{1} \left\{ \begin{array}{l} F_D(\theta R + G - S)\dfrac{\partial \omega(G,S)}{\partial G} \\ + \left[1 - F(\theta R + G)\right]\dfrac{\partial \omega(G,0)}{\partial G} \\ + \int_{\theta R + G - S}^{\theta R + G} \left[\begin{array}{l} \dfrac{\partial \omega(G, \theta R + G - L)}{\partial G} \\ + \dfrac{\partial \omega(G, \theta R + G - L)}{\partial s} \end{array} \right] f_D(L)dL \end{array} \right\} h(\theta)d\theta$$

$$= -(v-c)\int_{\underline{\theta}}^{1} \left\{ \begin{array}{l} F_D(\theta R + G - S)[1 - F_N(G + \eta S)] \\ +[1 - F_D(\theta R + G)][1 - F_N(G)] \\ +\int_{\theta R + G - S}^{\theta R + G} [1 - F_N(G + \eta(\theta R + G - L))] f_D(L)dL \end{array} \right\} h(\theta)d\theta$$

$$- \int_{\underline{\theta}}^{1} \left\{ \int_{\theta R + G - S}^{\theta R + G} \overline{\eta P}_N(\theta R + G - L) f_D(L)dL \right\} h(\theta)d\theta$$

$$= -(v-c)\int_{\underline{\theta}}^{1} \Pr(P_N = v \,|\, \theta) h(\theta)d\theta$$

$$- \int_{\underline{\theta}}^{1} \left\{ \int_{\theta R + G - S}^{\theta R + G} \overline{\eta P}_N(\theta R + G - L) f_D(L)dL \right\} h(\theta)d\theta,$$

(A.4d)

$$\partial\Omega / \partial S = \int_{\underline{\theta}}^{1} \left\{ F_D(\theta R + G - S)\frac{\partial \omega(G,S)}{\partial S} \right\} h(\theta)d\theta$$

$$= -\eta \overline{P}_N(S) \int_{\underline{\theta}}^{1} F_D(\theta R + G - S) h(\theta)d\theta.$$

Aside from the general substitution of (θR+G) for θR, the main difference between equations (A.4) and equations (4.4) in the text is the $\overline{P}_N$ integral in (A.4c) that does not appear in (4.4c). That term stems from case (c), in which competitive suppliers bid the price of energy above c, thus enabling gas generators to more than cover their marginal cost. (See Figure 3 in the text.)

Expected total cost is given by equation (4.5a) in the text, modified to reflect the fact that S affects daytime operating costs under Regime 2:

$$E\left(TC\right) = rR + gG + sS + \Lambda\left(R,G,S\right) + \Omega(G,S). \tag{A.5}$$

The first-order conditions for minimizing this quantity are given by

$$\partial E(TC)/\partial R = C_R - \int_{\underline{\theta}}^{1} \theta \left\{ \begin{array}{c} c[F(\theta R+G-S)-F(\theta R-S)] \\ +v[1-F(\theta R+G)] \\ +\int_{\theta R+G-S}^{\theta R+G} \eta \overline{P}_N(G,\theta R+G-L)]f_D(L)dL \end{array} \right\} h(\theta)d\theta = 0, \tag{A.6a}$$

$$\partial E(TC)/\partial G = C_G - \int_{\underline{\theta}}^{1} \left\{ \begin{array}{l} (v-c)\left\{\left[1-F(\theta R+G)\right]+\Pr(P_N=v|\theta)\right\} \\ +\int_{\theta R+G-S}^{\theta R+G} [\eta \overline{P}_N(G,\theta R+G-L)-c]f_D(L)dL \end{array} \right\} h(\theta)d\theta = 0, \tag{A.6b}$$

$$\partial E(TC)/\partial S = C_S + \int_{\underline{\theta}}^{1} \left\{ \begin{array}{l} c\left[F(\theta R+G-S)-F(\theta R-S)\right] \\ -[\eta \overline{P}_N(S)F_D(\theta R+G-S)] \end{array} \right\} h(\theta)d\theta = 0. \tag{A.6c}$$

These are, again, zero-expected-profit conditions. Condition (A.6a) compares capital cost per unit of renewable capacity to expected revenue per incremental unit of capacity when the market price is c (in case b), when it is v (in case d), and in case (c) when competition suppliers raises the market price above marginal generation cost. Similarly, condition (A.6b) reflects the fact that in case (c) competition among storage suppliers raises the market price above c, so that gas suppliers earn positive operating profits. Finally, condition (A.6c) says that per-unit capital cost of storage plus expected incremental daytime charging cost in case (b) must equal expected incremental nighttime revenue. Charging cost in case (a) is zero. Expected charging cost in case (c), which does not appear in (A.6c), is exactly offset by expected nighttime revenues in that case, which also do not appear.

As in the case of Regime 1, I have been unable to prove that the Hessian corresponding to equations (A.6) is always positive definite. The diagonal elements of that matrix are the following:

$$\partial^2 E(TC)/\partial R^2 = \int_{\underline{\theta}}^{1} \theta \left\{ \begin{array}{c} \theta f_D(\theta R + G - S)[\eta \overline{P}_N(S) - c] \\ +\theta f_D(\theta R + G)[v - \eta \overline{P}_N(0)] \\ +c\theta f_D(\theta R - S) \\ -\int_{\theta R+G-S}^{\theta R+G} \eta \dfrac{\partial \overline{P}_N(G, \theta R + G - L)}{\partial R} f_D(L)dL \end{array} \right\} h(\theta)d\theta > 0. \tag{A.7a}$$

(A.7b)

$$\partial^2 E(TC) / \partial G^2 =$$

$$(v-c)\int_{\underline{\theta}}^{1}\left\{\begin{array}{c} F_D(\theta R+G-S) f_N(G+\eta S) \\ +[1-F_D(\theta R+G)] f_N(G) \\ +(1+\eta)^2 \int_{\theta R+G-S}^{\theta R+G} f_N[G+\eta(\theta R+G-L)] f_D(L) dL \end{array}\right\} h(\theta) d\theta > 0,$$

(A.7c)

$$\partial^2 E(TC) / \partial S^2 =$$

$$\int_{\underline{\theta}}^{1}\left\{\begin{array}{c} f_D(\theta R+G-S)[\eta \overline{P}_N(S)-c] \\ +c f_D(\theta R-S) \\ -F_D(\theta R+G-S)\eta \dfrac{\partial \overline{P}_N(G,S)}{\partial S} \end{array}\right\} h(\theta) d\theta > 0.$$

The first and second terms on the right of (A.7a) and the first term on the right of (A.7c) are positive by the definition of Regime 2. Equation (3.2b) implies

that $\overline{P}_N(G,s)$ is a decreasing function of its second argument. As under Regime 1, these conditions imply that given values of any two of *R*, *G*, and *S*, the competitively determined value of the third variable minimizes expected total social cost.

B. OPTIMA AND EQUILIBRIA IN REGIME 3

The discussion of Regime 3 in the text implies the following equation for expected end-of-daytime charge:

(B.1a)

$$E(s)=\int_{\underline{\theta}}^{1}\left\{\begin{array}{c} F_D(\theta R-S)S+\int_{\theta R-S}^{\theta R-\tilde{s}}(\theta R-L)f_D(L)dL \\ +\tilde{s}[\tilde{\tilde{F_D}}(\theta R+G-s)-F_D(\theta R-s)] \\ +\int_{\theta R+G-\tilde{s}}^{\theta R+G}(\theta R+G-L)f_D(L)dL \end{array}\right\}h(\theta)d\theta.$$

Differentiation yields

(B.1b)

$$\partial E(s)/\partial R=\int_{\underline{\theta}}^{1}\theta\left\{\begin{array}{c} [F_D(\theta R-\tilde{s})-F_D(\theta R-S)] \\ +[F_D(\theta R+G)-F_D(\theta R+G-\tilde{s})] \end{array}\right\}h(\theta)d\theta$$

$$<\int_{\underline{\theta}}^{1}\theta h(\theta)d\theta,$$

(B.1c)

$$\partial E(s)/\partial G=\int_{\underline{\theta}}^{1}\left\{\begin{array}{c} \frac{\partial\tilde{s}}{\partial G}[F_D(\theta R+G-\tilde{s})-\tilde{F}_D(\theta R-s)] \\ +[F_D(\theta R+G)-F_D(\theta R+G-\tilde{s})] \end{array}\right\}h(\theta)d\theta,$$

(B.1d)

$$\partial E(s)/\partial S=\int_{\underline{\theta}}^{1}F_D(\theta R-S)h(\theta)d\theta<1.$$

Increasing R shifts the $s(L_D)$ curve to the right, and s is increased when L_D is in two distinct intervals. Increasing G decreases $\tilde{s}$ from (3.5b), shifting the curve down for L_D between $(\theta R-\tilde{s})$ and $(\theta R+G-\tilde{s})$, and shifting it to the right beyond $(\theta R + G - \tilde{s})$. Thus, in contrast to Regime 1 (under which $\partial E(s)/\partial G = 0$) and Regime 2 (in which $\partial E(s)/\partial G > 0$), here the impact of increases in G on s cannot be signed in general. Increasing S just shifts the $s(L_D)$ curve up to the left of $(\theta R - S)$.

Expected daytime operating cost under Regime 3 is given by

$$\Lambda(R,G) = \int_{\underline{\theta}}^{1} \left\{ \begin{array}{l} c \int_{\theta R - \tilde{s}}^{\theta R + G - \tilde{s}} [L - (\theta R - \tilde{s})] f_D(L) dL \\ + cG[1 - F_D(\theta R + G - \tilde{s})] \\ + v \int_{\theta R + G}^{\infty} [L - (\theta R + G)] f_D(L) dL \end{array} \right\} h(\theta) d\theta. \tag{B.2a}$$

Note that S does not affect daytime operating cost under this regime, since changes in S only affect charging in case (a), when marginal operating cost is zero. Differentiation yields

$$\partial \Lambda / \partial R = -\int_{\underline{\theta}}^{1} \theta \left\{ \begin{array}{c} c[F_D(\theta R + G - \tilde{s}) - \tilde{F}_D(\theta R - s)] \\ + v[1 - F_D(\theta R + G)] \end{array} \right\} h(\theta) d\theta, \tag{B.2b}$$

$$\partial \Lambda / \partial G = \int_{\underline{\theta}}^{1} \left\{ \begin{array}{c} c[1 - F_D(\theta R + G - \tilde{s})] \\ + c \dfrac{\partial \tilde{s}}{\partial G} [F_D(\theta R + G - \tilde{s}) - \tilde{F}_D(\theta R - s)] \\ - v[1 - F_D(\theta R + G)] \end{array} \right\} h(\theta) d\theta. \tag{B.2c}$$

If ηs MWh are sold from storage, conditional expected nighttime operating cost, $\omega(G,s)$, is given by equation (4.3a) in the text, as under Regimes 1 and 2:

(B.3)
$$\omega(G,s) \equiv c \int_{\eta s}^{\eta s+G} [L-\eta s] f_N(L)\,dL + cG\left[1-F_N(G+\eta s)\right] + v\int_{\eta s+G}^{\infty} [L-(\eta s+G)] f_N(L)\,dL.$$

Equations (4.3b) and (4.3c) are accordingly also valid under this regime.

Taking the expectation of ω over s, using the characterization of competitive behavior under this regime in Section 3, yields the unconditional expectation of nighttime operating costs:

(B.4a)
$$\Omega(R,G,S) = \int_{\underline{\theta}}^{1} \left\{ \begin{array}{c} F_D(\theta R - S)\omega(G,S) + \int_{\theta R - S}^{\theta R - \tilde{s}} \omega(G,\theta R - L) f_D(L)dL \\ + [F_D(\theta R + G - \tilde{s}) - F_D(\theta R - \tilde{s})]\omega(G,\tilde{s}) \\ + \int_{\theta R + G - \tilde{s}}^{\theta R + G} \omega(G,\theta R + G - L) f_D(L)dL \\ + [1 - F_D(\theta R + G)]\omega(G,0) \end{array} \right\} h(\theta)d\theta.$$

Differentiation of this expression yields

(B.4b)
$$\partial\Omega/\partial R = \int_{\underline{\theta}}^{1} \left\{ \begin{array}{l} \int_{\theta R - S}^{\theta R - \tilde{s}} \frac{\partial\omega(G,\theta R - L)}{\partial R} f_D(L)dL \\ + \int_{\theta R + G - \tilde{s}}^{\theta R + G} \frac{\partial\omega(G,\theta R + G - L)}{\partial R} f_D(L)dL \end{array} \right\} h(\theta)d\theta$$
$$= -\eta\theta \int_{\underline{\theta}}^{1} \left\{ \begin{array}{l} \int_{\theta R - S}^{\theta R - \tilde{s}} \overline{P}_N(\theta R - L) f_D(L)dL \\ + \int_{\theta R + G - \tilde{s}}^{\theta R + G} \overline{P}_N(\theta R + G - L) f_D(L)dL \end{array} \right\} h(\theta)d\theta,$$

(B.4c)

$$\partial\Omega/\partial G = -\int_{\underline{\theta}}^{1}\left\{\begin{array}{c}(v-c)\Pr(P_N = v|\theta)\\ +c\dfrac{\partial\tilde{s}}{\partial G}[F_D(\theta R+G-\tilde{s})-\tilde{F}_D(\theta R-s)]\\ +\eta\int_{\theta R+G-\tilde{s}}^{\theta R+G}\overline{P}_N(\theta R+G-L)f_D(L)dL\end{array}\right\}h(\theta)d\theta,$$

(B.4d)

$$\partial\Omega/\partial S = -\int_{\underline{\theta}}^{1}F_D(\theta R-S)\eta\overline{P}_N(G,S)h(\theta)d\theta.$$

Expected total cost is again given by a slight modification of equation (4.5a) in the text:

(B.5)

$$E\left(TC\right) = rR+gG+sS+\Lambda\left(R,G\right)+\Omega(R,G,S).$$

Differentiation yields the first-order necessary conditions for a minimum of expected total cost

(B.6a)

$$\partial E(TC)/\partial R = C_R$$

$$-\int_{\underline{\theta}}^{1}\theta\left\{\begin{array}{c}c[F_D(\theta R+G-\tilde{s})-\tilde{F}_D(\theta R-s)]\\ -v[1-F_D(\theta R+G)]\\ +\eta\int_{\theta R-S}^{\theta R-\tilde{s}}\overline{P}_N(\theta R-L)f_D(L)dL\\ +\eta\int_{\theta R+G-\tilde{s}}^{\theta R+G}\overline{P}_N(\theta R+G-L)f_D(L)dL\end{array}\right\}h(\theta)d\theta = 0,$$

(B.6b)

$$\partial E(TC)/\partial G = C_G$$

$$-\int_{\underline{\theta}}^{1}\left\{\begin{array}{l}(v-c)\{\Pr(P_N = v|\theta)+[1-F_D(\theta R+G)]\}\\ +\int_{\theta R+G-\tilde{s}}^{\theta R+G}[\eta\overline{P}_N(G,\theta R+G-L)-c]f_D(L)dL\end{array}\right\}h(\theta)d\theta,$$

$$\partial E(TC)/\partial S = C_S - \int_{\underline{\theta}}^{1} F_D(\theta R - S)\eta \overline{P}_N(G,S)h(\theta)d\theta = 0. \tag{B.6c}$$

These conditions once again imply zero expected profits for each technology. Condition (B.6a) compares unit capital cost for renewable generation with the sum of expected revenue per unit of capacity in cases (b)-(e). Similarly, condition (B.6b) compares unit capital cost for gas generation with the sum of expected net revenues above variable cost in cases (d) and (e) and in nighttime shortage conditions. (Comparing (B.2c) and (B.4c) reveals that the change in $\underset{S}{\sim}$ induced by a marginal increase in G has equal and opposite effects on expected gas generation costs in daytime and nighttime periods.) Finally, (B.6c) compares unit capital cost of storage with the marginal expected revenue from the increased charging in case (a) that a unit increase in storage capacity would induce. As in the other regimes, payments by storage suppliers above marginal generation costs, in cases (b) and (d) here, show up as revenues for renewable and gas generators but not as costs for storage suppliers, since those payments exactly equal the expected nighttime revenue from sales of the incremental stored energy.

As in the cases of Regimes 1 and 2, I have been unable to prove that the Hessian corresponding to equations (B.6) is always positive definite. The diagonal elements of that matrix are the following:

(B.7a)

$$\partial^2 E(TC)/\partial R^2 =$$

$$\int_{\underline{\theta}}^{1} \theta^2 \left\{ \begin{array}{l} \eta \tilde{P}_N(S) f_D(\theta R - S) + [v - \eta \overline{P}_N(0)] f_D(\theta R + G)] \\ +\eta^2 (v-c)\theta \int_{R-S}^{\theta R - \tilde{s}} f_N[G + \eta(\theta R - L)] f_D(L) dL \\ +\eta^2 (v-c) \int_{\theta R + G - \tilde{s}}^{\theta R + G} f_N[G + \eta(\theta R + G - L) f_D(L) dL \end{array} \right\} h(\theta) d\theta > 0,$$

(B.7b)

$$\partial^2 E(TC)/\partial G^2 =$$

$$(v-c)\int_{\underline{\theta}}^{1} \left\{ \begin{array}{c} F_D(\theta R - S) f_N(G + \eta S) \\ + \int_{\theta R - S}^{\theta R - \tilde{s}} f_N[G + \eta(\theta R - L)] f_D(L) dL \\ + f_N(G + \eta \tilde{s})[\tilde{\tilde{F}}_D(\theta R + G - s) - F_D(\theta R - s)] \\ +(1+\eta)^2 \int_{\theta R + G - \tilde{s}}^{\theta R + G} f_N[G + \eta(\theta R + G - L)] f_D(L) dL \\ + f_N(G)[1 - F_D(\theta R + G)] \end{array} \right\} h(\theta) d\theta$$

$$+ \int_{\underline{\theta}}^{1} [v - \eta \overline{P}_N(0)] f_D(\theta R + G) h(\theta) d\theta > 0,$$

(B.7c)

$$\partial^2 E(TC)/\partial S^2 =$$

$$\int_{\underline{\theta}}^{1} \left\{ \begin{array}{c} f_D(\theta R - S)\eta \overline{P}_N(S) \\ +F_D(\theta R - S)(v-c)\eta^2 f_N(G + \eta S) \end{array} \right\} h(\theta) d\theta > 0.$$

Equations (3.5) were used in the derivation of (B.7a) and (B.7b). Equation (B.7c) demonstrates that as under Operating Rules 1 and 2, in long-run competitive equilibrium, given values of any two of *R*, *G*, and *S*, the competitively determined value of the third variable minimizes expected total social cost.