5. APPENDIX

5.1 Look-Ahead Dispatch Implementations in Practice

Many liberalised wholesale power markets around the world have implemented a form of look-ahead dispatch, especially in the US.[27] Many of these markets already operate day-ahead and/or short-term unit commitment runs of the dispatch process which, by their nature, cover several dispatch intervals (usually over 24 hours) into the future. For these markets, the extension of the real-time market to cover several intervals in the future is straightforward.

For example, in **NYISO**, there is a day-ahead market which operates hourly and which optimises over the 24 hours of the following day. There is also a "Real-Time Commitment" (RTC) process which schedules in 15-minute intervals and which looks out over a nominal two-and-a-half hour period. The "Real-Time Dispatch" (RTD) process dispatches in 5- or 15-minute intervals and nominally looks out between 55- and 65-minutes. Source: NYISO (2019).

In **CAISO**, there is a short-term forward market with unit commitment known as the "fifteen-minute market" (FMM). The FMM runs for a horizon of up to four and a half hours ahead and as short as one hour ahead, and runs approximately 37.5 minutes ahead of the binding interval. The FMM unit commitment instructions are financially binding. There is also a five-minute "real-time dispatch market" (RTD). The RTD runs for a horizon of up to one hour and five minutes and runs 7.5 minutes ahead of the binding interval. Source: CAISO (2019).

The **PJM** market also features a short-term forward market with unit commitment, known as the "Intermediate Term Security Constrained Economic Dispatch" (IT SCED). This process solves a multi-interval, time-coupled dispatch problem which is used to make resource commitment decisions, amongst other things. In addition there is a "Real-Time Security Constrained Economic Dispatch" (RT SCED) which finds the optimal dispatch of energy and reserves over a near-term look-ahead period. We understand this near-term look-ahead period to be fifteen minutes, although there is discussion of this time period being reduced to ten minutes. Source: PJM (2019).

In the **Mid-Continent ISO**, the issue of implementation of look-ahead dispatch has been considered, but, at the time of writing, this policy issue has been put on hold (i.e., placed in the 'parking lot').[28]

Finally, in **ERCOT**, look-ahead dispatch has been considered but, to our knowledge, not yet implemented. See the discussion in Mickey (2015) and Xie et al. (2013).

5.2 Proofs of Theorems

Proof 1 (Proof of Theorem 1) *Let's suppose that, at time t,* $(g^t, p^t)_{t+1,\dots,T}$ *solves* $LAD(t, T|g_t^t)$ *(as defined in equations 1-3). This implies that there exist dual variables* $(\gamma^{t,U}, \gamma^{t,D})_{i,t+1,\dots,T}$ *such that the set of variables* $(g^t, p^t, \gamma^{t,U}, \gamma^{t,D})$ *minimises the following Lagrangian (while satisfying the primal feasibility, dual feasibility, and complementary slackness conditions):*

$$\begin{aligned}\mathcal{L}(g^t, p^t, \gamma^{t,U}, \gamma^{t,D}) = &\sum_{i,s} c_i(g_{i,s}^t) + \sum_s p_s^t (L_s^t - \sum_i g_{i,s}^t) \\ &- \sum_{i,s} \gamma_{i,s}^{t,U} (R_i - g_{i,s}^t + g_{i,s-1}^t) \\ &- \sum_{i,s} \gamma_{i,s}^{t,D} (g_{i,s}^t - g_{i,s-1}^t + R_i) \end{aligned} \quad (11)$$

[27] According to Mickey (2015), multi-interval real time markets have already been implemented by PJM, ISO-NE, NYISO, and CAISO, and, as of 2015, were under consideration in MISO and ERCOT

[28] See: https://www.misoenergy.org/stakeholder-engagement/issue-tracking/look-ahead-dispatch/

We can write this as:

$$\mathcal{L}(g^t, p^t, \gamma^{t,U}, \gamma^{t,D}) = -\sum_i \mathcal{L}_i^{PM}(g_i^t, p^t, \gamma_i^{t,U}, \gamma_i^{t,D}) + \sum_s p_s^t L_s^t \tag{12}$$

Where $\mathcal{L}_i^{PM}$ *is the Lagrangian for each generator's individual profit maximisation decision:*

$$\begin{aligned}\mathcal{L}_i^{PM}(g_i^t, p^t, \gamma_i^{t,U}, \gamma_i^{t,D}) = &\sum_s p_s^t g_{i,s}^t - c_i(g_{i,s}^t) \\ &+ \sum_{i,s} \gamma_{i,s}^{t,U}(R_i - g_{i,s}^t + g_{s-1}^t) \\ &+ \sum_{i,s} \gamma_{i,s}^{t,D}(g_{i,s}^t - g_{s-1}^t + R_i)\end{aligned} \tag{13}$$

It follows that (g_i^t, p^t) *are a solution to generator i's profit maximisation problem at time t (defined in equations 5-6).*

Moreover, by Bellman's Principle, if $(g_s^t, p_s^t)_{s=t+1,\ldots,T}$ *is a solution to* $LAD(t, T|g_t^t)$*, it follows that* $(g_s^t, p_s^t)_{s=\hat{t}+1,\ldots,T}$ *is a solution to the sub-problem* $LAD(\hat{t}, T|g_{\hat{t}}^t)$*. By the same argument as above,* $(g_{i,s}^t, p_s^t)_{s=\hat{t}+1,\ldots,T}$ *is also profit-maximising for generator i, for the sub-problem starting at time* $\hat{t}$*.*

Proof 2 (Proof of Theorem 2) *Let's suppose that, at time t, the power system is currently in a state given by* $(g_{i,t}^t)$*. The set of future forecast prices are* $(p_s^t)_{s\in[t+1,T]}$*.*

At some time $u \in [t, T-1]$*, each generator solves a profit-maximisation problem to find the optimal offer curve at time u, in the light of the future forecast prices* $(p_s^t)_{s\in[u+1,T]}$ *and the state of the power system* $(g_{i,u}^t)$*: The offer curve is denoted:* $\hat{c}_{i,u}^t(g|p^t, g_{i,u}^t)$*. The offer curve is the solution to the following profit-maximisation problem:*

$$\max_{g_{i,s}^t} \sum_{s\in[u+1,T]} [p_s^t g_{i,s}^t - c_i(g_{i,s}^t)] \tag{14}$$

$$s.t. \forall s \in [u+1, T] - R_i \le g_{i,s}^t - g_{i,s-1}^t \le R_i \tag{15}$$

For this problem there exist dual variables $(\gamma_{i,s}^{t,U}, \gamma_{i,s}^{t,D})_{s\in[u+1,T]}$ *which satisfy the following first order condition:*

$$p_u^t = c_i'(g_{i,u}^t) + (\gamma_{i,u}^{t,U} - \gamma_{i,u+1}^{t,U}) - (\gamma_{i,u}^{t,D} - \gamma_{i,u+1}^{t,D}) \tag{16}$$

Now let's define the profit-maximising offer curve for the generator as follows:

$$\hat{c}_{i,u}^t(g|p^t, g_{i,u}^t) = \begin{cases} -Mg, & \text{if } 0 \le g \le \max(-R_i + g_{i,u}^t, 0) \\ c_i(g) + (\gamma_{i,u}^{t,U} - \gamma_{i,u+1}^{t,U})g - (\gamma_{i,u}^{t,D} - \gamma_{i,u+1}^{t,D})g, & \max(-R_i + g_{i,u}^t, 0) \le g \le R_i + g_{i,u}^t \\ Mg, & g \ge R_i + g_{i,u}^t \end{cases} \tag{17}$$

Here M is a large number. Now let's consider the one-shot economic dispatch problem at time u. This problem is to minimise the cost, as announced by each generator, subject to the energy balance constraint:

$$\min_{g_{i,u}^t} \sum_i \hat{c}_{i,u}^t(g_{i,u}^t) \tag{18}$$

$$s.t. \sum_i g_{i,u}^t = L_u^t \tag{19}$$

The first-order condition for this problem is as follows:

$$\hat{c}'^{t}_{i,u}(g^t_{i,u}) = \pi^t_u \tag{20}$$

Using equation 17, this can be written as:

$$c'_i(g^t_{i,u}) + (\gamma^{t,U}_{i,u} - \gamma^{t,U}_{i,u+1}) - (\gamma^{t,D}_{i,u} - \gamma^{t,D}_{i,u+1}) = \pi^t_u \tag{21}$$

By the assumption on the forecast prices, we have that $p^t_u = \pi^t_u$. *It follows that* $(g^t, \pi^t, \gamma^{t,U}, \gamma^{t,D})_{u \in [t+1,T]}$ *is a solution to* $LAD(t, T|g^t_{i,t})$*, which (recall) is defined as follows:*

$$\min_{g^t_{i,s}} \sum_{i,s} c_i(g^t_{i,s}) \tag{22}$$

$$s.t. \forall s \in [t+1, T], \quad \sum_i g^t_{i,s} = L^t_s, \quad (\pi^t_s) \tag{23}$$

$$\forall i, s \in [t+1, T], \quad -R_i \leq g^t_{i,s} - g^t_{i,s-1} \leq R_i, \quad (\gamma^{t,D}_{i,s}, \gamma^{t,U}_{i,s}) \tag{24}$$

Moreover, if we assume that the problem is strictly convex, so that the dual variable on the energy balance constraint is unique, it follows that the prices that emerge from this process are the same as the prices which were forecast at the outset: $\pi^t_s = p^t_s$ *for* $s \in [t+1, T]$.

Finally, given perfect foresight we have that the prices that emerge from sequential runs of $LAD(t, T)$ *are the same as forecast at the outset:* $p^{s-1}_s = p^t_s$, *for* $s \in [t+1, T]$.