Crediting Wind and Solar Renewables in Electricity Capacity Markets: The Effects of Alternative Definitions upon Market Efficiency

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On-Line Appendix A:

Supporting proofs of social cost minimization and market equilibrium

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In our paper, we state that an optimization model with a capacity constraint can yield the social cost minimizing mix of generation capacity, if the constraint's reserve margin is chosen appropriately. The social cost minimizing solution involves only an energy market and price rises to VOLL at times of energy shortages. Due to space limitations in the paper, the proof of this claim is presented in this appendix as Proposition 1. Proposition 2, also proven here, then states that a power market equilibrium that includes a capacity market can be simulated using a cost minimization model with a capacity constraint.

PROPOSITION 1: A capacity market model (optimization version) with price caps can yield the same solution as social cost minimization if the correct reserve margin is chosen for the capacity constraint and generators are credited with their marginal capacity contribution.

A long-term equilibrium can be determined using a classical screening curve method for optimization (Stoft, Steven. (2002). *Power System Economics*, 44-45. Piscataway: IEEE Press.)

Nomenclature

Sets:

$g \in G$: Set of generators (g= 1, 2, 3 for the example)

$h \in H$: Set of all hours (e.g., h= 1, …,8760)

Variables:

x_g [MW]: Installed generator capacity

$e_{h,g}$ [MWh]: Hourly dispatch of generators

eue_h [MWh]: Hourly unserved energy

Parameters:

FC_g [\$/MW/year]: Generator levelized annual fixed costs (includes cost of capital)

VC_g [\$/MWh]: Generator variable cost (includes production and O&M)

DM_h [MWh]: Customer demand, also p

PD [MW]: Peak customer demand, highest annual demand for electricity

VOLL [$/MWh]: Value of Lost Load, highest amount customers are willing to pay for electricity

Approximation assumptions include:

- Annual load can be arranged in a load duration curve, *H(p)*, that provides the number of hours per year that each demand level, *p*, is exceeded (see Figure A1). *H(p)* is continuous and smooth ranging from *PD* (peak demand) to *MD* (minimum demand) where $MD \geq 0$ and has a negative but finite derivative, $dH/dp < 0$. Figure A1.
- Demand is inelastic – the load duration curve does not change with price.
- Generator cost can be described as a fixed per-unit investment cost (per MW of installed capacity) and linear variable costs (per MWh of output; this excludes number of starts, minimum runs in terms of MW or time, and non-linear ramp rates)
- Generators have 100% availability.
- The Value of Lost Load (*VOLL*) is higher than VC_g for all efficient generators.

Under these assumptions, the optimal (least cost) generation investment and operation generally involves a mix of generators that represent a range of tradeoffs between fixed and variable costs. Generators that are required to operate shorter periods of time have relatively low fixed costs but high variable costs while generators that are required to operate long periods of time (baseload) have relatively high fixed costs but low operating costs. Generators whose costs lie on the efficient cost frontier (defined as the set of generators for which no other generator has both lower fixed costs and lower variable costs) are arranged from g=1,…, G in increasing order of VC_g and decreasing order of FC_g. The solution can be obtained using the well-known screening curve method (Stoft, op. cit.), in which the least-cost generator is identified for each value of *h* on the x-axis of the load duration curve from 0 to 8760 (see Figure A1).

The proof involves comparing the first order conditions of two optimization models (one for total cost minimization, and one for cost minimization subject to a capacity constraint) and defining the assumptions under which they are equivalent.

Model 1: The socially optimal level of capacity considering VOLL is determined by maximizing social welfare (minimizing social cost) in an unconstrained optimization. There is no capacity requirement in the form of a reserve margin constraint.

Define $xcum_g = \sum_{i=1}^{g} x_g$ (cumulative installed generator capacity of plants) (A1)

Define $xcum_0 = 0$ (initialization) (A2)

Objective: Minimize Social Cost *SC* = investment cost + variable cost + total value of lost load

$$= \sum_{g=1}^{G} FC_g * x_g + \sum_{g=1}^{G} \int_{xcum_{g-1}}^{xcum_g} VC_g * H(p)dp + \int_{xcum_G}^{PD} VOLL * H(p)dp \quad \text{(A3)}$$

Where the total energy production from plant *g* can be calculated as:

$$\sum_{h \in H} e_{h,g} = \int_{xcum_{g-1}}^{xcum_g} H(p)dp \text{ and } \sum_{h \in H} eue_h = \int_{xcum_G}^{PD} H(p)dp \quad \text{(A4)}$$

That optimization is a function only of the amount of each type of capacity, and there are no constraints (other than non-negativity, which we disregard assuming that there is a strictly positive amount of each variable). As a result, the first order condition for optimization is:

$$\partial SC/\partial x_g = 0$$

$$= FC_g + VC_g * \mathrm{H}(xcum_g) + \sum_{i=g+1}^{G} VC_i * [\mathrm{H}(xcum_i) - \mathrm{H}(xcum_{i-1})] - VOLL * \mathrm{H}(xcum_G),\ \forall g \in G \quad \text{(A5)}$$

= Marginal investment cost + marginal total variable cost - variable cost savings from more expensive units – reduction in total *VOLL*

Equations (A1),(A5) define a system with $2G$ equations and $2G$ unknowns (x_g, $xcum_g$). Let the solution be x_g^*, $xcum_g^*$, from which each plant's expected energy production e_g* and the total expected unserved energy eue^* can be calculated. (Under above assumptions, all are strictly positive, and $xcum_G^* < PD$.)

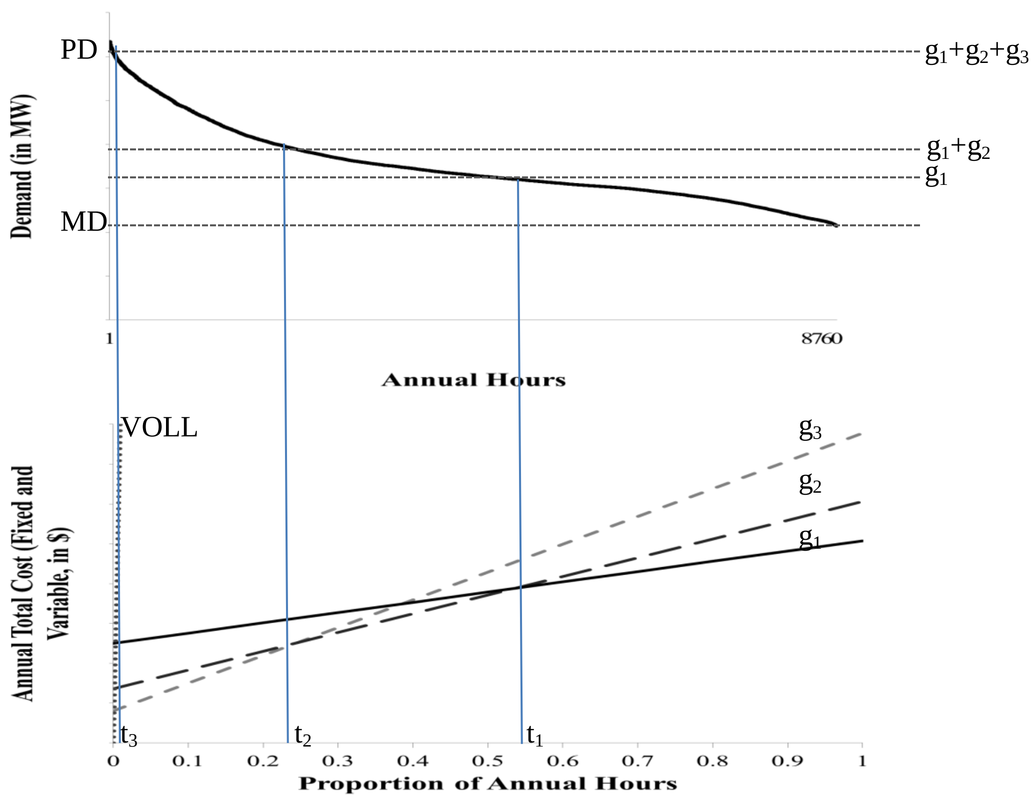

Figure A1: (Top) Load Duration Curve - Number of hours per year that load is at or below each demand level; (Bottom) Generation Screening Curve – Fixed and variable cost of operation

Model 2: The market model is a constrained optimization in which a price cap is imposed in the energy market and a policy constraint, in the form of a minimum reserve margin, is imposed to counteract capacity shortages that would otherwise occur due to the price cap. The market with price cap (*PC*) and installed capacity (*ICAP*) constraint is as follows. Let the required capacity $ICAP = PD*(1+RM)$, which we assume is less than or equal to the $xcum_G^*$ from the social cost minimizing solution. Let $PC < VOLL$.

Additional Parameters:

PC [\$/MWh]: Energy market price cap

RM [fraction]: Reserve margin, generation amount in excess of load needed to meet reliability criteria (note that this can be negative)

ICAP [MW]: Minimum capacity required to meet reliability criteria

Objective: Minimize (Adjusted) Social Cost = *ASC*

ASC = fixed cost+variable cost+total value of lost load (evaluated at price cap instead of *VOLL*)

$$= \sum_{g=1}^{G} FC_g * x_g + \sum_{g=1}^{G} \int_{xcum_{g-1}}^{xcum_g} VC_g * H(p)dp + \int_{xcum_G}^{PD} PC * H(p)dp \quad \text{(A6)}$$

$$\text{s.t.} \quad \sum_{i=1}^{G} x_g \geq ICAP \quad \text{(A7)}$$

Note: Since $PC < VOLL$, (A7) will bind in the optimal solution if $ICAP = \sum_{i=1}^{G} x_g^*$, so that condition is treated as an equality constraint in the following Lagrangian (A8).

MIN $\mathscr{L}$ =

$$\sum_{g=1}^{G} FC_g * x_g + \sum_{g=1}^{G} \int_{xcum_{g-1}}^{xcum_g} VC_g * H(p)dp + \int_{xcum_G}^{PD} PC * H(p)dp - \theta\left(\sum_{i=1}^{G} x_g - ICAP\right) \quad \text{(A8)}$$

Its first order conditions are:

$\partial\mathscr{L}/\partial x_g = 0$

$$= FC_g + VC_g * H(xcum_g) + \sum_{i=g+1}^{G} VC_i * [H(xcum_i) - H(xcum_{i-1})] - PC * H(xcum_G) - \theta$$

$$\forall\, g \in G \quad \text{(A9)}$$

$$\partial\mathscr{L}/\partial\theta = 0 = \sum_{i=1}^{G} x_g - ICAP \quad \text{(A10)}$$

These, plus the definition of $xcum_g$ (A1), define the optimal solution.

Now, let us assume that the regulator exogenously sets $ICAP = xcum_G^*$ (socially optimal total capacity). Now consider the candidate solution $x_g = x_g^*$ (from Model 1) and

$$\theta = (VOLL - PC) * H(xcum_G) \quad \text{(A11)}$$

Note that if we substitute (A11) into (A9), the original first order condition (A5) for Model 1 results. x_g^* will satisfy (A9) in that case and, by the definition of *ICAP*, x_g^* will also satisfy (A10). Thus x_g^* plus this θ are a solution to Model 2, which is unique under the above assumptions. Therefore, the market model with a price cap and a capacity market with an appropriately chosen reserve margin yield the same capacity (and thus operations and total cost) as the social cost minimizing problem. **Q.E.D.**

PROPOSITION 2: A capacity market model (optimization version, Model 2) with price caps yields the same solution as the capacity market model posed as an equilibrium problem.

Model 3: Equilibrium Model. Let each producer g be a price-taking profit maximizer, so it believes that its decisions will not change the number of hours for each market condition (and price) $H(xcum_{i-1}) - H(xcum_i)$ for any i (i.e., it treats $H(xcum_i)$ as fixed in each profit maximization). Each producer also believes it cannot change the price of capacity, θ.

For simplicity, assume the short run price in each hour equals the VC_g of the marginal generator in that hour (or *VOLL* if *eue* > 0, i.e., if producer capacity is less than load).

Each producer's profit is

$PROFIT_g =$

$$\sum_{i=g+1}^{G} VC_i * [H(xcum_{i-1}) - H(xcum_i)] * x_g + [PC * H(xcum_G) * x_g]$$
$$-VC_g * H(xcum_g) * x_g + (\theta - FC_g) * x_g \qquad \text{(A12)}$$

= (revenue from energy market to *g* minus its variable cost during periods when price > VC_g) + (revenue from capacity market – investment cost)

(Note, revenue and variable expenses during the time generator g is on the margin is not included, as price = VC_g during those times. Of course, *g* does not operate when demand is so low that some $i < g$ is on the margin. This assumes merit order operation (order of increasing variable cost), discussed above in Models 1, 2).

The market equilibrium is found by solving (A1), (A13) and (A14) for x_g , $xcum_g$, and θ:

Profit maximization: $\partial PROFIT_g/\partial x_g = 0$

$$= \sum_{i=g+1}^{G} VC_i * [H(xcum_{i-1}) - H(xcum_i)] + PC * H(xcum_G) - VC_g * H(xcum_g) + (\theta - FC_g)$$
$$\forall g \in G \qquad \text{(A13)}$$

Market clearing for capacity: $\sum_{g=1}^{G} x_g = \text{ICAP}$ (A14)

With some rearrangement, (A13) is the same as (A9); meanwhile (A14) is the same as (A10), and (A1) has to be satisfied in both cases, so the equilibrium problem is the same as the optimization problem Model 2. Therefore, a solution to Model 2 is also an equilibrium, and thus a single optimization model can be used to obtain the market equilibrium. **Q.E.D.**