Appendix 1.

1. The model

We develop a version of a neoclassical growth model. In this context, we represent the Spanish economy using a dynamic stochastic general equilibrium model, assuming rational expectations and the characteristics of a small open economy. In this model households, firms and the external sector interact by trading a final good, government bonds and three energy inputs. Finally, there are four possible sources of macroeconomic fluctuations: a productivity shock and three fossil fuel price shocks (crude oil, coal and natural gas). Time is discrete and infinite.

In this Appendix the production function of the economy is divided into three to make easier the resolution of the model: the production function of final goods and services, a production function for "aggregate energy" and a production function for "intermediate energy". However, the function "aggregate energy" and the function "intermediate energy" are instrumental variables and do not have an intuitive economic meaning. This is the reason why in the section of the paper titled "The Model" we only show the production function for final goods and services with five productive inputs, excluding instrumental variables.

1.1 The Household

The economy is made up of a representative household, that obtains utility from the consumption (C) and leisure $(L-N)$, where L is the time endowment of the members of household and N the time devoted to work in the economy. The representative household maximizes its expected utility defined over the stochastic sequences of consumption (C) and labor (N) subject to the budget constraint:

$$\mathrm{U} = E_0 \left\{ \sum_{t=0}^{\infty} \rho^t \frac{1}{1-\sigma} [(C_t - \Psi N_t^{\nu})^{1-\sigma}] - 1] \right\}, \tag{1}$$

where $\Psi > 0,\ \sigma > 0,\ \nu > 1$, E_0 denotes the expectation based upon the informational set available in the initial period, ρ is the intertemporal subjective discount rate, σ is the risk aversion parameter, Ψ is the disutility related to labor, and $1/(\nu - 1)$ is the intertemporal elasticity of substitution for the labor supply[1].

Resources not consumed during each period are used to increase the stock of private physical capital of the following period. The investment at period *t* accumulates productive capital available at period t+1, and there is a cost of adjustment that depends on the net investment[2]. The capital accumulation in each sector is given by:

$$I_t = K_{t+1} - (1-\delta)K_t + \frac{\Phi}{2}\left(\frac{K_{t+1} - K_t}{K_t}\right)^2 \quad , \tag{2}$$

where δ is the depreciation rate and Φ is the adjustment cost parameter.

The household total income consists of three components: i) labor income $(w_t N_t)$ where w_t is the real wage, ii) the return on the real capital stock $(r_t K_t)$, where r_t is the real return on physical capital, and iii) the real return on holdings of debt $(r_t^* D_t)$. On the other hand, the current income and financial wealth can be used for: i) consumption (C_t), ii) investment in physical capital (I_t), and iii) changes in his/her portfolio $(D_t - D_{t-1})$. Then the budget constraint is given by:

$$C_t + I_t + (1 + r_t^*)D_t = w_t N_t + r_t K_t + D_{t+1}\ . \tag{3}$$

[1] A well-known fact in the real business cycle literature for small open economies is that under Cobb-Douglas preferences in consumption and leisure, the consumption path is too smooth compared to the data. Correia, Neves and Rebelo (1995) show that under the preferences that we are using, this problem does not arise due to the absence of income effects on labor.

[2] The adjustment cost function is assumed to be homogeneous of degree zero. In particular we take the quadratic functional form from Bruno and Portier (1995).

bonds are one-period securities traded in international markets where r_t^* denotes the interest rate at which domestic residents can borrow from international markets in period *t*. We assume that the interest rate faced by domestic agents r_t is increasing in the aggregate stock of debt D_t.[3] More precisely, we assume that r_t evolves according to:

$$r_t = r_t^* + \Lambda(D_t) \ , \tag{4}$$

where r_t^* denoted the exogenous international interest rate and $\Lambda(D_t)$ is the risk premium of the domestic country assumed to be a strictly increasing function. For this purpose we follow the second strategy proposed by Schmitt-Grohé and Uribe (2003) in their seminal paper. We assume the following functional form for the risk premium: $\Lambda(D) = sign(D_{t-1})\varphi\left[e^{|D_{t-1}-D_{ss}|} - 1\right]$, where φ is a parameter and D_{ss} is the level of debt in the steady state[4].

Hence, the representative household chooses the paths $\{C_t, N_t, D_{t+1}, K_{t+1}\}$, taking prices $\{w_t, r_t, r_t^*\}$ and state variables $\{K_t, D_t\}$ as given, and maximizes (1) subject to (2), (3), and (4).

1.2 Firms

There are only three primary sources of energy in this economy: crude oil, natural gas and coal[5]. Natural gas and coal are used as inputs to produce another intermediate input, label 'intermediate energy.' Despite the fact that coal and natural gas are used in electricity production, we do not label this intermediate input as 'electricity' because natural gas and coal

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[3] The computation of real business cycle models for small open economies has been troublesome because the dynamics are such that the unconditional variance of debt or consumption is infinite. The specifications we adopt make such dynamics stationary.

[4] We follow Lim and McNelis (2008) for this formulation of the risk premium.

[5] We are aware that nuclear and renewable energy play a relevant role in the Spanish energy system. However, our model does not take these energy sources into consideration. We focus on the impact of unexpected fossil fuel price changes on the business cycle, rather than on energy policies. Renewable and nuclear energy consumption responds to political strategies that have to do with environmental concerns, energy security, industrial development, etc. In any case, these energies represent only around 18 percent of total energy consumption.

are also used in many industrial process[6]. The name electricity could be misleading. In a second stage, 'intermediate energy' and oil are combined to produce 'aggregate energy'. 'Aggregate energy', capital and labor are the inputs used to produce final goods and services. This approach gives us more flexibility to model the energy system.

1.2.1 Intermediate Energy Sector

The first firm produces an energy product labeled *intermediate energy* (E^*). *Intermediate energy* is produced using only two inputs, natural gas and coal. As mention before, coal and natural gas represent the bulk of electricity production, but are also used in many industrial processes. Intermediate energy is produced by a representative firm that operates in a competitive market according with a Constant Elasticity of Substitution (CES) technology of production with constant returns to scale. The function that relates the technology of production is expressed as:

$$E_t^* = \left(bE_{g_t}^{\delta_E} + (1-b)E_{c_t}^{\delta_E}\right)^{\frac{1}{\delta_E}} , \qquad (5)$$

where E_{g_t} and E_{c_t} are natural gas and coal, b is the participation of natural gas on E_t^* and $1/(1-\delta_E)$ is the interfuel elasticity of substitution between coal and natural gas. The representative firm in this sector solves the following problem[7]:

$$\max_{\{E_{g_t}, E_{c_t}\}} \Pi_{E^*} = P_{E_t^*}E_t^* - P_{g_t}E_{g_t} - P_{c_t}E_{c_t} ,$$

subject to: E_t^* given by (5),

where $P_{E_t^*}$ is the price of *intermediate energy* and P_{g_t} and P_{c_t} are the market prices of natural gas and coal.

[6] For this reason, we jointly model coal and natural gas instead of oil and coal or oil and natural gas.
[7] Spain imports all the crude oil and natural gas that it consumes, since its indigenous production is negligible. This paper assumes, for the sake of simplicity, that all fossil fuels are imported at international prices. Despite the fact Spain has some domestic coal production, this is a reasonable assumption since coal represents only 3% of the fossil fuel expenditures.

1.2.2 Final Energy Sector

There is another energy product that we define as *aggregate energy* or E. Aggregate energy are generated using two inputs, crude oil, E_{o_t} and intermediate energy. As for intermediate energy, a representative firm produces aggregate energy in a competitive environment according to the following production function,

$$E_t = \left(aE_{o_t}^{\gamma_E} + (1-a)E_t^{*\gamma_E}\right)^{\frac{1}{\gamma_E}} \ , \tag{6}$$

where a is the participation of oil on E and $1/(1-\gamma_E)$ is the interfuel elasticity of substitution between oil and intermediate energy. As the previous case, the representative firm in this sector solves the following problem:

$$\max_{\{E_t^*, E_{o_t}\}} \Pi_E = P_{E_t}E_t - P_{o_t}E_{o_t} - P_{E_t^*}E_t^* \ ,$$

subject to: E_t given by (6),

where P_{E_t} is the price of *aggregate energy* and P_{o_t} and $P_{E_t^*}$ are the market prices of oil and intermediate energy.

1.2.3 Final Good Sector

Finally, there is a sector that produces a final output according to a Cobb-Douglas production function, combining capital, labor and energy services as inputs,

$$Y_t = Z_t F(K_t, N_t, E_t) = \theta_t N_t^{\alpha} K_t^{\beta} E_t^{1-\alpha-\beta} \ , \tag{7}$$

where Y_t is the final good, N_t represents labor, K_t is the stock of capital, E_t is final energy and θ_t is the total factor productivity shock. The final goods and services producer is perfectly competitive and the representative firm in this sector solves the following problem:

$$\max_{\{E_t^*, E_{o_t}\}} \Pi_Y = P_t Y_t - w_t N_t - r_t K_t - P_{E_t} E_t$$

subject to: Y_t given by (7),

where P_t is the price of *final goods* that is normalized to one.

Figure 1 describes the theoretical framework of the economy

[Insert Figure 1]

2. The equilibrium

The equilibrium of this economy is summarized in the following equations system:

$$\nu \Psi N_t^{\nu-1} = \alpha \theta_t N_t^{\alpha-1} K_t^{\beta} E_t^{1-\alpha-\beta} \quad (8)$$

$$(C_t - \Psi N_t^{\nu})^{-\sigma} \left[1 + \Phi \frac{K_{t+1} - K_t}{K_t^2}\right] =$$

$$= \beta E_t \left[(C_{t+1} - \Psi N_{t+1}^{\nu})^{-\sigma} \left(\beta \theta_{t+1} K_{t+1}^{\beta-1} N_{t+1}^{\alpha} E_{t+1}^{1-\alpha-\beta} + 1 - \delta + \Phi \frac{K_{t+2} - K_{t+1}}{K_{t+1}} \frac{K_{t+2}}{K_{t+1}^2}\right)\right] \quad (9)$$

$$(C_t - \Psi N_t^{\nu})^{-\sigma} = \rho E_t \left[[(C_{t+1} - \Psi N_{t+1}^{\nu})^{-\sigma}]\left(1 + r_t^* + \varphi\left[e^{|D_{t-1} - D_{ss}|} - 1\right]\right)\right] \quad (10)$$

$$NX_t = -D_{t+1} + \left(1 + r_t^* + \varphi\left[e^{|D_t - D_{ss}|} - 1\right]\right) D_t - P_{o,t} E_{o,t} - P_{c,t} E_{c,t} - P_{g,t} E_{g,t} \quad (11)$$

$$E_{o,t} = \left[a \left(\frac{P_{E_t}}{P_{E_{o,t}}}\right)\right]^{\frac{1}{1-\gamma_E}} E_t \quad (12)$$

$$E_{c,t} = \left[(1-b) \left(\frac{P_{E_t^*}}{P_{E_{c,t}}}\right)\right]^{\frac{1}{1-\delta_E}} E_t^* \quad (13)$$

$$E_{g,t} = \left[b \left(\frac{P_{E_t^*}}{P_{E_{g,t}}}\right)\right]^{\frac{1}{1-\delta_E}} E_t^* \quad (14)$$

$$P_{E_t^*} = (1-a) \left(\frac{E_t}{E_t^*}\right)^{1-\gamma_E} P_{E_t} \quad (15)$$

$$w_t = \alpha \theta_t N_t^{\alpha-1} K_t^{\beta} E_t^{1-\alpha-\beta} \quad (16)$$

$$r_t = \beta \theta_t N_t^{\alpha} K_t^{\beta-1} E_t^{1-\alpha-\beta} \quad (17)$$

$$P_{E_t} = (1-\alpha-\beta) \theta_t N_t^{\alpha} K_t^{\beta} E_t^{-\alpha-\beta} \quad (18)$$

$$E_t = \left(a E_{o,t}^{\gamma_E} + (1-a) E_t^{*\gamma_E}\right)^{\frac{1}{\gamma_E}} \quad (19)$$

$$E_t^* = \left(b E_{g,t}^{\delta_E} + (1-b) E_{c,t}^{\delta_E}\right)^{\frac{1}{\delta_E}} \quad (20)$$

A competitive equilibrium is a set of allocations $\{C_t, N_t, D_t, K_t, E_t, E_t^*, E_{o,t}, E_{g,t}, E_{c,t}, NX_t\}$ and a system of prices $\{P_{E_t}, P_{E_t^*}, P_{E_{o,t}}, P_{E_{g,t}}, P_{E_{c,t}}, w_t, r_t\}$ such that, given the sequences of the technological shocks: i) $\{C_t, N_t, D_t, K_t\}$ solve the consumer problem; ii) $\{E_{g,t}, E_{c,t}\}$ solve the problem of the representative firm that produces intermediate energy; iii) $\{E_{o,t}, E_t^*\}$ solve the problem of the representative firm that produces final energy; iv) $\{N_t, K_t, E_t\}$ solve the problem of the representative firm that produces the final good iv) markets clear. The aggregate resources constraint is:

$$C_t + I_t + XN_t + P_{o,t}E_{0,t} + P_{g,t}E_{g,t} + P_{c,t}E_{c,t} = Y_t - P_{E_t}E_t \tag{21}$$

or $C_t + I_t + XN_t = Y_t$, since, in equilibrium $P_{E_t}E_t = P_{o,t}E_{0,t} + P_{g,t}E_{g,t} + P_{c,t}E_{c,t}$. The available supply of the final good is consumed by the representative household and exported to the rest of the world.

The trade balance can be obtained from the optimal evolution of the stock of assets D_t, as a function, among others, of external determinants such as the international interest rate[8]:

$$XN_t = -D_{t+1} + [1 + r_t^* + \Lambda(D_t)]D_t - P_{o,t}E_{o,t} - P_{g,t}E_{g,t} - P_{c,t}E_{c,t} \tag{22}$$

In this economic environment, there are four potential sources of uncertainty: 1) productivity shocks, 2) oil price shocks, 3) natural gas price shocks and 4) coal price shocks.

In the case of the productivity shocks, technology shocks follow an AR(1) process:

$$\begin{aligned} &\ln \theta_t = (1 - \rho_\theta)\bar{\theta} + \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta_t}, \\ &\varepsilon_{\theta_t} \sim iid\, N(0, \sigma_\theta^2) \end{aligned} \tag{23}$$

[8] The international interest rate, in the end, is conditioned by the evolution of foreign demand as long as monetary policy is frequently used as a tool to expand or contract aggregate demand.

As the prices for three fossil fuel are highly correlated, we assume that the exogenous variable given by the prices of the energy sources follow a multivariate vector autoregressive process (VAR) estimated as:

$$\tilde{P}_{t+1} = \widehat{\Theta}\tilde{P}_t + \hat{u}_t, \text{ with } E(\hat{u}_t\hat{u}_t') = \widehat{\Sigma}_u \tag{24}$$

where $\tilde{P}_{t+1} = [\tilde{P}_{0,t}, \tilde{P}_{g,t}, \tilde{P}_{c,t}]'$, $\tilde{P}_{j,t} = \ln(P_{j,t}/\bar{P}_j)$, $j = \{o, g, c\}$ and $\bar{P}_j$ is sample average. Estimated matrix $\widehat{\Theta}$ a matrix captures the persistence of the process and $\widehat{\Sigma}_u$denotes the matrix of variances and covariances of $\hat{u}_t$. Using a Cholesky decomposition, we identify the structural shocks by ordering the price shocks for oil, natural gas and coal. As $\widehat{\Sigma}_u$ is positive-definite matrix, there is a matrix $\mathbf{Q}$ triangular superior that verifies that $\widehat{\Sigma}_u = \mathbf{Q}'\mathbf{Q}$ where:

$$\mathbf{Q} = \begin{bmatrix} q_{11}q_{12}q_{13} \\ 0 \quad q_{22}q_{23} \\ 0 \quad 0 \quad q_{33} \end{bmatrix}$$

The above assumption implies that the structural model that generates estimated VAR is $\widehat{u_t} = \mathbf{Q}'\varepsilon_t$, with matrix variance and covariances $E(\varepsilon_t{\varepsilon_t}') = I_{3x3}$, i.e.,

$$\begin{cases} \hat{u}_{1,t} = q_{11}\varepsilon_{o,t} \\ \hat{u}_{2,t} = q_{21}\varepsilon_{o,t} + q_{22}\varepsilon_{g,t} \\ \hat{u}_{3,t} = q_{31}\varepsilon_{o,t} + q_{32}\varepsilon_{g,t} + q_{33}\varepsilon_{c,t} \end{cases}$$

This specification for energy price shocks implies that an oil shock $\varepsilon_{o,t}$ will also affect the prices of natural gas and coal; a natural gas price shock $\varepsilon_{g,t}$ will affect the coal price and a coal price shock will not affect oil and natural gas prices. This specification is based on the following evidence. First, the oil market is global, meaning that oil shocks are immediately transmitted all around the world. Natural gas and coal are regional markets. Second, natural gas markets are dominated by long-term contracts, usually linked to oil prices. For this reason, natural gas prices tend to react to changes in oil prices. Finally, a significant portion of natural gas and coal is consumed in electricity generation. In most electricity markets, natural gas prices determine the marginal cost of production, and thus the price of electricity. The demand for coal reacts to changes in those prices and, therefore, so do the prices of coal.

The results of the VAR estimation are key to understanding the results of the DSGE model. Academic literature generally focuses on oil prices as the main source of energy shocks, ignoring natural gas and coal prices. However, natural gas and coal prices do not necessarily move in parallel with oil.

3. The steady state

The steady state is a vector $\{C, N, D, K, NX, E, E^*, E_o, E_g, E_c, P_{E^*}, P_E\}$ that satisfies the optimality conditions of all the agents. This means that if this vector is reached in any period, in the absence of any perturbation, the system will stay at this point forever.

Given that our objective is to analyze the stochastic properties of the economy, we initially describe the steady state, which we use to characterize the long-run properties of the economy, and to estimate the structural parameters as described in the next section.

The computation of the steady state can be carried out as follows:

Step 1: The following system of equations characterizes the steady state for this economy for the allocations $\{C_{ss}, N_{ss}, D_{ss}, K_{ss}, E_{ss}, E_{ss}^*, E_{o,ss}, E_{g,ss}, E_{c,ss}, P_{E^*}, P_E\}$, and the trade balance to aggregate output ratio given exogenously ($NX/Y = \mu$):

$$\nu\Psi N_{ss}^{\nu-1} = \alpha\bar{\theta}N_{ss}^{\alpha-1}K_{ss}^{\beta}E_{ss}^{1-\alpha-\beta} \tag{25}$$

$$1 = \rho\left[\beta\bar{\theta}N_{ss}^{\alpha}K_{ss}^{\beta-1}E_{ss}^{1-\alpha-\beta} + 1 - \delta\right] \tag{26}$$

$$NX_{ss}/Y_{ss} = \mu \tag{27}$$

$$NX_{ss} = r^* D_{ss} - P_o E_{o,ss} - P_c E_{c,ss} - P_g E_{g,ss} \tag{28}$$

$$C_{ss} + \delta K_{ss} + NX_{ss} + P_o E_{o,ss} + P_c\, E_{c,ss} + P_g E_{g,ss} = \quad \bar{\theta} N_{ss}^{\alpha} K_{ss}^{\beta} E_{ss}^{1-\alpha-\beta} - P_E E_{ss} \tag{29}$$

$$E_{o_{ss}} = \left[a\left(\frac{P_E}{P_o}\right)\right]^{\frac{1}{1-\gamma_E}} E_{ss} \tag{30}$$

$$E_{c_{ss}} = \left[(1-b)\left(\frac{P_{E^*}}{P_c}\right)\right]^{\frac{1}{1-\delta_E}} E_{ss}^* \tag{31}$$

$$E_{g_{ss}} = \left[b\left(\frac{P_{E^*}}{P_g}\right)\right]^{\frac{1}{1-\delta_E}} E_{ss}^* \tag{32}$$

$$E_{ss} = \left(a E_{o_{ss}}^{\gamma_E} + (1-a) {E_{ss}^*}^{\gamma_E}\right)^{\frac{1}{\gamma_E}} \tag{33}$$

$$E_{ss}^* = \left(b E_{g_{ss}}^{\delta_E} + (1-b) E_{c_{ss}}^{\delta_E}\right)^{\frac{1}{\delta_E}} \tag{34}$$

$$P_{E^*} = \left[(1-a)\left(\frac{E_{ss}}{E_{ss}^*}\right)\right]^{1-\gamma_E} P_E \tag{35}$$

$$P_E = (1-\alpha-\beta)\bar{\theta} N_{ss}^{\alpha} K_{ss}^{\beta} E_{ss}^{-\alpha-\beta} \tag{36}$$

Step 2: i) from (25)-(34) we obtain $(E_{ss}/Y_{ss}), (E_{g,ss}/Y_{ss}), (E_{c,ss}/Y_{ss}), (E_{ss}^*/Y_{ss})$ and $(E_{o,ss}/Y_{ss})$ ii) $P_{E_{ss}}, P_{E_{ss}^*}$ is obtained from (34)-(36), iii) From (26) we obtain (K_{ss}/Y_{ss}) iv) from production function we obtain (N_{ss}/Y_{ss}) and from (25) taking $N_{ss} = 0.31$ we obtain Ψ, iv) from (28) we obtain (D_{ss}/Y_{ss}) and from (29) is obtained (C_{ss}/Y_{ss}), v) then we can obtain $\{C_{ss}, N_{ss}, K_{ss}, D_{ss}, E_{o,ss}, E_{g,ss}, E_{c,ss}, E_{ss}, E_{ss}^*\}$ and vi) given Ψ from (26) we obtain β.

Step 3: From the first order conditions derived from the problem faced by the firm that produces the final good, it is possible to find $\{w, r\}$.

Appendix 2.

4. Solving the Model

We solve the model through the Blanchard-Kahn (1980) procedure. First, we log linearized the optimality conditions and second, we solve the expectations to obtain the model's solution. Thus, the approximate solution can be written in state space as follows:

$$\xi_{t+1} = \mathbf{F}\xi_t + \mathbf{B}\tilde{P}_{\boldsymbol{t}} + \mathbf{G}\varepsilon_{\theta,t+1} \qquad \text{(state equation)}, \tag{37}$$

$$x_t = \mathbf{A}'\tilde{P}_{\boldsymbol{t}} + \mathbf{H}\xi_t \qquad \text{(observation equation)}, \tag{38}$$

for all $t = 0, 1, 2, \ldots,$ where $\xi_{t+1} = \left(\tilde{D}_{t+1,}, \tilde{K}_{t+1,}, \tilde{\theta}_{t+1,}\right)$ is the state variables vector of the system, $x_t = \left(\tilde{y}_{t,}, \tilde{E}_{o,t,}, \tilde{E}_{g,t}, \tilde{E}_{g,t}\right)'$ is the control variables vector, $\tilde{P}_{\boldsymbol{t}} = \left(\tilde{P}_{o,t,}, \tilde{P}_{g,t}, \tilde{P}_{g,t}\right)'$ is the exogenous variables vector, and $\mathbf{G} = (0,0,1)'$, with $\tilde{z}_t \equiv \ln(Z_t/Z_{ss})$ for $Z_t = D_t, K_t, \theta_t, Y_t, E_{o,t}, E_{g,t}, E_{c,t}$, and $\tilde{P}_{\boldsymbol{t}}$ is the price of the energy sources vector defined in deviations from the mean. Matrices **F**, **B**, **A**, and **H** are functions of the structural parameters of the model.

Given the observables $\{X\}_{t=1}^{T}$ with $X_t = \left(\tilde{y}_{t,}, \tilde{E}_{o,t,}, \tilde{E}_{g,t}, \tilde{E}_{g,t}\right)'$, (35)-(36) give rise to an empirical model of the form

$$\xi_{t+1} = \mathbf{F}\xi_t + \mathbf{B}\tilde{P}_{\boldsymbol{t}} + \mathbf{G}\varepsilon_{\theta,t+1} \quad \text{(state equation)}, \tag{39}$$

$$X_t = \mathbf{A}'\tilde{P}_{\boldsymbol{t}} + \mathbf{H}\xi_t + \Gamma_t \qquad \text{(observation equation)}, \tag{40}$$

where Γ_t denotes an observation error term. Given the stochastic singularity of the DSGE models, they cannot be estimated by maximum likelihood using more observable variables than the number of structural shocks specified in the model (see Ruge-Murcia, 2007). To address such singularity we add measurement errors given by Γ_t. We suppose that Γ_t is serially uncorrelated innovations $\varepsilon_{\theta,t}$, and that variance-covariance matrix $E(\Gamma_t \Gamma_t') = \mathbf{W} =$

diag$(\omega_1, \omega_2, \omega_3, \omega_4)$ is diagonal, implying that error terms are uncorrelated across observable variables. To estimate the structural parameters of the model first we construct the likelihood function for the sample $\{X\}_{t=1}^{T}$ as outlined by Hamilton (1994, Ch. 13) using Kalman Filter, and second, we use a Bayesian likelihood estimation approach.

Given the estimation of the structural parameters, we estimate Variance Decompositions and Impulse-Response functions for the model evaluation. We are interested in the effects of energy price shocks on macroeconomic variables and energy consumption. Thus, it is convenient to rewrite the system given by (39)-(40) as follows:

$$\begin{bmatrix} \xi_{t+1} \\ \tilde{P}_{\boldsymbol{t+1}} \end{bmatrix} = \begin{bmatrix} \mathbf{F} & \boldsymbol{B} \\ 0_{3x3} & \widehat{\Theta} \end{bmatrix} \begin{bmatrix} \xi_t \\ \tilde{P}_{\boldsymbol{t}} \end{bmatrix} + \mathbf{M} \begin{bmatrix} \varepsilon_{\theta,t+1} \\ \varepsilon_{t+1} \end{bmatrix}$$

$$X_t = [\mathbf{H}'\mathbf{A}'] \begin{bmatrix} \xi_t \\ \tilde{P}_{\boldsymbol{t}} \end{bmatrix} + \Gamma_t,$$

where $\mathbf{M} = \begin{bmatrix} 0_{2x4} \\ [1 \quad 0_{1x3}] \\ [0_{3x1} \quad \mathbf{Q}'] \end{bmatrix}$ and $\varepsilon_{t+1} = \begin{bmatrix} \varepsilon_{o,t+1} \\ \varepsilon_{g,t+1} \\ \varepsilon_{c,t+1} \end{bmatrix}$

Bayesian DSGE models combine microeconomic behavioral foundations with a full-system Bayesian likelihood estimation approach using key macroeconomic variables. In this sense, we confront our model with the data using Bayesian methods. Formally, we stack all the parameters in the model in the vector $\varsigma \in \chi$, $\varsigma = (\sigma, \nu, \phi, \varphi, \delta_E, b, \gamma_E, a, \alpha, \beta, \bar{\theta}, \rho_\theta, \sigma_{\varepsilon_\theta}, \omega_1, \omega_2, \omega_3, \omega_4)$ and $\chi = (\sigma, \nu, \phi, \varphi, \delta_E, b, \gamma_E, a, \alpha, \beta, \bar{\theta}, \rho_\theta, \sigma_{\varepsilon_\theta}, \omega_1, \omega_2, \omega_3, \omega_4, \rho, \delta)'$, given the observed data

$X^T = \{X_1, X_2, \dots, X_T\}$, where $X_t = (\hat{y}_t, \hat{e}_{o,t}, \hat{e}_{c,t}, \hat{e}_{g,t})'$. Then, we have the posterior distribution of ς:

$$p(\varsigma / X^T) \propto p(X^T / \varsigma) p(\varsigma),$$

where $\propto$ indicates proportionality. The posterior distribution summarizes the uncertainty regarding the parameter values and it can be used for point estimation once we have specified a loss function. We obtain the parameter values underlying the quantitative results presented in the remaining of the paper in two ways. Some structural parameters are calibrated to match specific macroeconomic ratios and the long run averages of data (ρ, δ, φ). The remaining structural parameters and the parameters of the stochastic process for the productivity shock (θ_t) are estimated using a Bayesian version of the standard maximum likelihood approach. In particular we use Random Walk *Markov Chain Monte Carlo* (MCMC) Metropolis-Hastings methods.

5. Calibration, priors and posteriors

The parameterization strategy consists of keeping some parameters fixed and estimating those related to model dynamics using Bayesian techniques. The Bayesian estimation process involves combining the estimation of the parameters by maximum likelihood, using an observed set of data with information obtained from prior distributions defined for those same parameters. The model has 20 parameters, 3 of which are calibrated and the remaining 17 are estimated.

The calibrated parameters are shown in Table 1. We selected these parameters before estimating the model in such a way that the balanced growth path of the model replicates the long run properties of the data for the period 1969-2013. The discount factor is consistent with an annualized real interest rate of 4 percent, following Martin-Moreno et al (2014).

[Insert Table 1]

The set of estimated parameters includes the relative risk aversion and the intertemporal elasticity of substitution of labor supply in the household preferences, the parameter associated with adjustment cost of investment, the risk premium, the interfuel elasticities of substitution between gas and coal and between oil and intermediate energy, the share parameter associated with natural gas and the parameter share linked to oil, the output elasticities in the production function and the stochastic processes driving shocks, including their first order autocorrelations and standard deviations.

This study assumes prior distributions that are standard in Bayesian estimations of DSGE models. In particular, we assume Inverse Gamma prior distributions for non-negative parameters, such as the risk aversion parameter, and Beta prior distributions for parameters between 0 and 1, such as output elasticity of labor.

Table 2 shows prior main parameters. Regarding the selection of the prior means and in general terms, we set the hiperparameters of the distribution to guarantee that the mean of the distribution is in line with the values found in academic literature, economic studies, microeconomic evidence, etc. The risk aversion parameter is 1.9, according to Prescott (1986). The ν parameter is 1.7 in accordance with Greenwood et al. (1988). We select the risk premium parameter taking into consideration the trade balance to GDP. The parameter α is 0.62, according to the average observed labor share in the Spanish National Accounts. The parameters for the energy production functions are chosen in line with Atallah and Blazquez (2015). Finally, the parameters ρ_θ and σ_θ, linked to productivity shocks, are 0.9 and 0.005 according to Martin-Moreno (1998).

[Insert Table 2]

We generate 20.000 draws from the posterior using a Metropolis-Hastings random walk. The posterior medians and the 5 and 95 percentile values of the 17 estimated parameters of the model are reported in Table 3. Figure 2 plots the histograms of each parameter. Figure 2 is a valid instrument to understand and to summarize the quality of the estimations and the strength of the model, according to Fernandez-Villaverde et al. (2008).

[Insert Table 3]

[Insert Figure 2]

This model is innovative in its approach to the energy system. The model considers three sources of fossil fuel at the macroeconomic level: oil, natural gas and coal. The key parameters to understanding the relationship among different kinds of energy are the short-run inter-fuel elasticity of substitution of each production function, $1/(1-\delta_E)$ and$1/(1-\gamma_E)$. According to the estimations in table 5, the elasticity of substitution between coal and gas ($1/(1-\delta_E)$) is 0.54 and the elasticity of substitution between oil and intermediate energy ($1/(1-\gamma_E)$) is 0.45. These elasticities are relatively high short-run elasticities. For example, the US Energy Information Administration (2012) estimates inter-fuel elasticities of substitution of around 0.1 to 0.2 among coal, gas and oil for different technologies but it does not include those of gas and oil. Switching from one fuel to another is technically impossible to do at a micro level. For example, a coal power plant cannot run with natural gas. Burniaux and Truong (2002) explains that the interfuel elasticity of substitution in the short run is very low (0.25) and it increases with time[9]. Our results suggest additional capacity to switch among fuels at a macro level because Spanish power plants do not run at full capacity. In any case, this is the first time that these energy production functions have been estimated at a macro level. This alone represents a

[9] After the first oil crisis, academic literature focused on inter-fuel elasticities of substitution among fossil fuels including Atkinson and Halvorsen (1976), Griffin (1977) and Pindyck (1979).

remarkable finding. These parameters are very different from the ones used in other DSGE models that take into account energy inputs, such as Golosov (2014)[10].

The estimated inter-fuel elasticities have a significant impact on the model dynamics and results. The negative value of δ_E implies that coal and natural gas are complementary inputs. Similarly, the negative value of γ_E implies that oil and intermediate energy are also complementary inputs. This means that oil, natural gas and coal behave as complements and according to economic theory, these inputs tend to be used together.

The results are as expected for the parameters of the production function of final goods and services, α and β. The estimation of the model shows that α is around 0.61 and β is around 0.33, while a standard calibration process of the production function shows a value of 0.62 for α and 0.38 for β.

The estimates for parameters σ and ν are quite close to other estimates in the academic literature. Posterior estimates of first-order autoregressive coefficients for productivity shock are quite similar to the estimated priors and show that the persistence of technology shock is 0.91, while the estimated posterior mean is 0.90. Finally, the posterior mean of the standard deviation of this shock is 0.007, also quite similar to the prior.

[10]Golosov (2014) uses an inter-fuel elasticity of substitution of 0.95, implying that fuels are very weak complements.

Figure 1

Description of the model

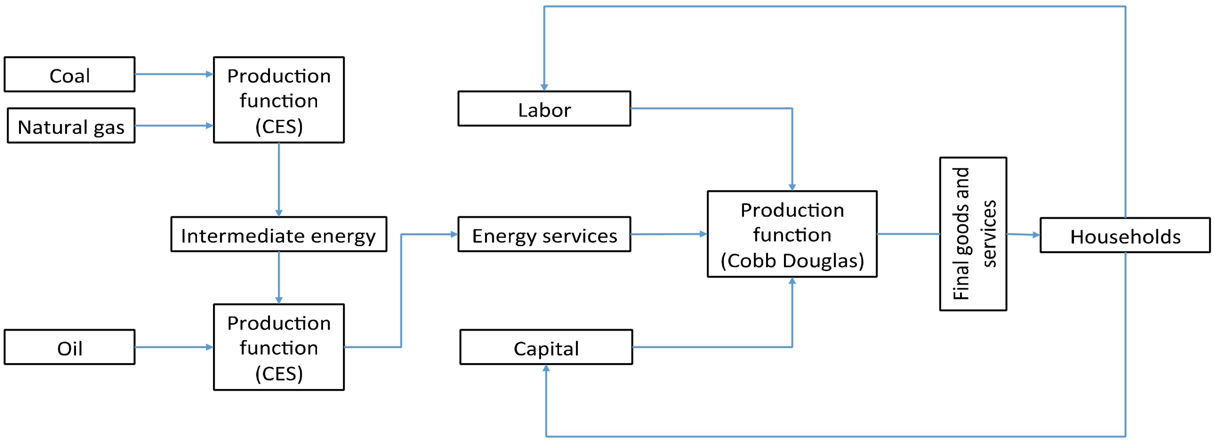

Table 1. Calibrated parameters

ρ	Discount factor	0.96
δ	Depreciacion rate	0.049
N	Fraction of hours worked	0.31

Table 2. Priors

Σ	ν	Φ	φ	δ_E	γ_E
Ga(3.61, 0.52)	*Ga*(5.66, 0.3)	*Ga*(5, 1)	*Ga*(0.06, 0.3)	*Ga*(0.476, 1)	*Ga*(0.37, 1)
B	a	α	$\alpha+\beta$	$\bar{\theta}$	ρ_θ
Be(1.38, 1)	*Be*(2.12, 1)	*Be*(1.63, 1)	*Be*(24, 1)	*Ga*(1, 2)	*Be*(9, 1)
σ_θ	ω_1	ω_2	ω_3	ω_4	
Ga(0.03, 0.3)	*Ga*(0.003, 0.3)	*Ga*(0.003, 0.3)	*Ga*(0.003, 0.3)	*Ga*(0.003, 0.3)	

Be(.) denotes beta distribution. *Ga*(.) denotes gamma distribution.

TABLE 3. Mean estimated parameters (5 and 95 percentage in brackets)

σ	ν	Φ		δ_E	γ_E
2.312	1.555	3.776	0.029	- 0.857	- 1.336
[1.710, 3.423]	[1.188, 1.911]	[1.912, 5.289]	[0.019, 0.046]	[-1.762, -0.335]	[-2.195, -0.715]
b	a	α	β	$\bar{\theta}$	ρ_θ
0.586	0.724	0.613	0.335	1.163	0.905
[0.513, 0.664]	[0.638, 0.843]	[0.490, 0.726]	[0.247, 0.418]	[0.618, 2.081]	[0.883, 0.927]
σ_θ	ω_1	ω_2	ω_3	ω_4	
0.007	0.0017	0.0009	0.0012	0.0010	
[0.005, 0.010]	[0.0009, .0031]	[0.0007, 0.0011]	[0.0009, 0.0015]	[0.0007, 0.0015]	

Figure 2. Prior and posterior distributions