Online Supplementary Materials

Energy R&D Investments and Emissions Abatement Policy

Appendix 1 Endogenous Variables

- U_t utility in period t
- C_t per capita consumption of goods and services (constant 2005 USD per person)
- Q_t total net output (trillions of constant 2005 USD)
- $Q_{i,t}$ net output in capital sector *i* (trillions of constant 2005 USD)
- Ω_t damage function (climate damages as a fraction of world output)
- Λ_t abatement cost function (abatement costs as a fraction of world output)
- μ_t emissions-control rate (fraction of uncontrolled emissions)
- K_t total physical capital stock (trillions of constant 2005 USD)
- $K_{i,t}$ physical capital stock in sector *i* (trillions of constant 2005 USD)
- I_t investments in physical capital (trillions of constant 2005 USD)
- $ES_{i,t}$ energy service used in sector *i*
- $ER_{i,t}$ raw energy used in sector *i*
- $P_{i,t}$ oil products used in sector *i*
- $W_{i,t}$ coal products used in sector *i*
- $G_{i,t}$ natural gas used in sector *i*
- $B_{i,t}$ backstop energy used in sector *i*
- $CE_{i,t}$ energy cost in sector *i*
- $z_{i,B,t}$ energy cost of backstop energy in sector *i*
- *b* the scale factor of the energy cost of backstop energy
- $R_{E,t}$ R&D investments in energy efficiency
- $R_{B,t}$ R&D investments in backstop technology
- $H_{E,t}$ knowledge stock on energy efficiency
- $H_{B,t}$ knowledge stock on backstop technology
- EM_t total carbon emissions
- E_t energy-related carbon emissions

$M_{A,t}$	atmospheric CO2 concentration
$M_{U,t}$	upper oceans/biosphere CO2 concentration
$M_{L,t}$	lower oceans CO2 concentration
F_t	radioactive forcing, increase over the preindustrial level
T_t	atmospheric temperature, increase over 1900 level
TL_t	lower ocean temperature, increase over 1900 level

Appendix 2 Exogenous Variables and Parameters

t	time period	30
i	1=capital-goods production sector, 2=consumption-goods	
	production sector	1,2
r	pure rate of social time preference (per year)	0.015
α	the elasticity of marginal utility of consumption (pure number)	1.45
L_0	the initial population level	7403
$L_{1.0}$	2015 initial labor percentage in the capital-goods sector	0.126
$L_{2.0}$	2015 initial labor percentage in the consumption-goods sector	0.874
$g_{L,t}$	growth rate to calibrate to 2050 pop projection	0.13449
L_{max}	asymptotic population	11500
Y_0	initial world gross output	105.5
$Y_{1,0}$	initial world gross output in the capital-goods sector	21.1
$Y_{2,0}$	initial world gross output in the consumption-goods sector	84.4
K_0	initial capital value 2015	223
$K_{1,0}$	initial capital value in capital-goods sector 2015	50
$K_{2,0}$	initial capital value in consumption-goods sector 2015	173
$A_{1,0}$	initial level of TFP in the capital-goods sector	
$A_{2,0}$	initial level of TFP in the consumption-goods sector	
$g_{A,0}$	initial growth rate of total factor productivity per five years	0.076
d_A	the rate of decline of the growth rate of total factor productivity	0.005
β_i	Capital elasticity in output function of sector i	0.2828/0.3258
γ _i	Energy elasticity in output function of sector i	0.0784/0.0806
$a_1 a_2$	parameters of damage function	0/0.00266

<i>H</i> _{<i>m</i>,0}	initial knowledge stock in energy efficiency/ backstop	
	technology	0.001/1
<i>R_{m,0}</i>	initial R&D investments in energy efficiency/ backstop	
	technology	0.00453/0.00429
$\varphi_{E,n}$	parameters of energy stock in energy efficiency, n=1,2,3	0.00213/0.20/0.55
$\varphi_{B,n}$	parameters of energy stock in backstop technology, n=1,2,3	0.00429/0.005/0.1
α_H	scale parameter	0.336
ρ	substitution parameter for raw energy and knowledge	0.38
δ_K	the rate of depreciation of physical capital (per period)	0.1
δ_H	the rate of depreciation of knowledge stock (per period)	0
<i>S</i> _{<i>J</i>,0}	proved reserve of fossil fuel J in 2005 (J=P, W, G, namely, oil,	9496.2/28750.8/7
	coal, and gas)	039.8
<i>Z</i> _{1,<i>J</i>,<i>t</i>}	energy cost of fossil fuel j used in the capital-goods sector (J=P,	0.004647/0.01398
	W, G, namely, oil, coal, and gas)	5/0.004375
Z _{2,J,t}	energy cost of fossil fuel j used in the consumption-goods sector	0.020225/0.02920
	(J=P, W, G, namely, oil, coal, and gas)	4/0.011776
<i>z_{i,B,0}</i>	the initial cost of backstop energy in sector i, i=1,2	0.125808/0.13848
		8
η_J	CO2-energy coefficient of fossil fuel j (J=P, W, G, namely, oil,	0.0211/0.0289/0.0
	coal, and gas)	153
LU ₀	initial land-use carbon emissions	2.6
δ_{LU}	the rate of decline of land-use carbon emissions	0.115
<i>M</i> _{<i>n</i>,0}	initial concentration in atmosphere/upper strata/lower strata	
	2015 (GtC), n=A, U, L	851/1540/10010
M _{n,eq}	equilibrium concentration in atmosphere/upper strata/lower	
	strata 2015 (GtC), n=A, U, L	588/1350/10000
$\phi_{33}\phi_{2}$	Carbon cycle transition matrix	0.08800/0.00250
æ	temperature dynamics parameters $n-1.2.3.4$	0.104/1.564/0.088
o_n	comperature dynamics parameters, n=1.2,3,4	/0.025

Appendix 3 Equation List

(A.1) max
$$V = \sum_{t=1}^{T} U_t(c_t, L_t) R_t$$

(A.2) $U_t(c_t, L_t) = L_t \frac{c_t^{1-\alpha}}{1-\alpha}$

$$\begin{aligned} (A.3) Y_{t} = Y_{1,t} + Y_{2,t} \\ (A.4) Y_{l,t} &= \Omega_{t} A_{l,t} K_{l,t}^{\beta_{l}} E S_{l,t}^{\gamma_{l}} L_{l,t}^{1-\beta_{l}-\gamma_{l}} - CE_{l,t}, i = 1,2 \\ (A.5) \Omega_{t} &= 1/(1 + a_{1}T_{t} + a_{2}T_{t}^{2}) \\ (A.6) L_{t} &= L_{0} \exp(g_{L,t}) \\ (A.7) g_{L,t} &= \left(\frac{g_{L,0}}{d_{L}}\right) * (1 - \exp(-d_{L} * t)) \\ (A.8) L_{t} &= L_{1,t} + L_{2,t} \\ (A.9) A_{l,t} &= A_{l,0} \exp(g_{A,t}) \\ (A.10) g_{A,t} &= \left(\frac{g_{A,0}}{d_{A}}\right) * (1 - \exp(-d_{A} * t)) \\ (A.11) K_{t} &= l_{t} + (1 - \delta_{K})K_{t-1} \\ (A.12) K_{t} &= K_{1,t} + K_{2,t} \\ (A.13) ES_{l,t} &= (\alpha_{H}H_{E,t}^{\beta} + ER_{l,t}^{\beta})^{1/\rho}, i = 1,2 \\ (A.14) ES_{l,t} &= P_{l,t} + W_{l,t} + G_{l,t} + B_{l,t}, i = 1,2 \\ (A.15) S_{J,0} &\geq \sum_{t=1}^{T} (J_{1,t}+J_{2,t}), J = P, W, G \\ (A.16) CE_{l,t} &= z_{l,p,t}P_{l,t} + z_{l,W,t}W_{l,t} + z_{l,G,t}G_{l,t} + z_{l,B,t}B_{l,t}, i = 1,2 \\ (A.17) z_{l,B,t} &= z_{l,B,0}B_{l,t}^{2b1}H_{B,t}^{2b2} \\ (A.18) H_{m,t} &= (1 - \delta_{H})H_{m,t-1} + \varphi_{m,1}H_{m,t-1}^{\varphi_{m,2}}R_{m,t}^{\varphi_{m,3}}, m = E, B \\ (A.19) Y_{1,t} &= I_{t} + R_{E,t} + R_{B,t} \\ (A.20) Y_{2,t} &= C_{t} \\ (A.21) EM_{t} &= E_{t} + LU_{0}(1 - \delta_{LU})^{t} \\ (A.23) M_{A,t} &= 5 * EM_{t} + \varphi_{33}M_{A,t-1} + \varphi_{23}M_{U,t-1} \\ (A.24) M_{U,t} &= \varphi_{32}M_{A,t-1} + \varphi_{21}M_{U,t-1} \\ (A.25) M_{L,t} &= \psi_{1,1}M_{L,t-1} + \varphi_{21}M_{U,t-1} \\ (A.26) F_{t} &= 4.1[log_{2}(M_{A,t}/M_{A,1750})] + F_{EX,t} \\ (A.27) T_{t} &= T_{t-1} + \sigma_{1}[F_{t} - \sigma_{2}T_{t-1} - \sigma_{3}(T_{t-1} - TL_{t-1})] \\ (A.28) TL_{t} &= TL_{t-1} + \sigma_{4}(T_{t-1} - TL_{t-1}) \end{aligned}$$