Directed Technical Change and Energy Intensity Dynamics: Structural Change vs. Energy Efficiency

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Online Technical Appendices

A. SOLVING FOR THE EQUILIBRIUM

In order to simplify notation, we drop the time index in Appendix A. Due to perfect competition on market for the final product, the profit-maximising behaviour of the final good producer results in the following relative demand for both sectoral goods:

$$\frac{p_l}{p_e} = \left(\frac{Y_l}{Y_e}\right)^{-\frac{1}{\epsilon}} . \tag{A.1}$$

This price ratio implies that the relative price is inversely related to the relative supply of both sectors. Defining the final good as numeraire, the price index can be written as

$$\left(p_l^{1-\epsilon} + p_e^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} = 1. \tag{A.2}$$

Sectoral producers maximise their profits by choosing the quantities of the respective sector specific machines and labour,

$$\max_{x_{li}, L_l} \left\{ \Pi_{Y_l} = p_l L_l^{1-\alpha} \int_0^1 A_{li}^{1-\alpha} x_{li}^{\alpha} di - w L_l - \int_0^1 p_{li} x_{li} di \right\}, \tag{A.3}$$

as well as, in the case of the e-sector, the amount of energy,

$$\max_{x_{ei}, L_e, E} \left\{ \Pi_{Y_e} = p_e E^{\alpha_2} L_e^{1-\alpha} \int_0^1 A_{ei}^{1-\alpha_1} x_{ei}^{\alpha_1} di - w L_e - \int_0^1 p_{ei} x_{ei} di - c_E E \right\}. \tag{A.4}$$

Profit-maximisation yields the sectoral demands for machine i in the labour-intensive sector,

$$x_{li} = \left(\frac{\alpha p_l}{p_{li}}\right)^{\frac{1}{1-\alpha}} L_l A_{li}, \tag{A.5}$$

and in the energy-intensive sector,

$$x_{ei} = \left(\frac{\alpha_1 p_e E^{\alpha_2} L_e^{1-\alpha}}{p_{ei}}\right)^{\frac{1}{1-\alpha_1}} A_{ei}. \tag{A.6}$$

The demands for machines increase in the price of the respective sector's output (p_j), employed labour in the sector (L_j), and the quality of the individual technology (A_{ji}).

Machines are produced under monopolistic competition. The producer of each variety maximises her profit ($\pi_{ji} = \left(p_{ji} - \psi\right) x_{ji}$) given the demand for her variety. The optimisations yield the price setting rules for monopolists in both sectors, that are $p_{li} = \psi/\alpha$ for machine producers in the l-sector and $p_{ei} = \psi/\alpha_1$ for machine producers in the e-sector. Using these prices and the demands for machines in both sectors, (A.5) and (A.6), the equilibrium profits of machine producers in the labour-intensive sector are

$$\pi_{li} = (1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}} \left(\frac{1}{\psi}\right)^{\frac{\alpha}{1-\alpha}} p_l^{\frac{1}{1-\alpha}} L_l A_{li}, \tag{A.7}$$

whereas the profits in the energy-intensive sector are

$$\pi_{ei} = (1-\alpha_1)\,\alpha_1^{\frac{1+\alpha_1}{1-\alpha_1}} \left(\frac{1}{\psi^{\alpha_1}}\right)^{\frac{1}{1-\alpha_1}} p_e^{\frac{1}{1-\alpha_1}} E^{\frac{\alpha_2}{1-\alpha_1}} L_e^{\frac{1-\alpha}{1-\alpha_1}} A_{ei}. \tag{A.8}$$

Profit maximisation in the energy-intensive and labour-intensive sectors yields the following first-order conditions:

$$L_l = \left(\frac{w}{(1-\alpha)\,p_l \int_0^1 A_{li}^{1-\alpha} x_{li}^{\alpha} di}\right)^{-\frac{1}{\alpha}}, \tag{A.9}$$

$$L_e = \left(\frac{w}{(1-\alpha)\,p_e E^{\alpha_2} \int_0^1 A_{ei}^{1-\alpha_1} x_{ei}^{\alpha_1} di}\right)^{-\frac{1}{\alpha}}, \text{ and} \tag{A.10}$$

$$E = \left(\frac{c_E}{p_e \alpha_2 L_e^{1-\alpha} \int_0^1 A_{ei}^{1-\alpha_1} x_{ei}^{\alpha_1} di}\right)^{\frac{1}{\alpha_2-1}}. \tag{A.11}$$

Plugging the equilibrium quantity of machines (A.5) into (3) yields the production of labour-intensive output:

$$Y_l = L_l A_l \left(\frac{\alpha^2 p_l}{\psi}\right)^{\frac{\alpha}{1-\alpha}}. \tag{A.12}$$

Plugging (A.6) into (A.11) yields the equilibrium quantity of energy:

$$E = \left(\frac{(\alpha_1)^2}{\psi}\right)^{\frac{\alpha_1}{1-\alpha}} \left(\frac{\alpha_2 A_e}{c_E}\right)^{\frac{1-\alpha_1}{1-\alpha}} p_e^{\frac{1}{1-\alpha}} L_e \tag{A.13}$$

Combining (A.13) and (A.6) with (4) yields the production of the energy-intensive good as:

$$Y_e = \left(\frac{(\alpha_1)^2}{\psi}\right)^{\frac{\alpha_1}{1-\alpha}} \left(\frac{\alpha_2 A_e}{c_E}\right)^{\frac{\alpha_2}{1-\alpha}} p_e^{\frac{\alpha}{1-\alpha}} L_e A_e. \tag{A.14}$$

Equilibrium on the labour market implies an identical wage in both sectors. Equating (A.9) and (A.10), together with (A.13), (A.6), and (A.5), yields the relative price:

$$\frac{p_l}{p_e} = \frac{\psi^{\alpha_2} (\alpha_1)^{2\alpha_1} (\alpha_2)^{\alpha_2} A_e^{1-\alpha_1}}{c_E^{\alpha_2} \alpha^{2\alpha} A_l^{1-\alpha}}. \tag{A.15}$$

The relative price (A.1) yields, together with the sectoral production quantities,(A.12) and (A.14), the relative supply in both sectors. Combining relative supply and with relative demands yields the relative employment as:

$$\frac{L_l}{L_e} = \left(\frac{c_E^{\alpha_2} \alpha^{2\alpha}}{\psi^{\alpha_2} (\alpha_1)^{2\alpha_1} (\alpha_2)^{\alpha_2}}\right)^{\epsilon-1} \frac{A_l^{-\varphi}}{A_e^{-\varphi_1}} \tag{A.16}$$

with $\varphi_1 \equiv (1-\alpha_1)(1-\epsilon)$ and $\varphi \equiv (1-\alpha)(1-\epsilon)$.

Finally, the equilibrium prices and quantities can be calculated. The price ratio (A.15), together with the price index (A.2), leads to the equilibrium prices in both sectors:

$$p_l = \frac{\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}A_e^{1-\alpha_1}}{\left(\left(\alpha^{2\alpha}c_E^{\alpha_2}\right)^{1-\epsilon}A_l^{\varphi}+\left(\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}\right)^{1-\epsilon}A_e^{\varphi_1}\right)^{\frac{1}{1-\epsilon}}}, \tag{A.17}$$

$$p_e = \frac{\alpha^{2\alpha}c_E^{\alpha_2}A_l^{1-\alpha}}{\left(\left(\alpha^{2\alpha}c_E^{\alpha_2}\right)^{1-\epsilon}A_l^{\varphi}+\left(\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}\right)^{1-\epsilon}A_e^{\varphi_1}\right)^{\frac{1}{1-\epsilon}}}. \tag{A.18}$$

Combining the prices with input demands yields the equilibrium employment of labour in both sectors

$$L_l = \frac{\left(\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}\right)^{1-\epsilon}A_e^{\varphi_1}}{\left(\left(\alpha^{2\alpha}c_E^{\alpha_2}\right)^{1-\epsilon}A_l^{\varphi}+\left(\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}\right)^{1-\epsilon}A_e^{\varphi_1}\right)}, \tag{A.19}$$

$$L_e = \frac{\left(c_E^{\alpha_2}\alpha^{2\alpha}\right)^{1-\epsilon}A_l^{\varphi}}{\left(\left(\alpha^{2\alpha}c_E^{\alpha_2}\right)^{1-\epsilon}A_l^{\varphi}+\left(\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}\right)^{1-\epsilon}A_e^{\varphi_1}\right)} \tag{A.20}$$

as well as equilibrium energy use in the energy-intensive sector

$$E = \frac{\left(\frac{\alpha_1^2}{\psi}\right)^{\frac{\alpha_1}{1-\alpha}}\alpha_2^{\frac{1-\alpha_1}{1-\alpha}}\alpha^{2\alpha\left(\frac{1}{1-\alpha}-\epsilon+1\right)}c_E^{\alpha_2-1-\epsilon\alpha_2}A_l^{1+\varphi}A_e^{\frac{1-\alpha_1}{1-\alpha}}}{\left(\left(\alpha^{2\alpha}c_E^{\alpha_2}\right)^{1-\epsilon}A_l^{\varphi}+\left(\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}\right)^{1-\epsilon}A_e^{\varphi_1}\right)^{\frac{1+\varphi}{\varphi}}}. \tag{A.21}$$

Plugging these optimal inputs into (A.12) and (A.14) yields the the equilibrium outputs in the labour- and energy-intensive sector as

$$Y_l = \frac{\alpha^{\frac{2\alpha}{1-\alpha}}\psi^{\frac{\alpha_1(\epsilon\alpha_2-1)}{1-\alpha}}\alpha_1^{\frac{2\alpha_1(1-\epsilon+\epsilon\alpha)}{1-\alpha}}\alpha_2^{\frac{\alpha_2(1-\epsilon-\epsilon\alpha)}{1-\alpha}}A_e^{\frac{1-\alpha_1}{1-\alpha}(\alpha+\varphi)}A_l}{\left(\left(\alpha^{2\alpha}c_E^{\alpha_2}\right)^{1-\epsilon}A_l^{\varphi}+\left(\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}\right)^{1-\epsilon}A_e^{\varphi_1}\right)^{\frac{\alpha+\varphi}{\varphi}}}, \tag{A.22}$$

$$Y_e = \frac{\left(\frac{\alpha_1^2}{\psi}\right)^{\frac{\alpha_1}{1-\alpha}}\alpha_2^{\frac{\alpha_2}{1-\alpha}}\alpha^{2\alpha\left(\frac{1}{1-\alpha}-\epsilon\right)}c_E^{-\epsilon\alpha_2}A_l^{\alpha+\varphi}A_e^{\frac{1-\alpha_1}{1-\alpha}}}{\left(\left(\alpha^{2\alpha}c_E^{\alpha_2}\right)^{1-\epsilon}A_l^{\varphi}+\left(\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}\right)^{1-\epsilon}A_e^{\varphi_1}\right)^{\frac{\alpha+\varphi}{\varphi}}}. \tag{A.23}$$

B. EQUILIBRIUM PROFIT RATIO AND ALLOCATION OF RESEARCHERS

B.1 Relative Profitability of Research

Since scientists only direct a sector and are randomly allocated to a specific machine variety, the average sectoral productivity is used as defined in (5). Combining (A.7) and (A.8) and taking into account the probabilities of a successful innovation, η_j, the expected firm value (i.e. expected profit) of an innovation in the l-sector, $\Pi_l(t)$, relative to an innovation in the e-sector, $\Pi_e(t)$, is:

$$\frac{\Pi_l(t)}{\Pi_e(t)} = \omega \frac{\eta_l}{\eta_e} \cdot \underbrace{\frac{p_l(t)^{\frac{1}{1-\alpha}}}{p_e(t)^{\frac{1}{1-\alpha_1}}}}_{\text{price effect}} \cdot \underbrace{\frac{L_l(t)}{E(t)^{\frac{\alpha_2}{1-\alpha_1}} L_e(t)^{\frac{1-\alpha}{1-\alpha_1}}}}_{\text{market size effect}} \cdot \underbrace{\frac{A_l(t)}{A_e(t)}}_{\substack{\text{direct} \\ \text{productivity effect}}} \tag{B.1}$$

with $\omega \equiv (1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}}\,(1-\alpha_1)^{-1}\,\alpha_1^{-\frac{1+\alpha_1}{1-\alpha_1}}\,\psi^{\frac{\alpha+\alpha_1}{(1-\alpha)(1-\alpha_1)}}$. Analogously to the Directed Technical Change literature (Acemoglu, 1998, 2002), relative profitability of innovating is affected by a price- and a market size effect. The *price effect* directs innovation in the sector with the higher price. The *market size effect* makes innovations more attractive in the sector, where more factors of production, labour and energy, are employed. Since a larger market size is associated with a lower price for the output of the respective sector, both effects are opposite forces. Finally, the term $A_l(t)/A_e(t)$ captures a *direct productivity effect* as introduced by Acemoglu et al. (2012). This effect directs innovation to the sector that is technologically further advanced and hence follows the concept of "building on the shoulders of giants". In addition to these three forces, the respective probabilities of successful research, η_l and η_e, affect the relative profits.

B.2 Allocation of Researchers

With strong positive (negative) energy price growth, i.e. $\eta_e \gamma\,(1-\alpha_1)\,/\alpha_2 < \gamma_{c_E} < (-\eta_l \gamma\,(1-\alpha)\,/\alpha_2)$, the direction of the change of relative profit is independent of research.

Proof: For $\epsilon > 1$, it follows with (8), (9), and (10) that,

$$\frac{d\left(\frac{\Pi_l(t)}{\Pi_e(t)}\right)}{dt} = \alpha_2(\epsilon-1)\gamma_{c_E} + \varphi_1 s_e \eta_e \gamma - \varphi s_l \eta_l \gamma > 0 \;\Leftrightarrow\; \gamma_{c_e} > \eta_e \gamma (1-\alpha_1)/\alpha_2$$

and

$$\frac{d\left(\frac{\Pi_l(t)}{\Pi_e(t)}\right)}{dt} = \alpha_2(\epsilon-1)\gamma_{c_E} + \varphi_1 s_e \eta_e \gamma - \varphi s_l \eta_l \gamma < 0 \;\Leftrightarrow\; \gamma_{c_e} < -\eta_l \gamma (1-\alpha)/\alpha_2.$$

□

From that it follows that for moderate energy price growth, i.e. $-\eta_l \gamma\,(1-\alpha)\,/\alpha_2 \leq \gamma_{c_E} \leq \eta_e \gamma\,(1-\alpha_1)\,/\alpha_2$, the direction of the change of relative profit is not independent of research.

Moderate energy price growth

In the case of substitutes ($\epsilon > 1$):

1. From equation (10) and with $s(t) \equiv s_l(t)$ it follows that

$$d\frac{\Pi_{li(t)}}{\Pi_{ei(t)}}/dt \gtreqless 0 \quad \text{if} \quad s(t) \gtreqless \frac{\alpha_2(\epsilon-1)\frac{\gamma_{c_E}}{\gamma} + \eta_l \varphi_1}{\varphi \eta_l + \varphi_1 \eta_e} \equiv s^{**}. \tag{B.2}$$

Proof:

$$d\frac{\Pi_{li(t)}}{\Pi_{ei(t)}}/dt \gtreqless 0 \Leftrightarrow 0 \lesseqgtr \frac{d\frac{\Pi_{li(t)}}{\Pi_{ei(t)}}/dt}{\frac{\Pi_{li(t)}}{\Pi_{ei(t)}}} = \frac{\alpha_2(\epsilon-1)}{c_E(t)}\frac{dc_E(t)}{dt} - \frac{\varphi}{A_l(t)}\frac{dA_l(t)}{dt} + \frac{\varphi_1}{A_e(t)}\frac{dA_e(t)}{dt}.$$

Using equation (8) and (9) yields:

$$0 \lesseqgtr \alpha_2(\epsilon-1)\gamma_c - \varphi s_l(t)\gamma\eta_l + \varphi_1 s_e(t)\gamma\eta_e$$
$$\Leftrightarrow s(t) \gtreqless \frac{\alpha_2(\epsilon-1)\frac{\gamma_{c_E}}{\gamma} + \eta_l\varphi_1}{\varphi\eta_l + \varphi_1\eta_e} \equiv s^{**}.$$

□

2. At time $t = z$ there exists a unique equilibrium research allocation $s^*(t = z)$ with research directed to sector l (e) only, i.e. $s(t = z) = 1$ ($s(t = z) = 0$), if

$$A(t = z) \equiv \frac{A_e(t = z)^{(1-\alpha_1)}}{A_l(t = z)^{(1-\alpha)}} \overset{(>)}{<} \left(\frac{\eta_e}{\kappa\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}}.$$

Proof: Using equation (10) yields:

$$\frac{\Pi_l(t)}{\Pi_e(t)} = \kappa\frac{\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}{\eta_e}\frac{A_l(t)^{-\varphi}}{A_e(t)^{-\varphi_1}} \overset{(<)}{>} 1 \Leftrightarrow \left(\frac{\eta_e}{\kappa\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}} \overset{(<)}{>} \frac{A_e(t)^{(1-\alpha_1)}}{A_l(t)^{(1-\alpha}} \equiv A(t).$$

□

If $s^*(t = z) \in \{0, 1\}$ is an equilibrium in $t = z$ than it is also an equilibrium in all $t > z$ (follows from (B.2)).

3. At time $t = z$ there exist multiple equilibria $s \in [0, 1]$ if

$$A(t = z) = \left(\frac{\eta_e}{\kappa\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}}.$$

Proof:

$$\frac{\Pi_l(t)}{\Pi_e(t)} = \kappa\frac{\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}{\eta_e}\frac{A_l(t)^{-\varphi}}{A_e(t)^{-\varphi_1}} = 1 \Leftrightarrow \left(\frac{\eta_e}{\kappa\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}} = \frac{A_e(t)^{(1-\alpha_1)}}{A_l(t)^{(1-\alpha}} \equiv A(t).$$

□

If $s^*(t = z) \in (0, 1)$ is an equilibrium in $t = z$ then $s^*(t = z) \in (0, 1)$ is also an equilibrium in $t > z$ if and only if $s^*(t) = s^{**}\ \forall\ t \geq z$. If $s^*(t) \overset{(<)}{>} s^{**}$ there will be research in sector l (e) only in all $t > z$ (follows from (B.2)).

In the case of complements ($\epsilon < 1$):

1. From equation (10) follows:

$$d\frac{\Pi_{li(t)}}{\Pi_{ei(t)}}/dt \gtreqless 0 \quad \text{if} \quad s(t = z) \lesseqgtr s^{**}.$$

Proof: Analogue to the case of substitutes ($\epsilon > 1$) and moderate energy price growth. □

2. At time $t = z$ there exists a unique equilibrium research allocation s^* with research directed to sector l (e) only, i.e. $s(t = z) = 1$ ($s(t = z) = 0$), if

$$A(t = z) \overset{(<)}{>} \left(\frac{\eta_e}{\kappa\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}}.$$

Proof: See proof for $\epsilon > 1$ and moderate energy price growth. □

With $s^*(t=z) \in \{0,1\}$, $\left|1 - \frac{\Pi_{li(t)}}{\Pi_{ei(t)}}\right|$ decreases over time and hence there exists a time $\tau > z$, where $\frac{\Pi_{li(t=\tau)}}{\Pi_{ei(t=\tau)}} = 1 \left(\Leftrightarrow A(t=\tau) = \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{\epsilon}-1} \right)$.

3. At time $t = z$ there exist multiple equilibria $s^* \in [0,1]$, if

$$A(t=z) = \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{\epsilon-1}}.$$

If $s^*(t=z) \in (0,1)$ is an equilibrium in $t = z$ than $s^*(t=z) \in (0,1)$ is also an equilibrium in all $t > z$ if and only if $s^*(t) = s^{**} \ \forall \ t \geq z$.
We assume $s^* = s^{**}$ (i.e. the dynamically stable equilibrium) in the case of an inner equilibrium. This is also the technical result for longer patent duration (see Appendix D).

Strong energy price growth

In the case of substitutes ($\epsilon > 1$):

1. At time $t = z$ there exists a unique equilibrium research allocation $s^*(t=z)$ with research directed to sector l (e) only, i.e. $s(t=z) = 1$ ($s(t=z) = 0$), if

$$A(t=z) = \frac{A_e(t=z)^{(1-\alpha_!)}}{A_l(t=z)^{(1-\alpha}} \overset{(>)}{<} \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{\epsilon}-1}. \tag{B.3}$$

Proof: Using equation (10) yields:

$$\frac{\Pi_l(t)}{\Pi_e(t)} = \kappa \frac{\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}{\eta_e} \frac{A_l(t)^{-\varphi}}{A_e(t)^{-\varphi_1}} \overset{(<)}{>} 1 \Leftrightarrow \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{\epsilon-1}} \overset{(<)}{>} \frac{A_e(t)^{(1-\alpha_!)}}{A_l(t)^{(1-\alpha}} \equiv A(t).$$

□

If $A(t=z) \overset{(>)}{<} \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{\epsilon-1}}$ and with strong positive (negative) energy price growth, $s^*(t=z) = 1$ (0) is an equilibrium in $t = z$ and in all $t > z$ (follows from Lemma 2). If $A(t=z) \overset{(<}{>} \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{\epsilon-1}}$ and with strong positive (negative) energy price growth, $s^*(t=z) = 0$ (1) is an equilibrium in $t = z$ and since $\left|1 - \frac{\Pi_{li(t)}}{\Pi_{ei(t)}}\right|$ increases over time, there exists a time $\tau > z$, such that $\frac{\Pi_{li(t=\tau)}}{\Pi_{ei(t=\tau)}} = 1$ and $\frac{\Pi_{li(t)}}{\Pi_{ei(t)}} \overset{(<)}{>} 1$ for all $t > \tau$, leading to an equilibrium with research directed only to sector l (e) for all $t > \tau$.

2. At time $t = z$ there exist multiple equilibria $s \in [0,1]$ if

$$A(t=z) = \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{\epsilon-1}}.$$

Proof:

$$\frac{\Pi_l(t)}{\Pi_e(t)} = \kappa \frac{\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}{\eta_e} \frac{A_l(t)^{-\varphi}}{A_e(t)^{-\varphi_1}} = 1 \Leftrightarrow \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{1-\epsilon}} = \frac{A_e(t)^{(1-\alpha_1)}}{A_l(t)^{(1-\alpha}} \equiv A(t).$$

□

With strong positive (negative) energy price growth, $s^*(t) = 1$ (= 0) is the unique equilibrium in all $t > z$ (follows from Lemma 2).

In the case of complements ($\epsilon < 1$):

1. At time $t = z$ there exists a unique equilibrium research allocation $s^*(t = z)$ with all research directed to sector l (e), i.e. $s(t = z) = 1$ ($s(t = z) = 0$), if

$$A(t = z) \overset{(>)}{<} \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}}. \tag{B.4}$$

If $A(t = z) \overset{(>)}{<} \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}}$ and with strong positive (negative) energy price growth, $s^*(t = z) = 0$ (1) is an equilibrium in $t = z$ and in all $t > z$ (follows from Lemma 2). If $A(t = z) \overset{(<}{>} \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}}$ and with strong positive (negative) energy price growth, $s^*(t = z) = 1$ (0) is a unique equilibrium in $t = z$ and since $\left|1 - \frac{\Pi_{li(t)}}{\Pi_{ei(t)}}\right|$ increases over time, there exists a time $\tau > z$, such that $\frac{\Pi_{li(t=\tau)}}{\Pi_{ei(t=\tau)}} = 1$ and $\frac{\Pi_{li(t)}}{\Pi_{ei(t)}} \overset{(<)}{>} 1$ for all $t > \tau$, leading to an equilibrium with all research directed to sector l (e) for all $t > \tau$.

2. At time $t = z$ there exist multiple equilibria $s^* \in [0, 1]$, if

$$A(t = z) = \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}}.$$

With strong positive (negative) energy price growth, $s^*(t) = 0$ (= 1) is the unique equilibrium in all $t > z$ (follows from (B.4)).

C. STRUCTURAL EFFECT AND EFFICIENCY EFFECT

Proof of Proposition 1

$$\gamma_{\frac{E}{Y}} = [(\alpha_2 S - 1) - S\epsilon\alpha_2]\gamma_{c_E} + [(1-\alpha)S - S(1-\alpha)\epsilon]\gamma_{A_l} + [-(1-\alpha_1)S + S\epsilon(1-\alpha_1)]\gamma_{A_e}$$

Proof:

i. Follows from equation (17) with $\gamma_{A_e} > 0$, $\gamma_{A_l} = 0$, and $\gamma_{c_E} = 0$:

$$\textit{structural effect} = (1-\alpha_1)S\epsilon\gamma_{A_e} > 0,$$
$$\textit{efficiency effect} = -(1-\alpha_1)S\gamma_{A_e} < 0,$$
$$\textit{structural effect} + \textit{efficiency effect} \equiv \gamma_{\frac{E}{Y}} = (\epsilon - 1)(1-\alpha_1)S\gamma_{A_e} \overset{(<)}{>} 0 \Leftrightarrow \epsilon \overset{(<)}{>} 1$$

ii. Follows from equation (17) with $\gamma_{A_e} = 0$, $\gamma_{A_l} > 0$, and $\gamma_{c_E} = 0$:

$$\textit{structural effect} = -(1-\alpha)S\epsilon\gamma_{A_l} < 0,$$
$$\textit{efficiency effect} = (1-\alpha)S\gamma_{A_l} > 0,$$
$$\textit{structural effect} + \textit{efficiency effect} \equiv \gamma_{\frac{E}{Y}} = (1-\epsilon)(1-\alpha)S\gamma_{A_l} \overset{(>)}{<} 0 \Leftrightarrow \epsilon \overset{(<)}{>} 1$$

iii. Follows from equation (17) with $\gamma_{A_e} = 0$, $\gamma_{A_l} = 0$, and $\gamma_{c_E} \neq 0$:

$$\begin{aligned} \textit{structural effect} &= -S\epsilon\alpha_2\gamma_{c_E} \overset{(>)}{<} 0 \Leftrightarrow \gamma_{c_E} \overset{(<)}{>} 0, \\ \textit{efficiency effect} &= -(1-\alpha_2 S)\gamma_{c_E} \overset{(>)}{<} 0 \Leftrightarrow \gamma_{c_E} \overset{(<)}{>} 0, \\ \textit{structural effect} + \textit{efficiency effect} \equiv \gamma_{\frac{E}{Y}} &\overset{(>)}{=<} 0 \Leftrightarrow \gamma_{c_E} \overset{(<)}{>} 0. \end{aligned}$$

□

Proof of Proposition 2

Proof:

i. Follows from equation (18):

$$\begin{aligned} \textit{total effect} &= \left[\frac{\alpha_2(1-\epsilon)\,A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}} - 1\right]\gamma_{c_E} + \left[\frac{\varphi\,A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma\eta_l < 0 \\ &\Leftrightarrow \gamma_{c_E} > \frac{\varphi A^{1-\epsilon}}{(\alpha_2(1-\epsilon)-1)\,A^{1-\epsilon} - \theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_l \equiv \Lambda_{l,TE} \overset{(<)}{>} 0 \Leftrightarrow \epsilon \overset{(<)}{>} 1, \end{aligned}$$

$$\frac{\partial \Lambda_{l,TE}}{\partial A} = -\frac{(1-\epsilon)\varphi A^{-\epsilon}\theta c_E^{\alpha_2(1-\epsilon)}}{\left[(\alpha_2(1-\epsilon)-1)\,A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}\right]^2}\gamma\eta_e < 0.$$

ii. Follows from equation (14):

$$\begin{aligned} \textit{efficiency effect} &= \frac{(\alpha_2-1)A^{1-\epsilon}-\theta c_E^{\alpha_2(1-\epsilon)}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\gamma_{c_E} + \frac{(1-\alpha)A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_l < 0, \\ &\Leftrightarrow \gamma_{c_E} > \frac{(1-\alpha)A^{1-\epsilon}}{(1-\alpha_2)A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_l \equiv \Lambda_{l,EE} > 0. \end{aligned}$$

$$\frac{\partial \Lambda_{l,EE}}{\partial A} = \frac{\varphi A^{-\epsilon}\theta c_E^{\alpha_2(1-\epsilon)}}{\left[(1-\alpha_2)A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}\right]^2}\gamma\eta_l \overset{(>)}{<} 0 \Leftrightarrow \epsilon \overset{(<)}{>} 1.$$

iii. Follows from equation (16):

$$\textit{structural effect} = \frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\epsilon(-\alpha_2\gamma_{c_E} - (1-\alpha)\gamma\eta_l) < 0 \Leftrightarrow \gamma_{c_E} > -\frac{(1-\alpha)}{\alpha_2}\eta_l\gamma.$$

□

Proof of Proposition 3

Proof:

i. Follows from equation (18):

$$\text{total effect} = \left[\frac{\alpha_2(1-\epsilon)\,A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}-1\right]\gamma_{c_E}+\left[\frac{-\varphi_1\,A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma\eta_e<0$$

$$\Leftrightarrow \gamma_{c_E} > \frac{\varphi_1 A^{1-\epsilon}}{(\alpha_2(1-\epsilon)-1)\,A^{1-\epsilon}-\theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_e \equiv \Lambda_{e,TE} \overset{(<)}{>} 0 \Leftrightarrow \epsilon \overset{(<)}{>} 1,$$

$$\frac{\partial \Lambda_{e,TE}}{\partial A} = -\frac{(1-\epsilon)\varphi_1 A^{-\epsilon}\theta c_E^{\alpha_2(1-\epsilon)}}{\left[(\alpha_2(1-\epsilon)-1)\,A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}\right]^2}\gamma\eta_e<0.$$

ii. Follows from equation (14):

$$\text{efficiency effect} = \left[\frac{\alpha_2\,A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}-1\right]\gamma_{c_E}+\left[\frac{-(1-\alpha_1)\,A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma\eta_e<0$$

$$\Leftrightarrow \gamma_{c_E} > -\frac{(1-\alpha_1)A^{1-\epsilon}}{(1-\alpha_2)A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_e \equiv \Lambda_{e,EE}<0,$$

$$\frac{\partial \Lambda_{e,EE}}{\partial A} = -\frac{\varphi_1 A^{-\epsilon}\theta c_E^{\alpha_2(1-\epsilon)}}{\left[(1-\alpha_2)A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}\right]^2}\gamma\eta_e \overset{(<)}{>} 0 \Leftrightarrow \epsilon \overset{(<)}{>} 1.$$

iii. Follows from equation (16):

$$\text{structural effect} = \left[-\frac{\epsilon\alpha_2 A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma_{c_E}+\left[\frac{\epsilon\,(1-\alpha_1)\,A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma\eta_e<0 \Leftrightarrow \gamma_{c_E} > \frac{(1-\alpha_1)}{\alpha_2}\eta_e\gamma.$$

□

Proof of Proposition 4

Proof:

i. Follows from (16):

$$\begin{aligned}
\text{structural effect} =& -\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\epsilon\alpha_2\gamma_{c_E}-\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}(1-\alpha)\epsilon\gamma\eta_l\frac{\alpha_2(\epsilon-1)\frac{\gamma_{c_E}}{\gamma}+\eta_e\varphi_1}{\eta_e\varphi_1+\eta_l\varphi}\\
&+\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\epsilon(1-\alpha_1)\gamma\eta_e\frac{-\alpha_2(\epsilon-1)\frac{\gamma_{c_E}}{\gamma}+\eta_l\varphi}{\eta_e\varphi_1+\eta_l\varphi}\\
=& -\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\epsilon\alpha_2\gamma_{c_E}+\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\alpha_2\epsilon\gamma_{c_E}\frac{(1-\alpha)\eta_l+(1-\alpha_1)\eta_e}{\eta_l(1-\alpha)+\eta_e(1-\alpha_1)}\\
&+\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\epsilon\gamma\frac{(1-\alpha)\eta_l\eta_e(1-\alpha_1)-(1-\alpha_1)\eta_e\eta_l(1-\alpha)}{\eta_l(1-\alpha)+\eta_e(1-\alpha_1)}\\
=&0.
\end{aligned}$$

Follows from (14):

$$\begin{aligned}
\textit{efficiency effect} =& \left(\alpha_2 \frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}} - 1\right)\gamma_{c_E} + (1-\alpha)\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_l \frac{\alpha_2(\epsilon-1)\frac{\gamma_{c_E}}{\gamma}+\eta_e\varphi_1}{\eta_e\varphi_1+\eta_l\varphi} \\
& - (1-\alpha_1)\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_e \frac{-\alpha_2(\epsilon-1)\frac{\gamma_{c_E}}{\gamma}+\eta_l\varphi}{\eta_e\varphi_1+\eta_l\varphi} \\
=& \left(\alpha_2 \frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}} - 1\right)\gamma_{c_E} + \frac{(-1)(1-\alpha)\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\eta_l\alpha_2\gamma_{c_E}}{\eta_e(1-\alpha_1)+\eta_l(1-\alpha)} \\
& - \frac{(1-\alpha_1)\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\eta_e\alpha_2\gamma_{c_E}}{\eta_e(1-\alpha_1)+\eta_l(1-\alpha)} \\
& + \frac{(1-\alpha)\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_e\eta_l(1-\alpha_1) - (1-\alpha_1)\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_e\eta_l(1-\alpha)}{\eta_e(1-\alpha_1)+\eta_l(1-\alpha)} \\
=& \left(\alpha_2 \frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}} - 1\right)\gamma_{c_E} - \frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\alpha_2\gamma_{c_E}\frac{(1-\alpha)\eta_l-(1-\alpha_1)\eta_e}{\eta_e(1-\alpha_1)+\eta_l(1-\alpha)} \\
=& -\gamma_{c_E}.
\end{aligned}$$

ii. As $\gamma_{c_E} > \eta_e\gamma(1-\alpha_1)/\alpha_2 > \Lambda_{e,EE} < 0$ (see Proposition 3), the efficiency effect is negative. For $\gamma_{c_E} > \eta_e\gamma(1-\alpha_1)/\alpha_2$, the structural effect is negative (see Proposition 3). Hence, the total effect must be negative.

iii. As $\gamma_{c_E} < \eta_l\gamma(1-\alpha)/\alpha_2 < \Lambda_{l,EE} > 0$ (see Proposition 2), the efficiency effect is positive. For $\gamma_{c_E} < \eta_l\gamma(1-\alpha)/\alpha_2$, the structural effect is positve (see Proposition 2). Hence, the total effect must be positive.

□

D. DIRECTION OF TECHNICAL CHANGE WITH INFINITE-DURATION PATENTS

Scientists choose to direct their research at the sector with higher expected firm value (discounted flow of future profits as entrepreneur):

$$E\left[V_{ji}(t=z)\right] = \int_z^\infty E\left[\pi_{ji}(t)\right] exp\left(-\int_z^t \left(1-E\left[s_j(t)\right]\eta_j\right)dt\right)dt \quad \text{with } j\in\{e,l\}.$$

The expected relative value of firm i in sector j at time $t=z$ comprises current (at time z) and discounted future ($t>z$) expected profits ($E\left[\pi_{ji}(t)\right]$). The expected discount rate $(1-E[s_j(t)]\eta_j)$ depends on the expected research effort in sector j at each time t ($E[s_j(t)]$) and the probability of successful research (η_j). Expected relative firm value at $t=z$ is defined as

$$V(t=z) \equiv \frac{E\left[V_{li}(t=z)\right]}{E\left[V_{ei}(t=z)\right]}.$$

Substitutes (i.e. $\epsilon>1$):
Since equilibrium research allocation depends crucially on the expected discount rate, the subsequent discussion of research equilibria is structured along three discount rate cases (for special cases see 1. & 3., general case 2.):

1. For $(1 - E[s_j(t)]\eta_j) \to 0$, $V(t = z) \to \frac{\Pi_{li}(t=z)}{\Pi_{ei}(t=z)}$, i.e. relative firm value reduces to current relative firm profits. Results of Appendix B can be applied.

2. For $0 < (1 - E[s_j(t)]\eta_j) < 1$ and since $\frac{\partial E[V_{ji}(t=z)]}{\partial \Pi_{ji}(t)} > 0$, $\frac{\partial E[\Pi_{ji}(t)]}{\partial A_j(t)} > 0$, $\frac{\partial E[\Pi_{ji}(t)]}{\partial A_j(t)} > \frac{\partial E[\Pi_{sector \neq j,i}(t)]}{\partial A_j(t)}$, $\lim_{A_j(t)\to 0} E[\Pi_{ji}(t)] = 0$, $\lim_{A_j(t)\to\infty} E[\Pi_{ji}(t)] = \infty$ for each set of parameters there exists a unique relative technology $(A_l(t = z)/A_e(t = z))^*$ such that $\frac{V_{li}(t=z)|_{s(t)=1}}{V_{ei}(t=z)|_{s(t)=0}}\Big|_{\frac{A_l(t=z)}{A_e(t=z)}=\left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*} =$ 1. With $\frac{A_l(t=z)}{A_e(t=z)} \overset{(<)}{>} \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*$, research will take place in the l-sector (e-sector) only. With $\frac{A_l(t=z)}{A_e(t=z)} = \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*$ there exists a unique equilibrium ($s^{**} \in (0, 1)$) with research directed to both sectors.

   (a) With moderate energy price growth the expected relative profit (and therefore the expected relative firm value ($V(t)$)) increases (decreases) if research is directed to sector l (e) only (Proof: see (B.2)). Therefore a research equilibrium $s^* \in \{0, 1\}$ at time z is always a research equilibrium in $t > z$. An inner equilibrium in $t = z$, $s^*(t = z) = s^{**}$, is an inner equilibrium if and only if $s^*(t) = s^{**}\ \forall\ t \geq z$. With $s^*(t = z) \overset{(<)}{>} s^{**}$ research will take place in sector l (e) for all $t > z$ (follows from (B.2)).

   (b) With strong positive (negative) energy price growth and $\frac{A_l(t=z)}{A_e(t=z)} \overset{(<)}{>} \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*$ research will occur in sector l (e) at $t = z$ and all $t > z$ (follows from (B.3)). If $\frac{A_l(t=z)}{A_e(t=z)} \overset{(>)}{<} \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*$ and with strong positive (negative) energy price growth, research will at $t = z$ take place in the e-sector (l-sector) only. Since strong positive (negative) energy price growth increases (decreases) $V(t)$, there exists a time $\tau > z$ where $V(\tau) = 1$ and $V(t > \tau) \overset{(<)}{>} 1$, leading to research equilibrium in sector l (e) for all $t > \tau$ (follows from (B.3)). There are multiple equilibria with $s^*(t = z) \in [0, 1]$ if $\frac{A_l(t=z)}{A_e(t=z)} = \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*$ and a unique equilibrium with all research in sector l (e) for all $t > z$ in the case of strong positive (negative) energy price growth.

3. For $(1 - E[s_j(t)]\eta_j) \to 1$ and moderate energy price growth, $V(t = z) \to 1$ and there exist two equilibria with all research directed to the e- or the l-sector and multiple equilibria with research directed to both sectors (i.e. $s \in (0, 1)$). With strong positive (negative) energy price growth there exists a unique equilibrium with all research directed to sector l (e), as $\frac{d\Pi_e i(t)}{dt} \to 0$ ($\frac{d\Pi_l i(t)}{dt} \to 0$) and therefore $V(t = z) \to \overset{(0)}{\infty}$.

For discount rates smaller than 1, i.e. $0 \leq (1 - E[s_j(t)]\eta_j) < 1$, from 1. and 2. it follows that alternative patent terms do not induce qualitative differences in the research equilibrium at $t = z$. Research takes place in the relatively more advanced sector; patent terms only affect the level of relative technology threshold $(A_l/A_e)^*$.

In the case of strong positive (negative) energy price growth and $\frac{A_l(t=z)}{A_e(t=z)} \overset{(>)}{<} \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*$, patent duration affects the timing of the redirection of technical change from the e- to the l-sector (l- to the e-sector). The lower the discount rate the earlier technical change is redirected.

Complements (i.e. $\epsilon < 1$):
With moderate energy price growth, in finite time $V(t) = 1$ holds and s^{**} is the equilibrium research allocation (analogue to moderate energy price growth and complements in AppendixB). With strong positive (negative) energy price growth, in finite time $V(t) \overset{(>)}{<} 1$ holds and $s = 0\,(= 1)$ is the equilibrium research allocation (analogue to strong energy price growth and complements in AppendixB).