Online Appendix for the Article "An Extended Analysis on the Remuneration of Capacity under Scarcity Conditions"

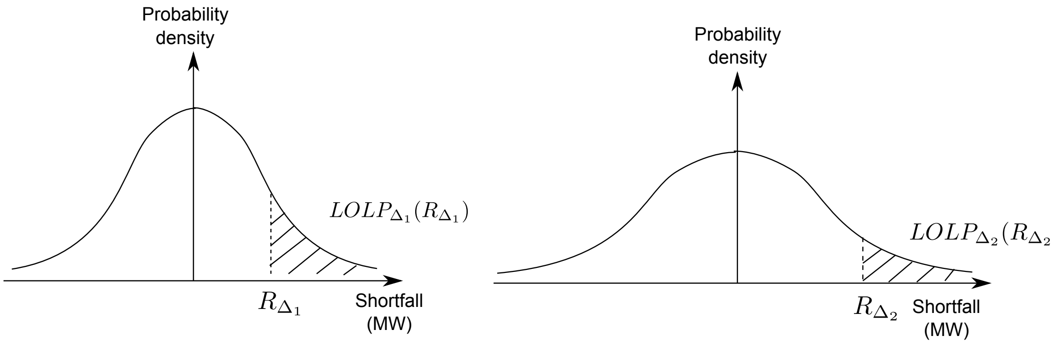
ANTHONY PAPAVASILIOU* AND YVES SMEERS** AND GILLES BERTRAND**

---

*Corresponding author. Center for Operations Research and Econometrics, Université catholique de Louvain. E-mail: anthony.papavasiliou@uclouvain.be. Telephone:+3210474325.

**Center for Operations Research and Econometrics, Université catholique de Louvain.

Figure 1: The loss of load probability as a function of reserve response time: for greater response time, $t_2 > t_1$, the system faces more uncertainty (note the greater variance of the distribution on the right), but more reserve can be made available ($R_{\Delta_2} > R_{\Delta_1}$).



1. COMPUTATION OF THE SCARCITY ADDER

The ORDC adder can be derived through an analysis of the KKT conditions of a two-stage stochastic dispatch model (Hogan, 2013; Papavasiliou and Smeers, 2017), and the resulting real-time energy price is computed as follows:

$$\begin{aligned}
&\lambda = MC_{g_m}(p_{g_m}) + \\
&\frac{T_1}{T_1 + T_2}(VOLL - \hat{MC}(\sum_g p_g)) \cdot LOLP_{\Delta_1}(R_{\Delta_1}) + \\
&\frac{T_2}{T_1 + T_2}(VOLL - \hat{MC}(\sum_g p_g)) \cdot LOLP_{\Delta_2}(R_{\Delta_2}).
\end{aligned} \tag{1}$$

In the above formula, $MC_{g_m}(p_{g_m})$ is the marginal cost of the marginal unit, as in an energy-only market without an adder. $VOLL$ represents the value of lost load. The function $\hat{MC}$ is introduced by Hogan (2013) as the incremental cost of operations of the system for meeting an increment in demand, for example one could employ the merit order function of the system when transmission constraints are ignored. The function $LOLP_{\Delta_i}(R_{\Delta_i})$ maps the amount of reserve that is available to the system and which can respond within Δ_i minutes (e.g. 7.5 minutes for secondary reserve, and 15 minutes for tertiary reserve) to the loss of load probability, given the uncertainty that the system is facing in a horizon of Δ_i minutes. This concept is illustrated graphically in figure 1. Finally, $T_1 = \Delta_1 < \Delta_2 = T_1 + T_2$.

2. FORMULAS FOR THE COMPUTATION OF THE ORDC ADDER WITH CORRELATED IMBALANCE INCREMENTS

Denote $LOLP_{7.5}(R_{7.5})$ as the loss of load probability in a 7.5-minute horizon given that a capacity of $R_{7.5}$ can respond within 7.5 minutes. We present formulas for $LOLP_{7.5}(R_{7.5})$ under three alternative assumptions. These alternatives are: *(i)* fully independent increments of imbalance (Papavasiliou and Smeers, 2017); *(ii)* fully correlated increments of imbalance (Hogan, 2013); and *(iii)* increments of imbalance which are calibrated against the empirically observed correlation. For all cases, it is assumed that the 15-minute uncertainty, X_t, is distributed as a normal distribution with mean μ_{15} and standard deviation σ_{15}.

Fully independent increments of imbalance. The 7.5-minute imbalance is obtained as a random variable with a distribution $N(\frac{1}{2}\mu_{15}, \sqrt{\frac{1}{2}}\sigma_{15})$, where $N(\cdot)$ denotes the normal distribution. The 7.5-minute contribution to the scarcity adder is computed as:

$$\begin{aligned}&LOLP_{7.5}(R_{7.5}) = \mathbb{P}[R_{7.5} < Y_t] = 1 - \mathbb{P}[R_{7.5} \geq Y_t] = \\ &1 - \mathbb{P}[\frac{R_{7.5} - \frac{1}{2}\mu_{15}}{\sqrt{\frac{1}{2}}\sigma_{15}} \geq \frac{Y_t - \frac{1}{2}\mu_{15}}{\sqrt{\frac{1}{2}}\sigma_{15}}] = 1 - \Phi(\frac{R_{7.5} - \frac{1}{2}\mu_{15}}{\sqrt{\frac{1}{2}}\sigma_{15}}),\end{aligned} \tag{2}$$

where Y_t is the 7.5-minute imbalance of interval t, and $\Phi(\cdot)$ is the cumulative density function of the standard normal distribution.

Fully correlated increments of imbalance. Suppose that the 15-minute imbalance X_t is the result of a linear evolution of the imbalance. The 7.5-minute contribution to the scarcity adder is computed as:

$$\begin{aligned}&LOLP_{7.5}(R_{7.5}) = \mathbb{P}[R_{7.5} < Y_t] = 1 - \mathbb{P}[R_{7.5} \geq Y_t] = \\ &1 - \mathbb{P}[R_{7.5} \geq \frac{1}{2}X_t] = 1 - \mathbb{P}[2R_{7.5} \geq X_t] = \\ &1 - \mathbb{P}[\frac{2R_{7.5} - \mu_{15}}{\sigma_{15}} \geq \frac{X_t - \mu_{15}}{\sigma_{15}}] = 1 - \Phi(\frac{2R_{7.5} - \mu_{15}}{\sigma_{15}}).\end{aligned} \tag{3}$$

Partially correlated increments of imbalance. We use kernel density estimation for estimating $G(\delta|x) = \mathbb{P}[\Delta_t \leq \delta|X_t = x]$, the cumulative distribution function of $\Delta_t = W_t - Y_t$ conditional on X_t, where W_t is the increment of imbalance from minute 7.5 to minute 15 for a given imbalance interval t. The detailed explanation of kernel density estimation is provided in section 3. Once the function $G(\cdot)$ has been estimated, we proceed with the computation of the new adders, which account for the correlation of the imbalance increments. Conditioning on the realization of the 15-minute imbalance, X_t, we obtain the following:

$$
\begin{aligned}
&LOLP_{7.5}(R_{7.5}) = \int_{-\infty}^{\infty} \mathbb{P}[Y_t \geq R_{7.5}|x]\mathbb{P}[X_t = x]dx = \\
&\int_{-\infty}^{\infty} \mathbb{P}[\Delta_t \leq x - 2 \cdot R_{7.5}|x]\mathbb{P}[X_t = x]dx = \\
&\int_{-\infty}^{\infty} G(x - 2 \cdot R_{7.5}|x)\phi(\frac{x - \mu_{15}}{\sigma_{15}})dx
\end{aligned} \tag{4}
$$

where $\phi(\cdot)$ represents the probability density function of the standard normal distribution. The second equality is obtained by observing that $\Delta_t = W_t - Y_t$ and $X_t = Y_t + W_t$ by definition.

3. KERNEL DENSITY ESTIMATION

In order to estimate the conditional distribution $\mathbb{P}[\Delta_t|X_t]$ of the difference of imbalance increments Δ_t given the 15-minute imbalance of the interval, X_t, we use kernel density estimation.

The idea of our approach is to customize the distribution of Δ_t not only to the month and the hour of the day (which is the current approach in ERCOT (Hogan, 2013)), but also to the imbalance of the interval, X_t. More specifically, we check how the feature vector x, which consists of three explanatory factors (hour, month, X_t), maps to the output (change in imbalance increments, Δ_t). We do this for the k 'closest' observations in the data, in the sense of the k historical observations of factors x_i whose Euclidean distance from the current-period conditions is the smallest[1]. This process of selecting the historical data with the closest explanatory factors in order to predict the output of the current period is called the k *nearest neighbors* algorithm in machine learning. We use $k = 1000$ neighbors in the results presented in the paper.

Once the k nearest neighbors of the current interval t have been identified, we use the historically observed imbalance of these neighbors in order to estimate a probability distribution for the change in imbalance increments, Δ_t. For this purpose, we use kernel density estimation (KDE). The basic idea of the KDE estimator is to create a distribution by placing a normal distribution, referred to as a Gaussian kernel, around the historically observed output (change in imbalance increments δ_i) resulting from the explanatory factors x_i of the k nearest neighbors. Mathematically, the kernel

[1] We normalize data so as to have the same standard deviation for the hour and month, and twice the standard deviation for the imbalance.

density estimator can be described as follows:

$$g(\delta|x) = \sum_{i=1}^{k} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\delta - \delta_i)^2}{2\sigma^2}\right), \tag{5}$$

where $g(\delta|x)$ is the probability density function of the change in imbalance δ, k is the number of nearest neighbors, σ is a free parameter which determines the width of the Gaussian kernel, and δ_i is the observed change in imbalance increments of the i-th nearest neighbor. The dependence on x, the explanatory factors, is implicit, since the output points δ_i which we choose in order to build the KDE are those of the k nearest neighbors, and therefore depend on the explanatory factors x.

The integral of $g(\delta|x)$ is the cumulative distribution function used in the computation of the 7.5-minute adder.

References

Hogan, W., 2013. Electricity scarcity pricing through operating reserves. Economics of Energy and Environmental Policy 2 (2), 65–86.

Papavasiliou, A., Smeers, Y., 2017. Remuneration of flexibility using operating reserve demand curves: A case study of Belgium. The Energy Journal, 105–135.