Fuel-Switching Capability

Alain Bousquet[*] and Norbert Ladoux[†]
University of Toulouse, IDEI and CEA

June 3, 2003

Abstract

Taking into account the link between energy demand and equipment choice, leads to a new model of energy demand where the key variables are energy prices, price volatility and the costs of equipment. In particular we show that adjustment to changes in relative energy prices are generally asymmetric, as the adjustment involves adaptation of equipment costs. As a consequence, the optimal equipment choice in the long run could imply short-term allocative inefficiency in energy use. This model can be seen as a mix or a generalization of the Putty-Putty versus Putty-Clay models of energy use. Depending on the technology and in particular costs of energy equipment, the model gives drastically different predictions about the effect of energy prices and uncertainty on energy demand.

[*] University of Toulouse, CEA and LEERNA, 21 allée de Brienne, 31000 Toulouse Cédex, tel: (33) 5 61 12 85 91, fax: (33) 5 61 12 86 37, Email : bousquet@cict.fr.

[†] University of Toulouse, IDEI and LEERNA, 21 allée de Brienne, 31000 Toulouse Cédex, tel: (33) 5 61 12 85 90, fax: (33) 5 61 12 86 37, Email : ladoux@cict.fr.

1 Introduction

This paper provides a micro-economic foundation of the concept of fuel-switching capability. Fuel switching is the extend to witch a producer can reduce the use of one type of energy and uptake of another source of energy in its place. We develop a micro-economic model where energy consumption and energy equipment capacity are both endogenous. In many situations the amount of input that will be used in the future is necessarily less than an upper limit determined by the installed equipment capacity. Energy demand and electricity generation are good examples of such a situation. Once the …rm has chosen an equipment, there is a maximal amount of energy use, determined by the power and energy e¢ciency characteristics of the equipment. As a direct consequence the amount of one kind of energy that could be substituted by another kind of energy is necessarily limited.

This paper is a mix of previous model of …rm behavior under input rationing and uncertainty. The main feature of our analysis is that a …rm has to decide on its machine characteristics determining a maximum level of use for some input before the demand conditions are completely known. In short, the producer trade-o¤ is to choose ex ante the optimal level of ‡exibility of the technology through di¤erent input capacities allowing to reach ex post e¢ciency given market conditions. In our paper we extend the analysis of the …rm behavior under price uncertainty when input use is subject to endogenous rationing.

Uncertainty and risk play a huge role in energy supply as well as in energy demand, especially on equipment choice. Understanding the e¤ects of price uncertainty on …rm's production and input choice decisions has long been an important issue in economics (see for example Sandmo (1971), Abel (1983) or Dixit and Pindyck (1994)). Real options approach to investment identi…es three characteristics of most investment decisions, …rst uncertainty over future pro…t streams, second irreversibility and …nally the choice of timing.

Most of existing models consider and study a situation where capital is …xed or quasi-…xed factor and other inputs are variable factors. Plant capacity is then de…ned by the maximum amount that can be produced per unit of time with existing plant and equipment, i.e. given the level of capital.

In the energy sector, as long as energy could not be used or generated without some equipment, we are in a situation where variable factors are linked to …xed factors. This is precisely what we want to develop in this paper. We consider that input use for variable factors could be constrained by …xed factors, according to the size and more generally the characteristics of the equipment.

2 The model

This section provides a simple model of a competitive firm with input price uncertainty. The time horizon is two periods. In our model the production decision is made in two stages : an ex ante plan and an ex post plan. Ex ante, the price of input is unknown. The firm makes an investment and chooses an equipment size which introduces an upper limit in the total amount of input that could be used in second period. Ex post, the equipment and therefore the maximal amount of input that could be used is given and the price of input is known. The firm cannot adjust the size of the equipment but can choose a low rate of use of its equipment if the realized price of the input is too high. The two-stage production decision problem is solved backwards.

2.1 Unrestricted cost function

We solve our model in the particular case of a CES production function. Part of the following results could be derived from a more general production function. Nevertheless, as we will focus mainly on the role of substitution possibility between inputs, and show its great importance in the choice of equipment capacity, the entire model is solved for this particular functional form, which is

$$y = \gamma\,[\delta x_1^{\rho} + (1-\delta)x_2^{\rho}]^{1/\rho};$$

where y represents output, and x_i for $i = 1;2$ represents input use.

The CES production function is defined for $\rho \in\,]-\infty;1]$, and $0 \leq \delta \leq 1$.

Moreover we know that the CES production function leads to the

Leontief production function, as $\rho \to -\infty$, the Cobb-Douglas production function as $\rho = 0$, and the linear production function, as $\rho = 1$.

We denote by $\sigma = \frac{1}{1-\rho}$ the substitution elasticity between the two inputs.

We start from the usual cost minimization program of the firm and we will introduce later on exogenous and then endogenous rationing on input use.

The optimal input demand are the solution of the following unconstrained cost minimization program,

$$\begin{aligned} &\underset{x_1;x_2}{Min} \quad p_1x_1 + p_2x_2; \\ &\text{Subject to } y = \gamma\,[\delta x_1^{\rho} + (1-\delta)x_2^{\rho}]^{1/\rho} \end{aligned};$$

and could be expressed as

$$
\begin{aligned}
x_1 &= \frac{y}{\gamma}\left[\delta + (1-\delta)\left(\frac{(1-\delta)p_1}{\delta p_2}\right)^{\frac{\rho}{1-\rho}}\right]^{-\frac{1}{\rho}};\\
x_2 &= \frac{y}{\gamma}\left[(1-\delta) + \delta\left(\frac{(1-\delta)p_1}{\delta p_2}\right)^{\frac{-\rho}{1-\rho}}\right]^{-\frac{1}{\rho}}.
\end{aligned}
$$

2.2 Restricted cost function

Consider now the firm behavior under exogenous rationing, as in Lee and Pitt (1987) and Squires (1994). We limit ourself to the case where only input 1 is subject to a quantity constraint, but the model can be generalized when input 2 is also subject to some quantity constraint. We consider and denote the constraint on input 1 as, follows:

$$x_1 \leq \overline{x_1}.$$

The cost minimization program is

$$\underset{x_1,x_2}{Min} \quad p_1x_1 + p_2x_2,$$

$$\text{Subject to } y = \gamma\left[\delta x_1^{\rho} + (1-\delta)x_2^{\rho}\right]^{1/\rho}, \text{ and } x_1 \leq \overline{x_1}.$$

Optimal input demand are

$$
\begin{cases}
\overline{x_1} \\ \underline{x_2} & \text{if } p_1 \leq \pi_1, \\
x_1^*(p_1,p_2,y) \\ x_2^*(p_1,p_2,y) & \text{if } p_1 > \pi_1.
\end{cases}
$$

Where π_1 is the virtual price of input 1 (see Heckman (1974) and Neary and Roberts (1980) for a complete treatment of this concept) at which the unconstrained demand for input 1 is exactly equal to $\overline{x_1}$, and

$$\pi_1 = p_2\left[\left(\frac{y}{\gamma\overline{x_1}}\right)^{\rho} - \delta\right]^{\frac{1-\rho}{\rho}}(1-\delta)^{\frac{-1}{\rho}}\delta. \tag{1}$$

Moreover when the constraint on x_1 is binding we have

$$\underline{x_2} = \left[\frac{\left(\frac{y}{\gamma}\right)^{\rho} - \delta\overline{x_1}^{\rho}}{1-\delta}\right]^{\frac{1}{\rho}}.$$

2.3 Optimal input capacity

Suppose now that the threshold $\overline{x_1}$ is endogenously determined. Assume first that some equipment is necessary to use input 1. Second, assume that the characteristics of the equipment, in particular the size of this equipment, induce a constraint on the maximal amount of input such that $x_1 \leq \overline{x_1}$. The cost of the equipment is a function of $\overline{x_1}$, and consider the case where the cost of the equipment could be written as $c_1\overline{x_1}$. Here, c_1 is the constant marginal cost of the equipment capacity. Finally assume that the firm faces an uncertain price for input 1. So p_1 is assumed to be a random variable with density function $\varphi(p_1)$, cumulative density function $\Phi(p_1)$ and with $p_1 \in [0; +\infty[$.

We assume that the firm has to decide about the level of the capacity $\overline{x_1}$ prior to the knowledge of the input price. For each possible value of $\overline{x_1}$, we know that there exist a price threshold, denoted by η_1 such that if $p_1 \leq \eta_1$ the equipment capacity constraint will be binding while for $p_1 > \eta_1$ input use is such that $x_1^* < \overline{x_1}$.

The ex ante problem corresponding to the choice of the input capacity constraint $\overline{x_1}$, for a risk neutral firm is,

$$\underset{\overline{x_1}}{Min} \int_0^{\eta_1} (p_1\overline{x_1} + p_2\underline{x_2})\varphi(p_1)dp_1 + \int_{\eta_1}^{+\infty} (p_1x_1^* + p_2x_2^*)\varphi(p_1)dp_1 + c_1\overline{x_1}.$$

The first order condition is

$$\int_0^{\eta_1} (p_1 - \eta_1)\varphi(p_1)dp_1 + c_1 = 0.$$

In general it is not possible to obtain an analytical solution for $\overline{x_1}^*$, this will depend on the shape of the statistical distribution φ.

However, remark that it is possible to solve the last FOC with respect to the price threshold and denote the solution by η_1^*, which is independent of the technology. As a consequence, given η_1^*, the parameters of the technology plays now a role in the optimal capacity level which is determined, according equation (1), by

$$\overline{x_1}^* = \frac{y}{\gamma}\left[\delta + (1-\delta)\left(\frac{(1-\delta)\eta_1^*}{\delta p_2}\right)^{\frac{\rho}{1-\rho}}\right]^{-\frac{1}{\rho}}.$$

The following figure illustrates the solution. In the short term the maximum amount of input 1 the firm can use is limited by $\overline{x_1}$ and the isoquant is an arc with an extrema points $\left(\overline{x_1}; \underline{x_2}\right)$. For a given capacity level, the ability to switch between the two inputs depends on the realized price and is limited. We distinguish substitution possibilities and switching capacity simply to keep in mind that the marginal rate of technical substitution is a local measure while the switching capacity is a global one and represents the extent to which substitution possibilities may occur in the short run.

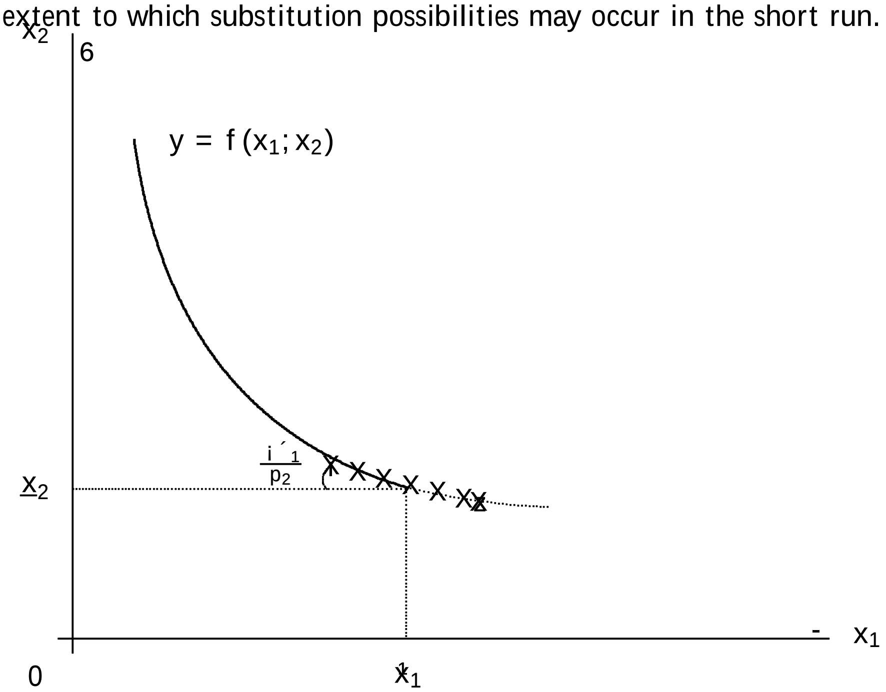

Figure 1 : Switching capability

3 Comparative statics

3.1 Technology and optimal capacity

From the theoretical model, it is possible to determine how the optimal equipment capacity level, depends on the price distribution and production function parameters. In this paper, we consider only the property of the chosen capacity with respect to technology characteristics.

In the appendix, we show that the relationship between optimal equipment capacity level and the substitution elasticity ¾, is highly non-linear. Figure 2 and 3, illustrates how the optimal input capacity level changes with respect to ¾ and ±. In this …gures, y and ° are set equal to 1 by convention. Here, the ratio $\frac{´¤_1}{p_2}$ is assumed to be equal to 1.

For ¾ = 0, the CES production function corresponds to the Leontief production function with perfect complementaries between inputs. It is easy to show that the optimal capacity is equal to $\frac{y}{°}$.

For large values of ¾, the CES production function corresponds to the linear technology with perfect substitutability. It can be shown that when ¾ ! +1, the optimal capacity is equal to $\frac{y}{°±}$ for ± > $\frac{1}{2}$ and equal to 0 for ± < $\frac{1}{2}$, as long as we assume in this simulation that, $\frac{´¤_1}{p_2} = 1$.

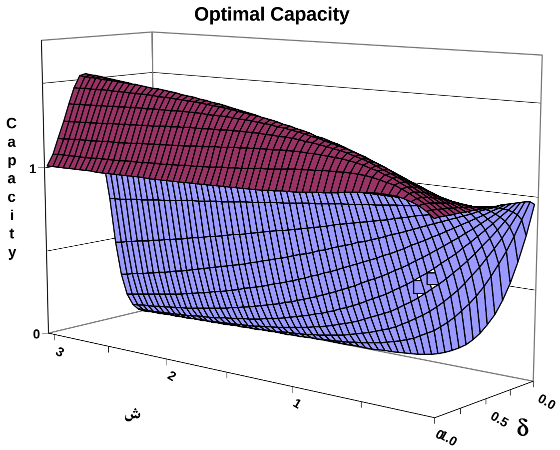

Figure 2: Technology characteristics and optimal capacity

The simulation shows clearly, that the optimal equipment capacity level increase or decrease with respect to the substitution elasticity parameter ¾. Moreover the ...gure illustrates that we could not expect a monotone relationship in general between capacity and ¾, since for example when ± = 0:9 the capacity ...rst increases with ¾ and then decreases. At least, this illustrates the di¢culty to determine the optimal equipment capacity in the energy context where it is well known that the di¤erent kinds of energy are more or less substitute. Moreover, since the model derived from the CES production function leads to a complex relationship between capacity and substitution elasticity, there is no chance to ...nd simple results in the case of a general production function.

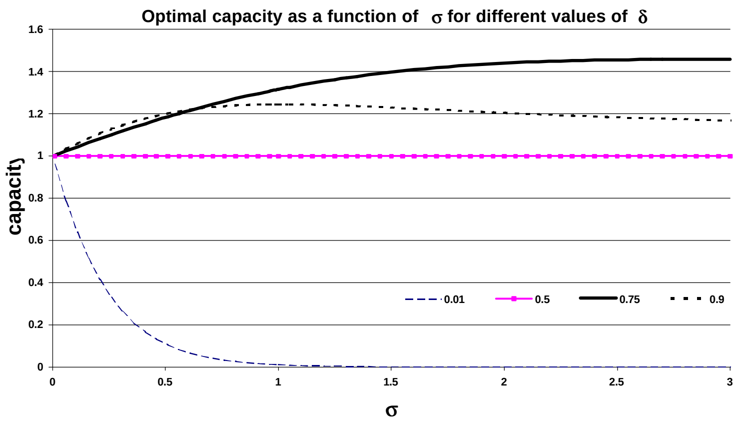

Figure 3: Optimal capacity and substitution elasticity

3.2 Consequences for energy demand modeling

The standard way to estimate the parameters of a CES production function, consists of testing the following simple a linear regression :

$$\ln\left(\frac{x_1}{x_2}\right) = \theta - \sigma \ln\left(\frac{p_1}{p_2}\right);$$

where the estimated slope gives the elasticity of substitution $\sigma = \frac{1}{1-\rho}$, and the value of α could be derived form the constant through the relation $\theta = \sigma \ln\left(\frac{\alpha}{1-\alpha}\right)$.

Demand elasticity for input i, $i = 1, 2$, is $-(1-w_i)\sigma$, where w_i is the share of input i in total cost.

The estimation of a CES in the presence of equipment choice must be derived from the estimation of the following Tobit model (Cragg (1971), Blundell and Meghir (1987)), associated to the short run demand :

$$\begin{cases} \ln\left(\frac{x_1}{x_2}\right) = \theta - \sigma \ln\left(\frac{p_1}{p_2}\right) & p_1 > c_1 \\ \ln\left(\frac{\overline{x_1}}{\overline{x_2}}\right) = \theta - \sigma \ln\left(\frac{c_1}{p_2}\right) & p_1 \leq c_1 \end{cases}$$

Note that it is easy to verify the value of the threshold in the previous censored regression. Using the fact that :

$$\frac{\overline{x_1}}{\overline{x_2}} = \frac{\overline{x_1}}{\left[\frac{\left(\frac{y}{\gamma}\right)^{\rho} - \alpha \overline{x_1}^{\rho}}{1-\alpha}\right]^{\frac{1}{\rho}}};$$

$$\begin{aligned} \overline{x_1} &= \frac{y}{\gamma}[A]^{-\frac{1}{\rho}}; \\ \text{with } A &= \alpha + (1-\alpha)\left(\frac{(1-\alpha)c_1}{\alpha p_2}\right)^{\frac{\rho}{1-\rho}} \end{aligned}$$

For a given set of parameters, $\alpha = 0.5$, $y = 1$, $\gamma = 1$, $\rho = -0.5$, $p_2 = 1$, $c_1 = 1$, and considering a Log normal distribution for the price of input 1 with parameter $\mu = 3$ and $\sigma^2 = 1$, we estimate the corresponding CES production function when the existence of energy equipment is ignored.

In this example the estimated elasticity is $\hat{\sigma} = 0.25$, while the correct value in this case is $\sigma = 0.66$. This very simple estimation show us that the bias in the estimated elasticity could be very large when we do not estimate the energy demand model derived from the CES, and according to the demand threshold induced by the equipment capacity. This estimation has been done only to illustrate the importance of a complete model of energy demand including energy use and equipment choice. Unfortunately we could not provide an empirical test of our model based on real data. At least, we could use general results provided by the MECS (1991) and (1994) to justify the importance of fuel switching capability in the short run and, as a consequence, the relative importance of energy equipment and more generally energy technology on the shape on energy demand.

4 Conclusion

Fuel costs are only one of several criteria that shape energy equipment decisions. In this paper, we embed the micro-economic decisions associated with investment under uncertainty, installed capacity, capacity utilization and energy use. We show that the combination of input price uncertainty and production technology, yields to a complex relationship between energy equipment purchasing behavior and energy demand. This model is consistent with the empirical observation provided by the Manufacturing Energy Consumption Survey (EIA 1994). In the electricity generation sector, inevitable trade-off between price level, price volatility and fixed costs of power plants, leads to a mix of capacity over different technologies and short-run fuel flexibility will be of a great importance in the future.

From a theoretical point of view, our approach provides a very simple and natural framework to understand asymmetric responses to energy price changes and the existence of threshold effects of energy price changes. Considering the fact that investment in capital goods affects not only output, but also input use for adjustable factors, our analysis contributes to enlarge theoretical literature that identifies channels through which uncertainty may influence investment.

5 Appendix

The relationship between the optimal level of the energy equipment capacity for input 1 and the parameters of the technology, is not straightforward in this case.

Deriving the $\overline{x_1}^*$ with respect to the substitution elasticity σ leads to the following non-linear expression,

$$\frac{\partial \overline{x_1}^*}{\partial \sigma} = \frac{\overline{x_1}^*}{1-\sigma}\left[\frac{1}{\sigma}\ln\left(\frac{\gamma \overline{x_1}^*}{y}\right) + \sigma \ln(\Psi)\left(1-\delta\left(\frac{\gamma \overline{x_1}^*}{y}\right)^{\frac{\sigma-1}{\sigma}}\right)\right];$$

where $\Psi = \frac{\delta \eta_1^*}{(1-\delta)p_2}$:

It is easy to show that if $\Psi = 1$, then $\frac{\partial \overline{x_1}^*}{\partial \sigma} = 0$. In this case, corresponding to a particular value of the virtual price associated to the equipment capacity, the optimal capacity is independent of σ and equal to $\frac{y}{\gamma}$.

6 References

Abel, A.B, 1983, "Energy Price Uncertainty and Optimal Factor Intensity: a mean-variance analysis", Econometrica, 51, N°6, 1839, 1845.

Blundell, R. W. and C. Meghir, 1987, "Bivariate Alternatives to the Tobit Model", Journal of Econometrics, 34, 179-200

Cragg, J. G., 1971, "Some Statistical Models for Limited Dependent Variables with Application to the Demand for Durable Goods", Econometrica, 39, 829-844.

Dixit, A., and R.S, Pindyck, 1994, Investment under Uncertainty, Princeton University Press, Princeton, NJ.

Energy Information Administration, 1994, Manufacturing Energy Consumption Survey of 1991, DOE/EIA-0512(91).

Heckman, J., 1974, "Shadow Prices, Market Wages, and Labor Supply", Econometrica, Vol 42, N°4, 679-693.

Lee, L.F. and M.M. Pitt, 1987, "Microeconometric Models of Rationing, Imperfect Markets, and Non-Negativity Constraints", Journal of Econometrics, 36, 89-110.

Squires, D., 1994, "Firm Behavior Under Input Rationing", Journal of Econometrics, 61, 235-257.

Neary, J. P. and K.W.S, Roberts, 1980, "The theory of household behavior under rationing", European Economic Review, 13, 25-42.

Sandmo, A., 1971, "On the Theory of the Competitive Firm Under Price Uncertainty", American Economic Review, 78, N°5, 65-73.