Market Power with Combined Heat and Power Production in the Nordic Energy System

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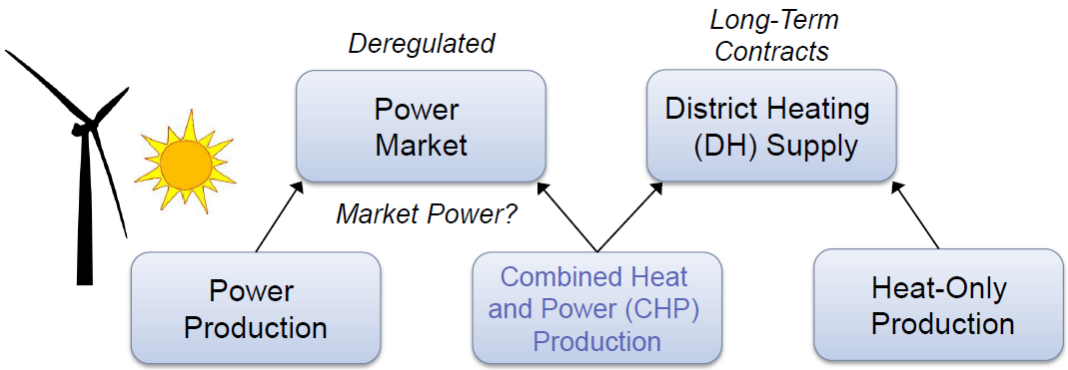
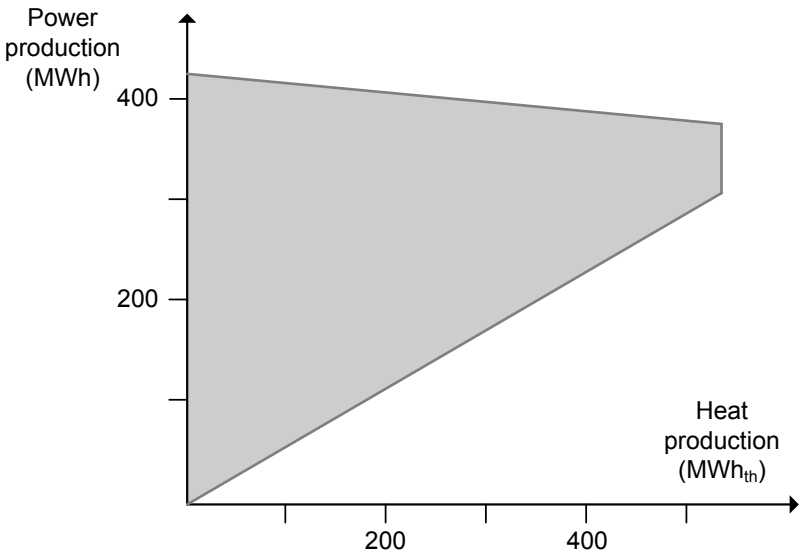
Introduction

Asymmetrically Linked Markets and the Feasible Operating Region (FOR)





Impact of CHP on System Operations

- Wu and Rosen (1999) develop an equilibrium model with CHP in a conventional power system
- Lund et al. (2005) demonstrate how CHP could facilitate integration of wind power
- Chen et al. (2015) examine how CHP with electric boilers and heat storage can provide system flexibility with VRE integration by developing a linear model with a convex CHP FOR
- Potential for exertion of market power (Hobbs, 2001)?
- Fridolfsson and Tangerås (2009) do not find systematically higher-than-marginal-cost prices in Nord Pool, but there could be local market power due to transmission constraints and strategic use of CHP
- Mitridati and Pinson (2016) develop a hierarchical stochastic setup to model CHP operations under uncertainty

Research Objective and Findings

- Examine the system-wide effects of CHP given VRE output, transmission constraints, and market power
- How is market power is impacted by CHP? How can market power can affect district heating (DH) supply?
- Equilibrium model: profit-maximising firms and a grid owner
- Decoupling CHP increases electricity prices with a higher impact under perfect competition than under Cournot oligopoly
- CHP facilitates exercise of market power because the decrease in revenue from withholding is offset by DH sales
- Market power shifts production toward CHP from heat-only plants because CHP makes it easier to withhold generation

Numerical Experiments

Model	Perfect Competition	Cournot Oligopoly
CHP status quo	Case 1 (SQ-PC)	Case 2 (SQ-CO)
CHP decoupling	Case 3 (DE-PC)	Case 4 (DE-CO)

	Case 1 (SQ-PC)	Case 2 (SQ-CO)	Δ^{SQ}	Case 3 (DE-PC)	Case 4 (DE-CO)	Δ^{DE}
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CS	57 697	51 439	- 6 258	53 393	48 607	- 4 785
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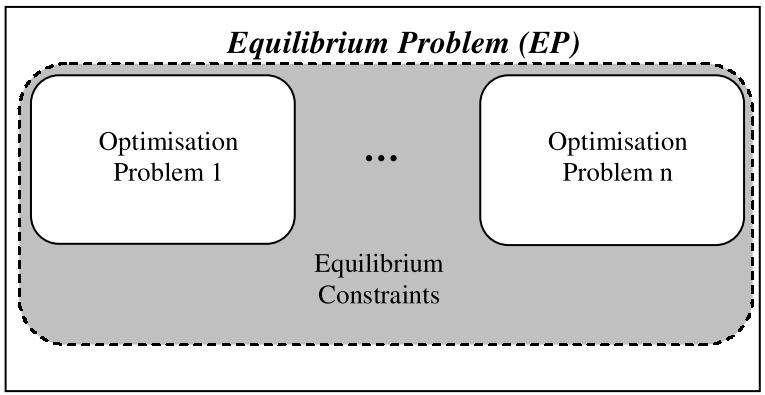
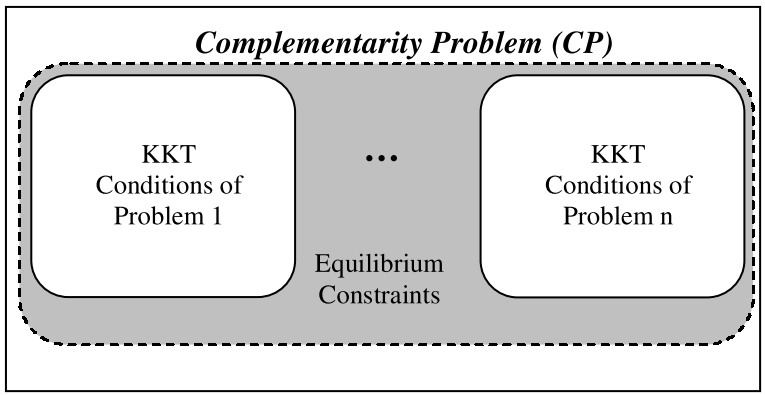
Problem Formulation

Assumptions

- **Economic and technological attributes**
  - **Linearised DC power flow**
  - **CHP marginal cost allocated to heat and power proportionally**
  - **VRE output: periodic availability and zero marginal costs**
  - **Hydropower variations: periodic maximum available capacity**
  - **Linear inverse demand:** $D^{\text{int}}_{n,t} - D^{\text{slp}}_{n,t} \sum_{i' \in \mathcal{I}} q^{\text{power}}_{n,i',t}$
- **Each firm i owns both conventional and VRE capacity plus CHP**
  - **Maximises profit by determining production and sales**
  - **Generates electricity at node n and sells it at node n'**
  - **Generation is constrained by installed capacity and FOR for CHP**
  - **Heat storage respects capacities and rates**
- **Grid owner**
  - **Controls transmission flows in order to maximise revenues from congestion charges on net imports**
  - **Must respect thermal limits of transmission lines**

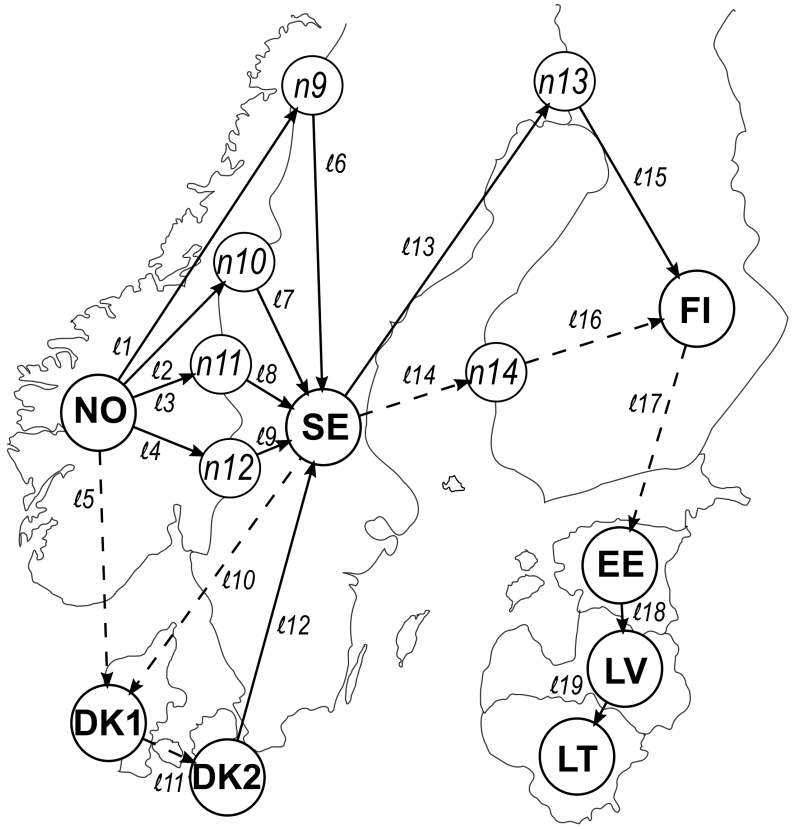
Equilibrium Model





Numerical Examples

Transmission Lines



Line	Type	Positive direction	Negative direction
$\ell 1$	AC	650	450
$\ell 2$	AC	150	250
$\ell 3$	AC	600	1000
$\ell 4$	AC	2145	2095
$\ell 5$	DC	950	1000
$\ell 6$	AC	650	450
$\ell 7$	AC	150	250
$\ell 8$	AC	600	1000
$\ell 9$	AC	2145	2095
$\ell 10$	DC	680	740
$\ell 11$	DC	590	600
$\ell 12$	AC	1700	1300
$\ell 13$	AC	1480	1120
$\ell 14$	DC	1200	1200
$\ell 15$	AC	1480	1120
$\ell 16$	DC	1200	1200
$\ell 17$	DC	860	1016
$\ell 18$	AC	750	779
$\ell 19$	AC	1234	684

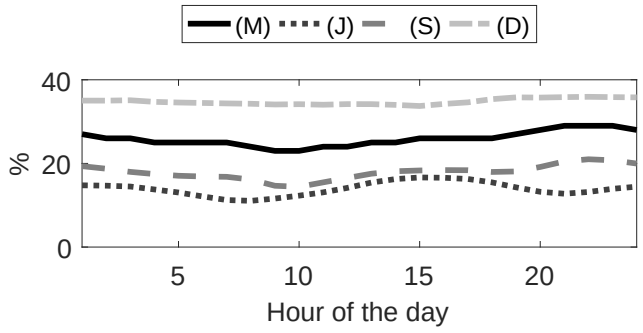
Generation Costs

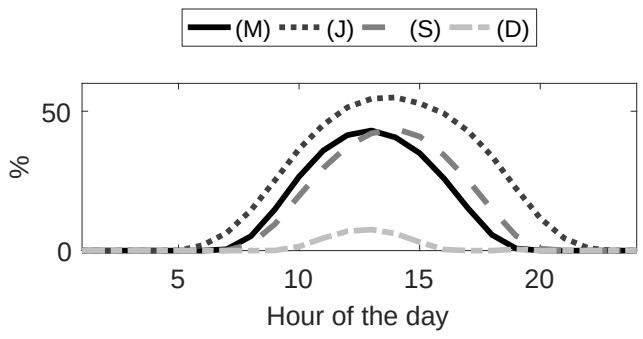
Fuel	Types	$C^{power}_{n,u,t}$	$C^{chp,p}_{y^B}$ / $C^{chp,h}_{y^B}$	$C^{chp,p}_{y^E}$ / $C^{chp,h}_{y^E}$	C^{heat}_{x}
Coal	$u1$, $y1$, $x1$	29	-	15 / 30	47
Natural gas	$u2$, $y2$, $x2$	85	45 / 50	46 / 51	53
CCGT	$u3$	47,*	-	-	-
Oil	$u4$, $y4$, $x4$	78	37 / 51	38 / 52	54
Biomass	$u5$, $y5$, $x5$	62	-	28 / 26	27
Oil shale	$u6$	33	-	-	-
Nuclear	$u7$	9	-	-	-
Hydro	$u8$	*,**	-	-	-
Waste	$y9$, $x9$	-	-	25 / 3	24
Peat	$y10$, $x10$	-	-	22 / 25	25
Bio oil	$x11$	-	-	-	51

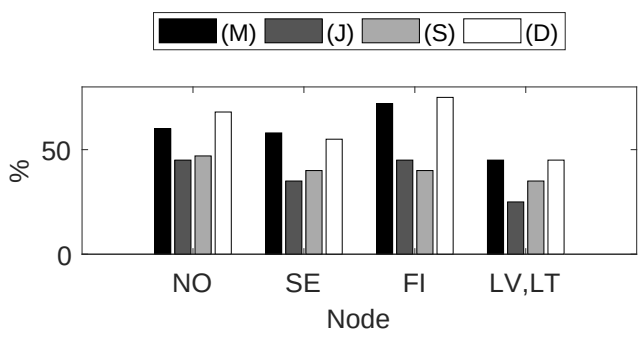
Installed Power-Only Capacity by Producer and at Node (GW)

Node	Producer	u1	u2	u3	u4	u5	u6	u7	u8	Wind	PV
SE	i1	-	-	-	-	-	-	3.2	8	0.4	-
	i2	-	0.1	0.3	0.3	0.1	-	-	2.6	0.2	-
	i3	-	-	-	-	-	-	3.7	-	-	-
	i4	-	-	-	-	-	-	2.6	-	-	-
	i5	-	-	-	-	-	-	-	2.9	-	-
	i6	0.1	0.1	0.1	1.8	0.1	-	-	3.9	4.8	0.1
FI	i5	0.6	-	-	-	-	-	1.0	0.8	-	-
	i7	-	-	-	-	-	-	1.8	-	-	-
	i8	0.5	-	-	0.2	-	-	-	0.4	-	-
	i9	-	-	-	0.1	-	-	-	-	-	-
	i10	-	-	-	-	-	-	-	1.1	-	-
	i11	-	0.1	-	1.1	-	-	-	0.8	0.6	-
DK1	i12	-	-	-	-	-	-	-	-	0.4	-
	i1	-	-	-	-	-	-	-	-	0.4	-
	i13	-	-	-	-	-	-	-	-	3.0	0.4
DK2	i12	-	-	-	0.7	-	-	-	-	0.2	-
	i13	-	-	-	-	-	-	-	-	0.8	0.2
NO	i14	-	-	0.2	-	-	-	-	9.9	0.2	-
	i15	-	-	-	-	-	-	-	2.5	-	-
	i16	-	-	-	-	-	-	-	2.0	-	-
	i17	-	-	-	-	-	-	-	1.8	-	-
	i18	-	-	-	-	-	-	-	1.7	-	-
	i19	-	-	-	-	-	-	-	1.4	-	-
	i20	-	-	-	-	-	-	-	1.2	-	-
	i21	-	-	0.2	0.1	-	-	-	9.6	0.7	-
EE*	i22	-	0.2	-	-	0.1	1.8	-	-	0.3	-
LV*	i22	-	0.5	-	-	-	-	-	1.5	0.1	-
LT*	i22	-	1.6	0.5	0.8	-	-	-	1.0	0.3	-
%	Available	90	95	80	86	95	90	90	See fig.	See fig.	See fig.

VRE and Hydropower Profiles







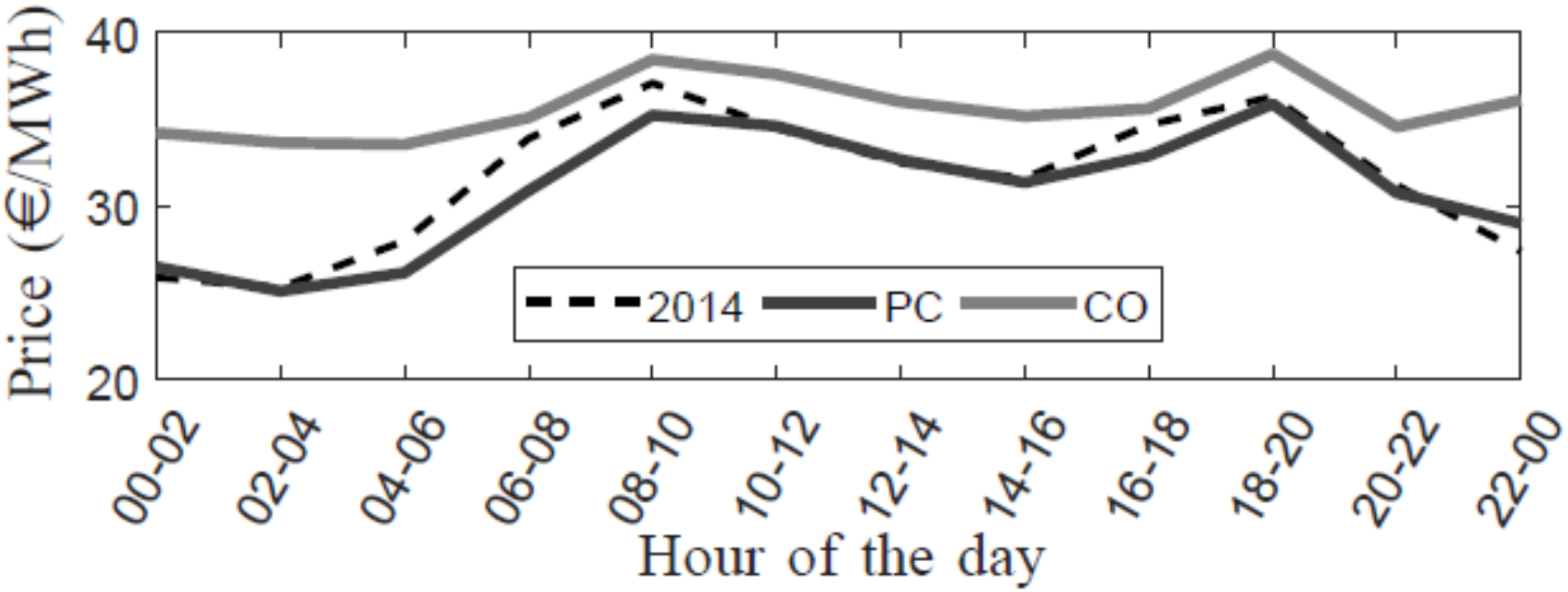
Installed CHP Power Capacity by Producer and at Node (GW)

Node	Producer	$y1^E$	$y9^E$	$y2^B$	$y2^E$	$y4^B$	$y4^E$	$y10^E$	$y5^E$
SE	*i*1	-	-	-	-	-	-	-	0.2
	*i*2	-	-	-	-	0.1	-	-	0.2
	*i*5	0.1	0.1	-	-	-	0.2	-	0.2
	*i*6	0.2	0.2	0.1	0.1	0.1	0.4	-	1.8
FI	*i*5	0.1	-	0.3	-	-	-	0.1	-
	*i*9	0.4	-	0.6	-	-	-	-	-
	*i*11	0.9	-	0.7	0.2	-	0.1	1.1	0.5
*DK*1	*i*12	1.1	-	0.1	0.4	-	-	-	0.1
	*i*1	0.7	-	-	-	-	-	-	-
	*i*13	-	0.1	0.5	0.4	-	-	-	-
*DK*2	*i*12	1.1	-	-	0.3	-	-	-	0.5
	*i*13	0.3	0.1	0.1	0.3	-	0.1	-	0.1
%	Available	90	90	95	95	95	95	95	95
	$R_{y^{B/E}}^{p\text{-}t\text{-}h}$	0.58	0.28	0.78	1.18	0.78	1.18	0.43	0.43

Installed Heat-Only Capacity by Producer and at Node (GW)

Node	Producer	x1	x9	x2	x4	x10	x5	x11
SE*	i1	-	-	-	-	-	0.5	-
	i2	-	0.3	0.1	-	-	0.2	-
	i5	0.2	0.4	-	-	-	0.2	0.1
	i6	0.6	6.4	0.6	0.6	0.8	12.2	0.6
FI	i5	0.2	-	0.6	0.4	-	-	-
	i9	0.2	-	1.0	1.2	-	-	-
	i11	-	-	2.3	5.3	0.9	1.3	-
DK1*	i13	1.7	1.7	1.5	0.1	-	2.9	-
DK2*	i13	1.1	1.1	1.0	0.1	-	1.9	-
NO*	i14	-	0.3	-	-	-	0.1	-
	i21	-	3.3	0.3	0.1	-	1.2	-
%	Available	90	90	95	95	95	95	95

Calibration for March Scenario



Test Cases and Main Results (in k€)

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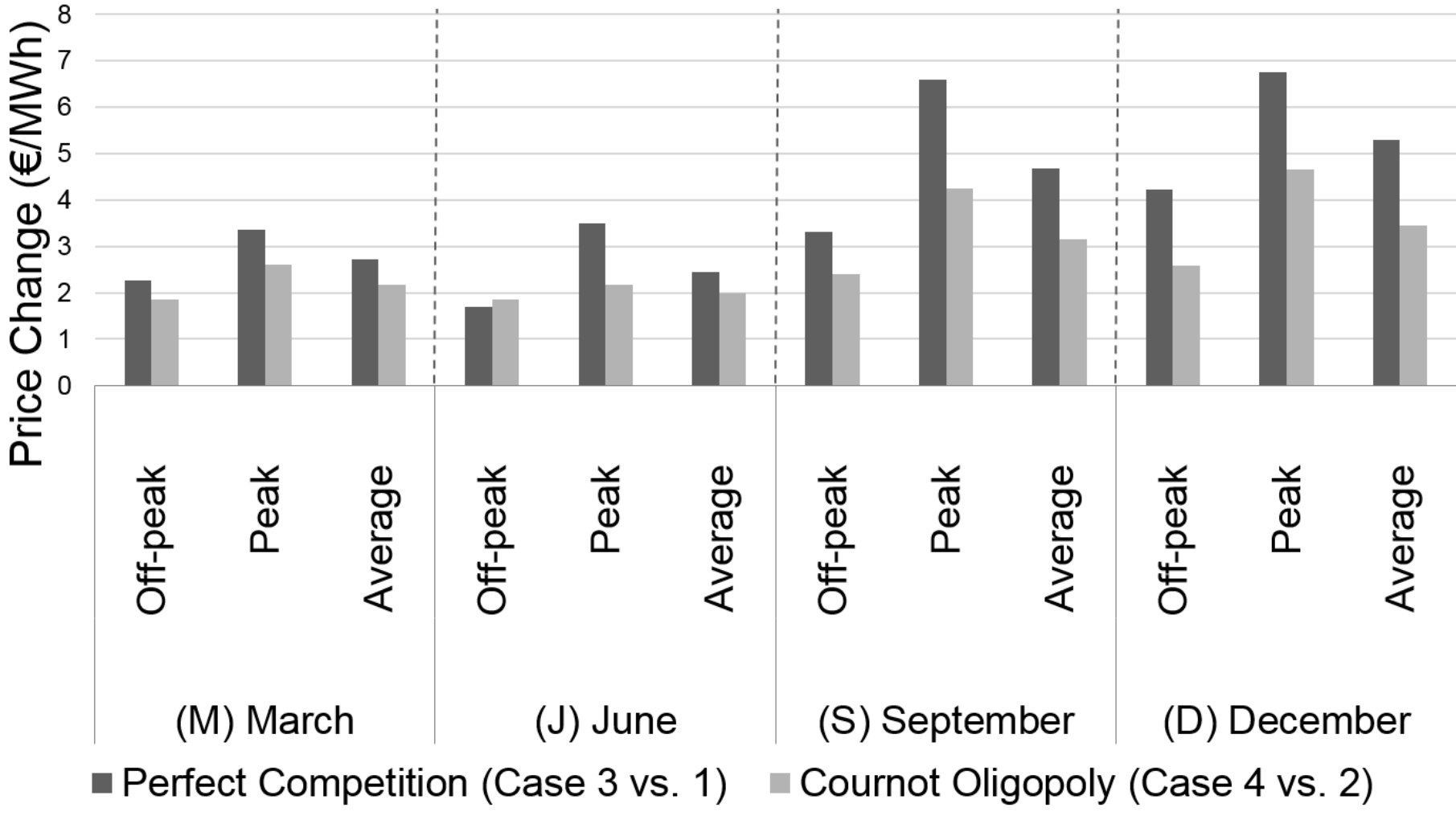
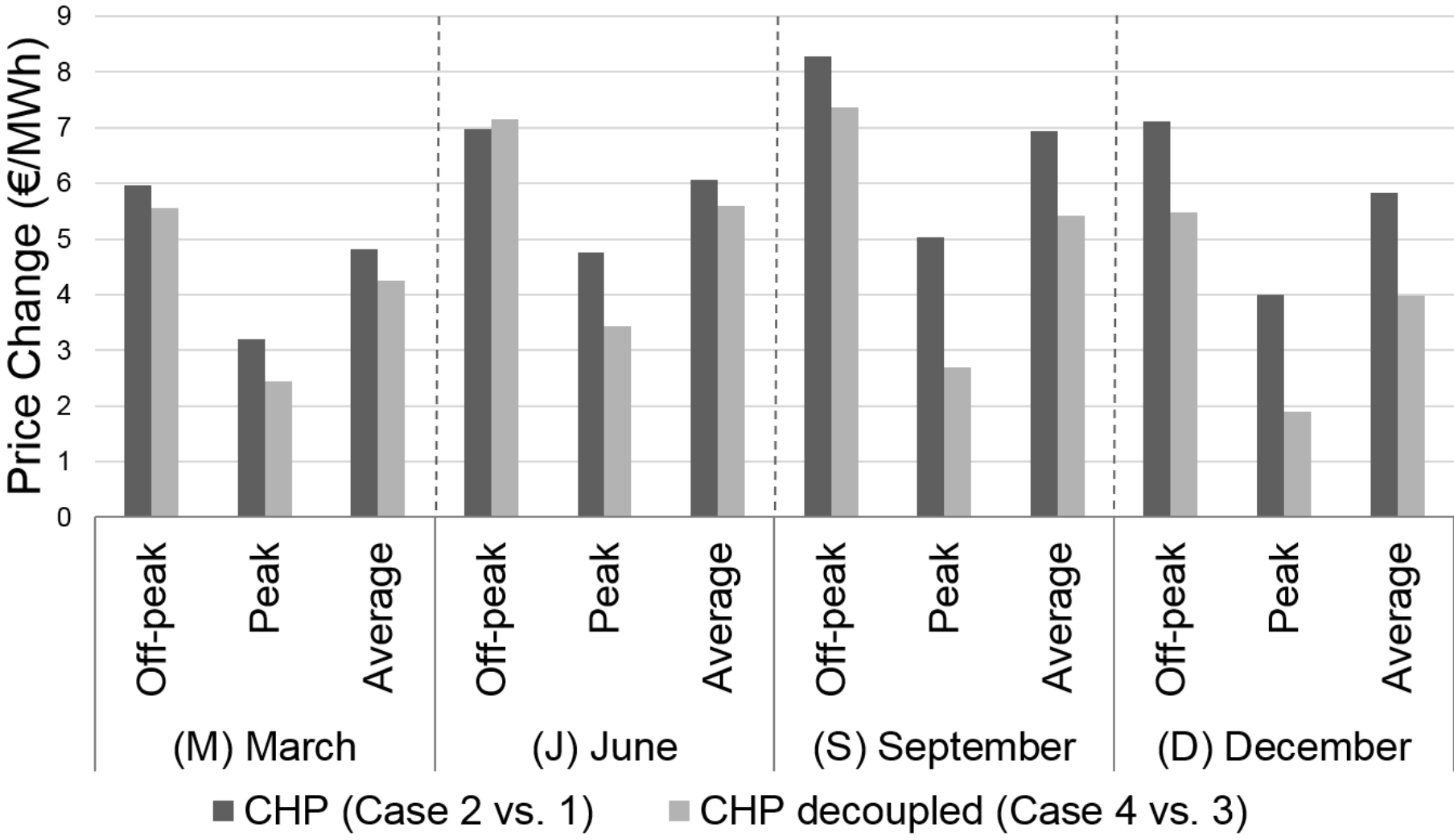
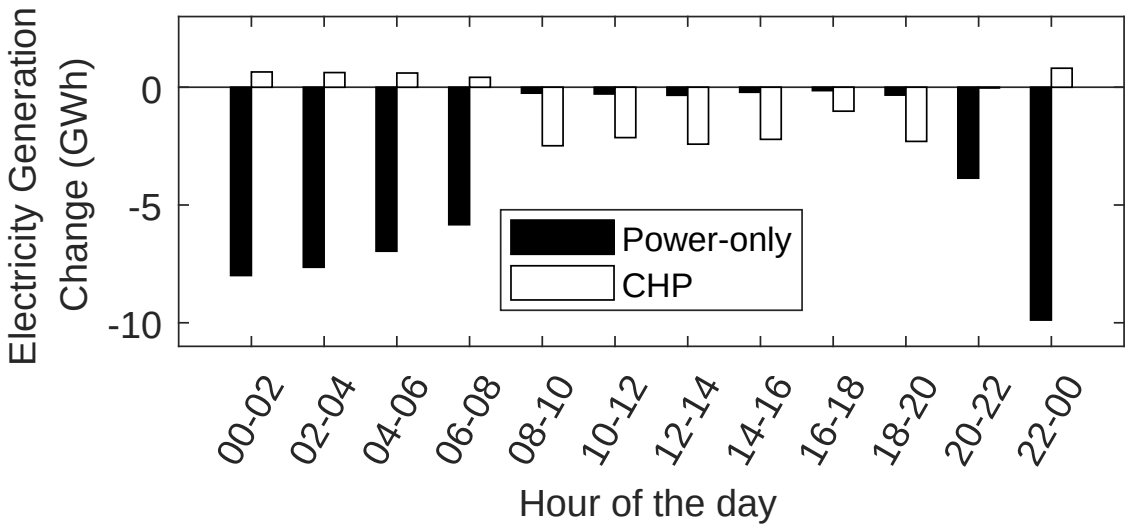
Impact of CHP Decoupling on Electricity Prices

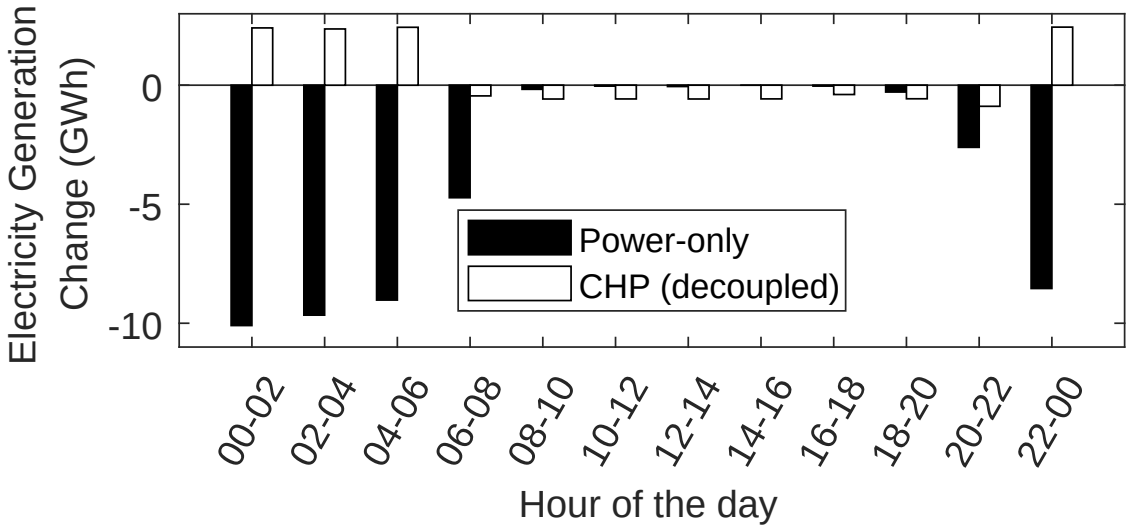


Impact of Market Power on Electricity Prices

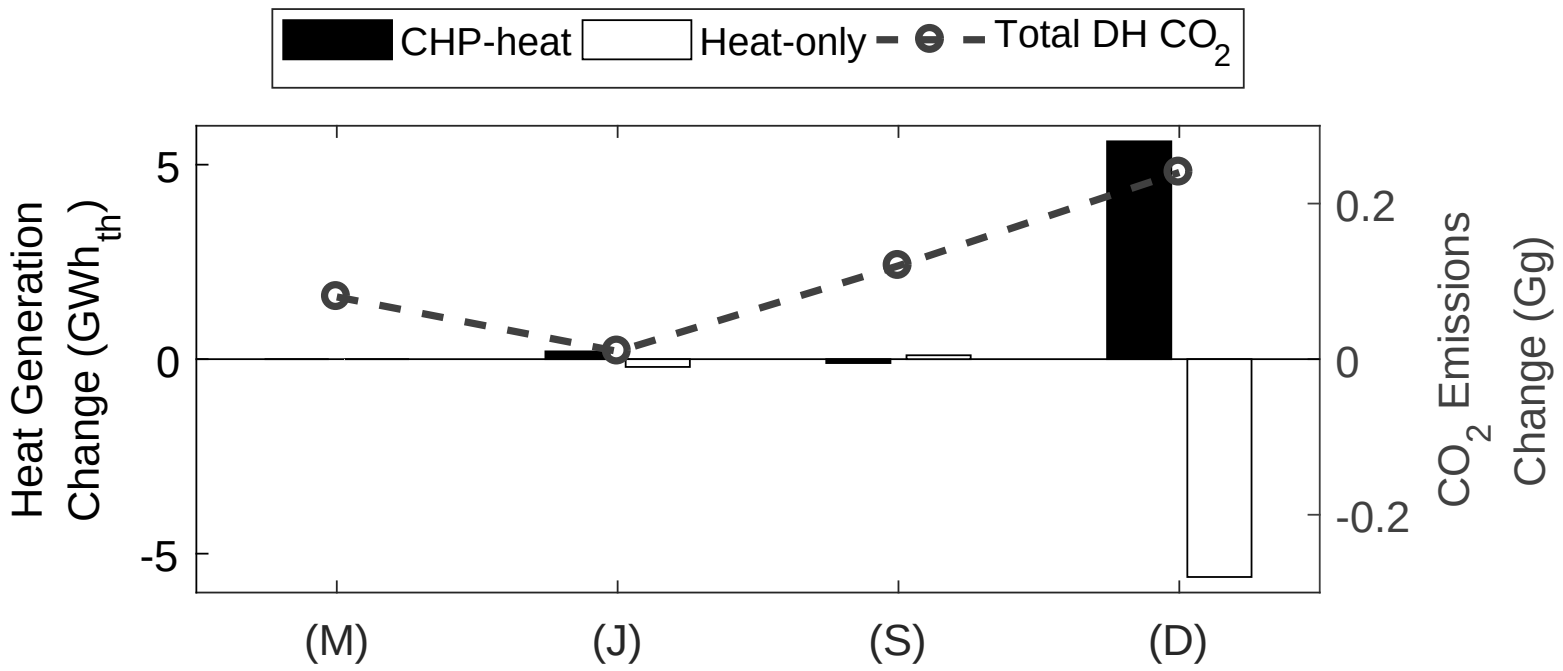


Analysis of Withholding via CHP (December Scenario)





Impact of Market Power on DH Sector and CO_2 Emissions



Conclusions

Summary

- High penetration of VRE in Nordic countries may encourage expansion of CHP capacity
- CHP can intensify producers' market power by reducing the opportunity cost of withholding output
- Market power may shift DH production to CHP from heat-only because of the additional leverage it provides during peak hours
- Even with fixed DH sales, market power may impact CO_2 emissions from the DH sector due to production mix changes
- Further policy analysis with VRE uncertainty and bi-level model for optimal CHP investment

Mathematical Appendix

Firm $i \in \mathcal{I}$

$$\min_{\Gamma} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} \Bigg[-\left(D^{\text{int}}_{n,t} - D^{\text{slp}}_{n,t} \sum_{i'\in\mathcal{I}} q^{\text{power}}_{n,i',t} - \omega_{n,t}\right) q^{\text{power}}_{n,i,t} - P^{\text{heat}}_{n} Q^{\text{heat}}_{n,i,t} + \sum_{u\in\mathcal{U}_{n,i}} \left(C^{\text{power}}_{n,u,t} - \omega_{n,t}\right) g^{\text{power}}_{n,i,u,t}$$

$$+ \sum_{x\in\mathcal{X}_{n,i}} C^{\text{heat}}_{x}\, g^{\text{heat}}_{n,i,x,t} + \sum_{y\in\mathcal{Y}_{n,i}} \left(C^{\text{chp,p}}_{y} - \omega_{n,t}\right) g^{\text{chp,p}}_{n,i,y,t} + \sum_{y\in\mathcal{Y}_{n,i}} C^{\text{chp,h}}_{y}\, g^{\text{chp,h}}_{n,i,y,t} - \omega_{n,t} \sum_{e\in\mathcal{E}} g^{e}_{n,i,t} \Bigg] \tag{1}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} q^{\text{power}}_{n,i,t} - \sum_{n\in\mathcal{N}} \sum_{u\in\mathcal{U}_{n,i}} g^{\text{power}}_{n,i,u,t} - \sum_{n\in\mathcal{N}} \sum_{y\in\mathcal{Y}_{n,i}} g^{\text{chp,p}}_{n,i,y,t} - \sum_{n\in\mathcal{N}} \sum_{e\in\mathcal{E}} g^{e}_{n,i,t} = 0 \quad \left(\delta^{\text{power}}_{i,t}\right), \ \forall t \tag{2}$$

$$Q^{\text{heat}}_{n,i,t} - \sum_{x\in\mathcal{X}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{heat}}_{n,i,x,t} - \sum_{y\in\mathcal{Y}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{chp,h}}_{n,i,y,t} + r^{\text{in}}_{n,i,t} - r^{\text{out}}_{n,i,t} = 0 \quad \left(\delta^{\text{heat}}_{n,i,t}\right), \ \forall n, t \tag{3}$$

$$g^{\text{chp,p}}_{n,i,y^{\text{B}},t} - R^{\text{p-t-h}}_{y^{\text{B}}}\, g^{\text{chp,h}}_{n,i,y^{\text{B}},t} = 0 \quad \left(\phi^{\text{B}}_{n,i,y^{\text{B}},t}\right), \ \forall n, y^{\text{B}} \in \mathcal{Y}^{\text{B}}{}_{n,i}, t \tag{4}$$

$$R^{\text{p-t-h}}_{y^{\text{E}}}\, g^{\text{chp,h}}_{n,i,y^{\text{E}},t} - g^{\text{chp,p}}_{n,i,y^{\text{E}},t} \leq 0 \quad \left(\phi^{\text{E,p-t-h}}_{n,i,y^{\text{E}},t}\right), \ \forall n, y^{\text{E}} \in \mathcal{Y}^{\text{E}}{}_{n,i}, t \tag{5}$$

$$R^{\text{f-t-h}}_{y^{\text{E}}}\, g^{\text{chp,h}}_{n,i,y^{\text{E}},t} + R^{\text{f-t-p}}_{y^{\text{E}}}\, g^{\text{chp,p}}_{n,i,y^{\text{E}},t} - \overline{F}_{y^{\text{E}}} \leq 0 \quad \left(\phi^{\text{E,ub-fuel}}_{n,i,y^{\text{E}},t}\right), \ \forall n, y^{\text{E}} \in \mathcal{Y}^{\text{E}}{}_{n,i}, t \tag{6}$$

$$R^{\text{heat-only}}_{n,i}\, Q^{\text{heat}}_{n,i,t} - \sum_{x\in\mathcal{X}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{heat}}_{n,i,x,t} \leq 0 \quad \left(\delta^{\text{heat-only}}_{n,i,t}\right), \ \forall n, t \tag{7}$$

$$g^{\text{power}}_{n,i,u,t} - T_t \overline{G}^{\text{power}}_{n,i,u} \leq 0 \quad \left(\lambda^{\text{power}}_{n,i,u,t}\right), \ \forall n, u \in \mathcal{U}_{n,i}, t \tag{8}$$

$$g^{\text{chp,h}}_{n,i,y,t} - T_t \overline{G}^{\text{chp,h}}_{n,i,y} \leq 0 \quad \left(\lambda^{\text{chp,h}}_{n,i,y,t}\right), \ \forall n, y \in \mathcal{Y}_{n,i}, t \tag{9}$$

$$g^{\text{heat}}_{n,i,x,t} - T_t \overline{G}^{\text{heat}}_{n,i,x} \leq 0 \quad \left(\lambda^{\text{heat}}_{n,i,x,t}\right), \ \forall n, x \in \mathcal{X}_{n,i}, t \tag{10}$$

$$g^{e}_{n,i,t} - T_t A^{e}_{n,t} \overline{G}^{e}_{n,i} = 0 \quad \left(\lambda^{e}_{n,i,t}\right), \ \forall n, e, t \tag{11}$$

Firm $i \in \mathcal{I}$

$$\min_{\Gamma} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} \Bigg[-\left(D^{\text{int}}_{n,t} - D^{\text{slp}}_{n,t} \sum_{i'\in\mathcal{I}} q^{\text{power}}_{n,i',t} - \omega_{n,t}\right) q^{\text{power}}_{n,i,t} - P^{\text{heat}}_{n} Q^{\text{heat}}_{n,i,t} + \sum_{u\in\mathcal{U}_{n,i}} \left(C^{\text{power}}_{n,u,t} - \omega_{n,t}\right) g^{\text{power}}_{n,i,u,t}$$

$$+ \sum_{x\in\mathcal{X}_{n,i}} C^{\text{heat}}_{x}\, g^{\text{heat}}_{n,i,x,t} + \sum_{y\in\mathcal{Y}_{n,i}} \left(C^{\text{chp,p}}_{y} - \omega_{n,t}\right) g^{\text{chp,p}}_{n,i,y,t} + \sum_{y\in\mathcal{Y}_{n,i}} C^{\text{chp,h}}_{y}\, g^{\text{chp,h}}_{n,i,y,t} - \omega_{n,t} \sum_{e\in\mathcal{E}} g^{e}_{n,i,t} \Bigg] \tag{1}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} q^{\text{power}}_{n,i,t} - \sum_{n\in\mathcal{N}} \sum_{u\in\mathcal{U}_{n,i}} g^{\text{power}}_{n,i,u,t} - \sum_{n\in\mathcal{N}} \sum_{y\in\mathcal{Y}_{n,i}} g^{\text{chp,p}}_{n,i,y,t} - \sum_{n\in\mathcal{N}} \sum_{e\in\mathcal{E}} g^{e}_{n,i,t} = 0 \quad \left(\delta^{\text{power}}_{i,t}\right), \ \forall t \tag{2}$$

$$Q^{\text{heat}}_{n,i,t} - \sum_{x\in\mathcal{X}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{heat}}_{n,i,x,t} - \sum_{y\in\mathcal{Y}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{chp,h}}_{n,i,y,t} + r^{\text{in}}_{n,i,t} - r^{\text{out}}_{n,i,t} = 0 \quad \left(\delta^{\text{heat}}_{n,i,t}\right), \ \forall n, t \tag{3}$$

$$g^{\text{chp,p}}_{n,i,y^{\text{B}},t} - R^{\text{p-t-h}}_{y^{\text{B}}}\, g^{\text{chp,h}}_{n,i,y^{\text{B}},t} = 0 \quad \left(\phi^{\text{B}}_{n,i,y^{\text{B}},t}\right), \ \forall n, y^{\text{B}} \in \mathcal{Y}^{\text{B}}{}_{n,i}, t \tag{4}$$

$$R^{\text{p-t-h}}_{y^{\text{E}}}\, g^{\text{chp,h}}_{n,i,y^{\text{E}},t} - g^{\text{chp,p}}_{n,i,y^{\text{E}},t} \leq 0 \quad \left(\phi^{\text{E,p-t-h}}_{n,i,y^{\text{E}},t}\right), \ \forall n, y^{\text{E}} \in \mathcal{Y}^{\text{E}}{}_{n,i}, t \tag{5}$$

$$R^{\text{f-t-h}}_{y^{\text{E}}}\, g^{\text{chp,h}}_{n,i,y^{\text{E}},t} + R^{\text{f-t-p}}_{y^{\text{E}}}\, g^{\text{chp,p}}_{n,i,y^{\text{E}},t} - \overline{F}_{y^{\text{E}}} \leq 0 \quad \left(\phi^{\text{E,ub-fuel}}_{n,i,y^{\text{E}},t}\right), \ \forall n, y^{\text{E}} \in \mathcal{Y}^{\text{E}}{}_{n,i}, t \tag{6}$$

$$R^{\text{heat-only}}_{n,i}\, Q^{\text{heat}}_{n,i,t} - \sum_{x\in\mathcal{X}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{heat}}_{n,i,x,t} \leq 0 \quad \left(\delta^{\text{heat-only}}_{n,i,t}\right), \ \forall n, t \tag{7}$$

$$g^{\text{power}}_{n,i,u,t} - T_t \overline{G}^{\text{power}}_{n,i,u} \leq 0 \quad \left(\lambda^{\text{power}}_{n,i,u,t}\right), \ \forall n, u \in \mathcal{U}_{n,i}, t \tag{8}$$

$$g^{\text{chp,h}}_{n,i,y,t} - T_t \overline{G}^{\text{chp,h}}_{n,i,y} \leq 0 \quad \left(\lambda^{\text{chp,h}}_{n,i,y,t}\right), \ \forall n, y \in \mathcal{Y}_{n,i}, t \tag{9}$$

$$g^{\text{heat}}_{n,i,x,t} - T_t \overline{G}^{\text{heat}}_{n,i,x} \leq 0 \quad \left(\lambda^{\text{heat}}_{n,i,x,t}\right), \ \forall n, x \in \mathcal{X}_{n,i}, t \tag{10}$$

$$g^{e}_{n,i,t} - T_t A^{e}_{n,t} \overline{G}^{e}_{n,i} = 0 \quad \left(\lambda^{e}_{n,i,t}\right), \ \forall n, e, t \tag{11}$$

Firm $i \in \mathcal{I}$

$$\min_{\Gamma} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} \Bigg[-\left(D^{\text{int}}_{n,t} - D^{\text{slp}}_{n,t} \sum_{i'\in\mathcal{I}} q^{\text{power}}_{n,i',t} - \omega_{n,t}\right) q^{\text{power}}_{n,i,t} - P^{\text{heat}}_{n} Q^{\text{heat}}_{n,i,t} + \sum_{u\in\mathcal{U}_{n,i}} \left(C^{\text{power}}_{n,u,t} - \omega_{n,t}\right) g^{\text{power}}_{n,i,u,t}$$

$$+ \sum_{x\in\mathcal{X}_{n,i}} C^{\text{heat}}_{x}\, g^{\text{heat}}_{n,i,x,t} + \sum_{y\in\mathcal{Y}_{n,i}} \left(C^{\text{chp,p}}_{y} - \omega_{n,t}\right) g^{\text{chp,p}}_{n,i,y,t} + \sum_{y\in\mathcal{Y}_{n,i}} C^{\text{chp,h}}_{y}\, g^{\text{chp,h}}_{n,i,y,t} - \omega_{n,t} \sum_{e\in\mathcal{E}} g^{e}_{n,i,t} \Bigg] \tag{1}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} q^{\text{power}}_{n,i,t} - \sum_{n\in\mathcal{N}} \sum_{u\in\mathcal{U}_{n,i}} g^{\text{power}}_{n,i,u,t} - \sum_{n\in\mathcal{N}} \sum_{y\in\mathcal{Y}_{n,i}} g^{\text{chp,p}}_{n,i,y,t} - \sum_{n\in\mathcal{N}} \sum_{e\in\mathcal{E}} g^{e}_{n,i,t} = 0 \quad \left(\delta^{\text{power}}_{i,t}\right), \ \forall t \tag{2}$$

$$Q^{\text{heat}}_{n,i,t} - \sum_{x\in\mathcal{X}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{heat}}_{n,i,x,t} - \sum_{y\in\mathcal{Y}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{chp,h}}_{n,i,y,t} + r^{\text{in}}_{n,i,t} - r^{\text{out}}_{n,i,t} = 0 \quad \left(\delta^{\text{heat}}_{n,i,t}\right), \ \forall n, t \tag{3}$$

$$g^{\text{chp,p}}_{n,i,y^{\text{B}},t} - R^{\text{p-t-h}}_{y^{\text{B}}}\, g^{\text{chp,h}}_{n,i,y^{\text{B}},t} = 0 \quad \left(\phi^{\text{B}}_{n,i,y^{\text{B}},t}\right), \ \forall n, y^{\text{B}} \in \mathcal{Y}^{\text{B}}{}_{n,i}, t \tag{4}$$

$$R^{\text{p-t-h}}_{y^{\text{E}}}\, g^{\text{chp,h}}_{n,i,y^{\text{E}},t} - g^{\text{chp,p}}_{n,i,y^{\text{E}},t} \leq 0 \quad \left(\phi^{\text{E,p-t-h}}_{n,i,y^{\text{E}},t}\right), \ \forall n, y^{\text{E}} \in \mathcal{Y}^{\text{E}}{}_{n,i}, t \tag{5}$$

$$R^{\text{f-t-h}}_{y^{\text{E}}}\, g^{\text{chp,h}}_{n,i,y^{\text{E}},t} + R^{\text{f-t-p}}_{y^{\text{E}}}\, g^{\text{chp,p}}_{n,i,y^{\text{E}},t} - \overline{F}_{y^{\text{E}}} \leq 0 \quad \left(\phi^{\text{E,ub-fuel}}_{n,i,y^{\text{E}},t}\right), \ \forall n, y^{\text{E}} \in \mathcal{Y}^{\text{E}}{}_{n,i}, t \tag{6}$$

$$R^{\text{heat-only}}_{n,i}\, Q^{\text{heat}}_{n,i,t} - \sum_{x\in\mathcal{X}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{heat}}_{n,i,x,t} \leq 0 \quad \left(\delta^{\text{heat-only}}_{n,i,t}\right), \ \forall n, t \tag{7}$$

$$g^{\text{power}}_{n,i,u,t} - T_t \overline{G}^{\text{power}}_{n,i,u} \leq 0 \quad \left(\lambda^{\text{power}}_{n,i,u,t}\right), \ \forall n, u \in \mathcal{U}_{n,i}, t \tag{8}$$

$$g^{\text{chp,h}}_{n,i,y,t} - T_t \overline{G}^{\text{chp,h}}_{n,i,y} \leq 0 \quad \left(\lambda^{\text{chp,h}}_{n,i,y,t}\right), \ \forall n, y \in \mathcal{Y}_{n,i}, t \tag{9}$$

$$g^{\text{heat}}_{n,i,x,t} - T_t \overline{G}^{\text{heat}}_{n,i,x} \leq 0 \quad \left(\lambda^{\text{heat}}_{n,i,x,t}\right), \ \forall n, x \in \mathcal{X}_{n,i}, t \tag{10}$$

$$g^{e}_{n,i,t} - T_t A^{e}_{n,t} \overline{G}^{e}_{n,i} = 0 \quad \left(\lambda^{e}_{n,i,t}\right), \ \forall n, e, t \tag{11}$$

Firm $i \in \mathcal{I}$

$$\min_{\Gamma} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} \Bigg[-\left(D^{\text{int}}_{n,t} - D^{\text{slp}}_{n,t} \sum_{i'\in\mathcal{I}} q^{\text{power}}_{n,i',t} - \omega_{n,t}\right) q^{\text{power}}_{n,i,t} - P^{\text{heat}}_{n} Q^{\text{heat}}_{n,i,t} + \sum_{u\in\mathcal{U}_{n,i}} \left(C^{\text{power}}_{n,u,t} - \omega_{n,t}\right) g^{\text{power}}_{n,i,u,t}$$

$$+ \sum_{x\in\mathcal{X}_{n,i}} C^{\text{heat}}_{x}\, g^{\text{heat}}_{n,i,x,t} + \sum_{y\in\mathcal{Y}_{n,i}} \left(C^{\text{chp,p}}_{y} - \omega_{n,t}\right) g^{\text{chp,p}}_{n,i,y,t} + \sum_{y\in\mathcal{Y}_{n,i}} C^{\text{chp,h}}_{y}\, g^{\text{chp,h}}_{n,i,y,t} - \omega_{n,t} \sum_{e\in\mathcal{E}} g^{e}_{n,i,t} \Bigg] \tag{1}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} q^{\text{power}}_{n,i,t} - \sum_{n\in\mathcal{N}} \sum_{u\in\mathcal{U}_{n,i}} g^{\text{power}}_{n,i,u,t} - \sum_{n\in\mathcal{N}} \sum_{y\in\mathcal{Y}_{n,i}} g^{\text{chp,p}}_{n,i,y,t} - \sum_{n\in\mathcal{N}} \sum_{e\in\mathcal{E}} g^{e}_{n,i,t} = 0 \quad \left(\delta^{\text{power}}_{i,t}\right), \ \forall t \tag{2}$$

$$Q^{\text{heat}}_{n,i,t} - \sum_{x\in\mathcal{X}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{heat}}_{n,i,x,t} - \sum_{y\in\mathcal{Y}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{chp,h}}_{n,i,y,t} + r^{\text{in}}_{n,i,t} - r^{\text{out}}_{n,i,t} = 0 \quad \left(\delta^{\text{heat}}_{n,i,t}\right), \ \forall n, t \tag{3}$$

$$g^{\text{chp,p}}_{n,i,y^{\text{B}},t} - R^{\text{p-t-h}}_{y^{\text{B}}}\, g^{\text{chp,h}}_{n,i,y^{\text{B}},t} = 0 \quad \left(\phi^{\text{B}}_{n,i,y^{\text{B}},t}\right), \ \forall n, y^{\text{B}} \in \mathcal{Y}^{\text{B}}{}_{n,i}, t \tag{4}$$

$$R^{\text{p-t-h}}_{y^{\text{E}}}\, g^{\text{chp,h}}_{n,i,y^{\text{E}},t} - g^{\text{chp,p}}_{n,i,y^{\text{E}},t} \leq 0 \quad \left(\phi^{\text{E,p-t-h}}_{n,i,y^{\text{E}},t}\right), \ \forall n, y^{\text{E}} \in \mathcal{Y}^{\text{E}}{}_{n,i}, t \tag{5}$$

$$R^{\text{f-t-h}}_{y^{\text{E}}}\, g^{\text{chp,h}}_{n,i,y^{\text{E}},t} + R^{\text{f-t-p}}_{y^{\text{E}}}\, g^{\text{chp,p}}_{n,i,y^{\text{E}},t} - \overline{F}_{y^{\text{E}}} \leq 0 \quad \left(\phi^{\text{E,ub-fuel}}_{n,i,y^{\text{E}},t}\right), \ \forall n, y^{\text{E}} \in \mathcal{Y}^{\text{E}}{}_{n,i}, t \tag{6}$$

$$R^{\text{heat-only}}_{n,i}\, Q^{\text{heat}}_{n,i,t} - \sum_{x\in\mathcal{X}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{heat}}_{n,i,x,t} \leq 0 \quad \left(\delta^{\text{heat-only}}_{n,i,t}\right), \ \forall n, t \tag{7}$$

$$g^{\text{power}}_{n,i,u,t} - T_t \overline{G}^{\text{power}}_{n,i,u} \leq 0 \quad \left(\lambda^{\text{power}}_{n,i,u,t}\right), \ \forall n, u \in \mathcal{U}_{n,i}, t \tag{8}$$

$$g^{\text{chp,h}}_{n,i,y,t} - T_t \overline{G}^{\text{chp,h}}_{n,i,y} \leq 0 \quad \left(\lambda^{\text{chp,h}}_{n,i,y,t}\right), \ \forall n, y \in \mathcal{Y}_{n,i}, t \tag{9}$$

$$g^{\text{heat}}_{n,i,x,t} - T_t \overline{G}^{\text{heat}}_{n,i,x} \leq 0 \quad \left(\lambda^{\text{heat}}_{n,i,x,t}\right), \ \forall n, x \in \mathcal{X}_{n,i}, t \tag{10}$$

$$g^{e}_{n,i,t} - T_t A^{e}_{n,t} \overline{G}^{e}_{n,i} = 0 \quad \left(\lambda^{e}_{n,i,t}\right), \ \forall n, e, t \tag{11}$$

Firm $i \in \mathcal{I}$

$$\min_{\Gamma} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} \Bigg[-\left(D^{\text{int}}_{n,t} - D^{\text{slp}}_{n,t} \sum_{i'\in\mathcal{I}} q^{\text{power}}_{n,i',t} - \omega_{n,t}\right) q^{\text{power}}_{n,i,t} - P^{\text{heat}}_{n} Q^{\text{heat}}_{n,i,t} + \sum_{u\in\mathcal{U}_{n,i}} \left(C^{\text{power}}_{n,u,t} - \omega_{n,t}\right) g^{\text{power}}_{n,i,u,t}$$

$$+ \sum_{x\in\mathcal{X}_{n,i}} C^{\text{heat}}_{x}\, g^{\text{heat}}_{n,i,x,t} + \sum_{y\in\mathcal{Y}_{n,i}} \left(C^{\text{chp,p}}_{y} - \omega_{n,t}\right) g^{\text{chp,p}}_{n,i,y,t} + \sum_{y\in\mathcal{Y}_{n,i}} C^{\text{chp,h}}_{y}\, g^{\text{chp,h}}_{n,i,y,t} - \omega_{n,t} \sum_{e\in\mathcal{E}} g^{e}_{n,i,t} \Bigg] \tag{1}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} q^{\text{power}}_{n,i,t} - \sum_{n\in\mathcal{N}} \sum_{u\in\mathcal{U}_{n,i}} g^{\text{power}}_{n,i,u,t} - \sum_{n\in\mathcal{N}} \sum_{y\in\mathcal{Y}_{n,i}} g^{\text{chp,p}}_{n,i,y,t} - \sum_{n\in\mathcal{N}} \sum_{e\in\mathcal{E}} g^{e}_{n,i,t} = 0 \quad \left(\delta^{\text{power}}_{i,t}\right), \ \forall t \tag{2}$$

$$Q^{\text{heat}}_{n,i,t} - \sum_{x\in\mathcal{X}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{heat}}_{n,i,x,t} - \sum_{y\in\mathcal{Y}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{chp,h}}_{n,i,y,t} + r^{\text{in}}_{n,i,t} - r^{\text{out}}_{n,i,t} = 0 \quad \left(\delta^{\text{heat}}_{n,i,t}\right), \ \forall n, t \tag{3}$$

$$g^{\text{chp,p}}_{n,i,y^{\text{B}},t} - R^{\text{p-t-h}}_{y^{\text{B}}}\, g^{\text{chp,h}}_{n,i,y^{\text{B}},t} = 0 \quad \left(\phi^{\text{B}}_{n,i,y^{\text{B}},t}\right), \ \forall n, y^{\text{B}} \in \mathcal{Y}^{\text{B}}{}_{n,i}, t \tag{4}$$

$$R^{\text{p-t-h}}_{y^{\text{E}}}\, g^{\text{chp,h}}_{n,i,y^{\text{E}},t} - g^{\text{chp,p}}_{n,i,y^{\text{E}},t} \leq 0 \quad \left(\phi^{\text{E,p-t-h}}_{n,i,y^{\text{E}},t}\right), \ \forall n, y^{\text{E}} \in \mathcal{Y}^{\text{E}}{}_{n,i}, t \tag{5}$$

$$R^{\text{f-t-h}}_{y^{\text{E}}}\, g^{\text{chp,h}}_{n,i,y^{\text{E}},t} + R^{\text{f-t-p}}_{y^{\text{E}}}\, g^{\text{chp,p}}_{n,i,y^{\text{E}},t} - \overline{F}_{y^{\text{E}}} \leq 0 \quad \left(\phi^{\text{E,ub-fuel}}_{n,i,y^{\text{E}},t}\right), \ \forall n, y^{\text{E}} \in \mathcal{Y}^{\text{E}}{}_{n,i}, t \tag{6}$$

$$R^{\text{heat-only}}_{n,i}\, Q^{\text{heat}}_{n,i,t} - \sum_{x\in\mathcal{X}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{heat}}_{n,i,x,t} \leq 0 \quad \left(\delta^{\text{heat-only}}_{n,i,t}\right), \ \forall n, t \tag{7}$$

$$g^{\text{power}}_{n,i,u,t} - T_t \overline{G}^{\text{power}}_{n,i,u} \leq 0 \quad \left(\lambda^{\text{power}}_{n,i,u,t}\right), \ \forall n, u \in \mathcal{U}_{n,i}, t \tag{8}$$

$$g^{\text{chp,h}}_{n,i,y,t} - T_t \overline{G}^{\text{chp,h}}_{n,i,y} \leq 0 \quad \left(\lambda^{\text{chp,h}}_{n,i,y,t}\right), \ \forall n, y \in \mathcal{Y}_{n,i}, t \tag{9}$$

$$g^{\text{heat}}_{n,i,x,t} - T_t \overline{G}^{\text{heat}}_{n,i,x} \leq 0 \quad \left(\lambda^{\text{heat}}_{n,i,x,t}\right), \ \forall n, x \in \mathcal{X}_{n,i}, t \tag{10}$$

$$g^{e}_{n,i,t} - T_t A^{e}_{n,t} \overline{G}^{e}_{n,i} = 0 \quad \left(\lambda^{e}_{n,i,t}\right), \ \forall n, e, t \tag{11}$$

Firm $i \in \mathcal{I}$

$$\min_{\Gamma} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} \Bigg[-\left(D^{\text{int}}_{n,t} - D^{\text{slp}}_{n,t} \sum_{i'\in\mathcal{I}} q^{\text{power}}_{n,i',t} - \omega_{n,t}\right) q^{\text{power}}_{n,i,t} - P^{\text{heat}}_{n} Q^{\text{heat}}_{n,i,t} + \sum_{u\in\mathcal{U}_{n,i}} \left(C^{\text{power}}_{n,u,t} - \omega_{n,t}\right) g^{\text{power}}_{n,i,u,t}$$

$$+ \sum_{x\in\mathcal{X}_{n,i}} C^{\text{heat}}_{x}\, g^{\text{heat}}_{n,i,x,t} + \sum_{y\in\mathcal{Y}_{n,i}} \left(C^{\text{chp,p}}_{y} - \omega_{n,t}\right) g^{\text{chp,p}}_{n,i,y,t} + \sum_{y\in\mathcal{Y}_{n,i}} C^{\text{chp,h}}_{y}\, g^{\text{chp,h}}_{n,i,y,t} - \omega_{n,t} \sum_{e\in\mathcal{E}} g^{e}_{n,i,t} \Bigg] \tag{1}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} q^{\text{power}}_{n,i,t} - \sum_{n\in\mathcal{N}} \sum_{u\in\mathcal{U}_{n,i}} g^{\text{power}}_{n,i,u,t} - \sum_{n\in\mathcal{N}} \sum_{y\in\mathcal{Y}_{n,i}} g^{\text{chp,p}}_{n,i,y,t} - \sum_{n\in\mathcal{N}} \sum_{e\in\mathcal{E}} g^{e}_{n,i,t} = 0 \quad \left(\delta^{\text{power}}_{i,t}\right), \ \forall t \tag{2}$$

$$Q^{\text{heat}}_{n,i,t} - \sum_{x\in\mathcal{X}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{heat}}_{n,i,x,t} - \sum_{y\in\mathcal{Y}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{chp,h}}_{n,i,y,t} + r^{\text{in}}_{n,i,t} - r^{\text{out}}_{n,i,t} = 0 \quad \left(\delta^{\text{heat}}_{n,i,t}\right), \ \forall n, t \tag{3}$$

$$g^{\text{chp,p}}_{n,i,y^{\text{B}},t} - R^{\text{p-t-h}}_{y^{\text{B}}}\, g^{\text{chp,h}}_{n,i,y^{\text{B}},t} = 0 \quad \left(\phi^{\text{B}}_{n,i,y^{\text{B}},t}\right), \ \forall n, y^{\text{B}} \in \mathcal{Y}^{\text{B}}{}_{n,i}, t \tag{4}$$

$$R^{\text{p-t-h}}_{y^{\text{E}}}\, g^{\text{chp,h}}_{n,i,y^{\text{E}},t} - g^{\text{chp,p}}_{n,i,y^{\text{E}},t} \leq 0 \quad \left(\phi^{\text{E,p-t-h}}_{n,i,y^{\text{E}},t}\right), \ \forall n, y^{\text{E}} \in \mathcal{Y}^{\text{E}}{}_{n,i}, t \tag{5}$$

$$R^{\text{f-t-h}}_{y^{\text{E}}}\, g^{\text{chp,h}}_{n,i,y^{\text{E}},t} + R^{\text{f-t-p}}_{y^{\text{E}}}\, g^{\text{chp,p}}_{n,i,y^{\text{E}},t} - \overline{F}_{y^{\text{E}}} \leq 0 \quad \left(\phi^{\text{E,ub-fuel}}_{n,i,y^{\text{E}},t}\right), \ \forall n, y^{\text{E}} \in \mathcal{Y}^{\text{E}}{}_{n,i}, t \tag{6}$$

$$R^{\text{heat-only}}_{n,i}\, Q^{\text{heat}}_{n,i,t} - \sum_{x\in\mathcal{X}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{heat}}_{n,i,x,t} \leq 0 \quad \left(\delta^{\text{heat-only}}_{n,i,t}\right), \ \forall n, t \tag{7}$$

$$g^{\text{power}}_{n,i,u,t} - T_t \overline{G}^{\text{power}}_{n,i,u} \leq 0 \quad \left(\lambda^{\text{power}}_{n,i,u,t}\right), \ \forall n, u \in \mathcal{U}_{n,i}, t \tag{8}$$

$$g^{\text{chp,h}}_{n,i,y,t} - T_t \overline{G}^{\text{chp,h}}_{n,i,y} \leq 0 \quad \left(\lambda^{\text{chp,h}}_{n,i,y,t}\right), \ \forall n, y \in \mathcal{Y}_{n,i}, t \tag{9}$$

$$g^{\text{heat}}_{n,i,x,t} - T_t \overline{G}^{\text{heat}}_{n,i,x} \leq 0 \quad \left(\lambda^{\text{heat}}_{n,i,x,t}\right), \ \forall n, x \in \mathcal{X}_{n,i}, t \tag{10}$$

$$g^{e}_{n,i,t} - T_t A^{e}_{n,t} \overline{G}^{e}_{n,i} = 0 \quad \left(\lambda^{e}_{n,i,t}\right), \ \forall n, e, t \tag{11}$$

Firm $i \in \mathcal{I}$

$$\min_{\Gamma} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} \Bigg[-\left(D^{\text{int}}_{n,t} - D^{\text{slp}}_{n,t} \sum_{i'\in\mathcal{I}} q^{\text{power}}_{n,i',t} - \omega_{n,t}\right) q^{\text{power}}_{n,i,t} - P^{\text{heat}}_{n} Q^{\text{heat}}_{n,i,t} + \sum_{u\in\mathcal{U}_{n,i}} \left(C^{\text{power}}_{n,u,t} - \omega_{n,t}\right) g^{\text{power}}_{n,i,u,t}$$

$$+ \sum_{x\in\mathcal{X}_{n,i}} C^{\text{heat}}_{x}\, g^{\text{heat}}_{n,i,x,t} + \sum_{y\in\mathcal{Y}_{n,i}} \left(C^{\text{chp,p}}_{y} - \omega_{n,t}\right) g^{\text{chp,p}}_{n,i,y,t} + \sum_{y\in\mathcal{Y}_{n,i}} C^{\text{chp,h}}_{y}\, g^{\text{chp,h}}_{n,i,y,t} - \omega_{n,t} \sum_{e\in\mathcal{E}} g^{e}_{n,i,t} \Bigg] \tag{1}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} q^{\text{power}}_{n,i,t} - \sum_{n\in\mathcal{N}} \sum_{u\in\mathcal{U}_{n,i}} g^{\text{power}}_{n,i,u,t} - \sum_{n\in\mathcal{N}} \sum_{y\in\mathcal{Y}_{n,i}} g^{\text{chp,p}}_{n,i,y,t} - \sum_{n\in\mathcal{N}} \sum_{e\in\mathcal{E}} g^{e}_{n,i,t} = 0 \quad \left(\delta^{\text{power}}_{i,t}\right), \ \forall t \tag{2}$$

$$Q^{\text{heat}}_{n,i,t} - \sum_{x\in\mathcal{X}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{heat}}_{n,i,x,t} - \sum_{y\in\mathcal{Y}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{chp,h}}_{n,i,y,t} + r^{\text{in}}_{n,i,t} - r^{\text{out}}_{n,i,t} = 0 \quad \left(\delta^{\text{heat}}_{n,i,t}\right), \ \forall n, t \tag{3}$$

$$g^{\text{chp,p}}_{n,i,y^{\text{B}},t} - R^{\text{p-t-h}}_{y^{\text{B}}}\, g^{\text{chp,h}}_{n,i,y^{\text{B}},t} = 0 \quad \left(\phi^{\text{B}}_{n,i,y^{\text{B}},t}\right), \ \forall n, y^{\text{B}} \in \mathcal{Y}^{\text{B}}{}_{n,i}, t \tag{4}$$

$$R^{\text{p-t-h}}_{y^{\text{E}}}\, g^{\text{chp,h}}_{n,i,y^{\text{E}},t} - g^{\text{chp,p}}_{n,i,y^{\text{E}},t} \leq 0 \quad \left(\phi^{\text{E,p-t-h}}_{n,i,y^{\text{E}},t}\right), \ \forall n, y^{\text{E}} \in \mathcal{Y}^{\text{E}}{}_{n,i}, t \tag{5}$$

$$R^{\text{f-t-h}}_{y^{\text{E}}}\, g^{\text{chp,h}}_{n,i,y^{\text{E}},t} + R^{\text{f-t-p}}_{y^{\text{E}}}\, g^{\text{chp,p}}_{n,i,y^{\text{E}},t} - \overline{F}_{y^{\text{E}}} \leq 0 \quad \left(\phi^{\text{E,ub-fuel}}_{n,i,y^{\text{E}},t}\right), \ \forall n, y^{\text{E}} \in \mathcal{Y}^{\text{E}}{}_{n,i}, t \tag{6}$$

$$R^{\text{heat-only}}_{n,i}\, Q^{\text{heat}}_{n,i,t} - \sum_{x\in\mathcal{X}_{n,i}} E^{\text{trans}}_{n}\, g^{\text{heat}}_{n,i,x,t} \leq 0 \quad \left(\delta^{\text{heat-only}}_{n,i,t}\right), \ \forall n, t \tag{7}$$

$$g^{\text{power}}_{n,i,u,t} - T_t \overline{G}^{\text{power}}_{n,i,u} \leq 0 \quad \left(\lambda^{\text{power}}_{n,i,u,t}\right), \ \forall n, u \in \mathcal{U}_{n,i}, t \tag{8}$$

$$g^{\text{chp,h}}_{n,i,y,t} - T_t \overline{G}^{\text{chp,h}}_{n,i,y} \leq 0 \quad \left(\lambda^{\text{chp,h}}_{n,i,y,t}\right), \ \forall n, y \in \mathcal{Y}_{n,i}, t \tag{9}$$

$$g^{\text{heat}}_{n,i,x,t} - T_t \overline{G}^{\text{heat}}_{n,i,x} \leq 0 \quad \left(\lambda^{\text{heat}}_{n,i,x,t}\right), \ \forall n, x \in \mathcal{X}_{n,i}, t \tag{10}$$

$$g^{e}_{n,i,t} - T_t A^{e}_{n,t} \overline{G}^{e}_{n,i} = 0 \quad \left(\lambda^{e}_{n,i,t}\right), \ \forall n, e, t \tag{11}$$

Firm $i \in \mathcal{I}$

$$g^{\text{power}}_{n,i,u,t} - g^{\text{power}}_{n,i,u,t-1} - T_t R^{\text{power-up}}_u \overline{G}^{\text{power}}_{n,i,u} \leq 0 \ \left(\rho^{\text{power-up}}_{n,i,u,t}\right), \ \forall n, u \in \mathcal{U}_{n,i}, t \tag{12}$$

$$g^{\text{power}}_{n,i,u,t-1} - g^{\text{power}}_{n,i,u,t} - T_t R^{\text{power-down}}_u \overline{G}^{\text{power}}_{n,i,u} \leq 0 \ \left(\rho^{\text{power-down}}_{n,i,u,t}\right), \ \forall n, u \in \mathcal{U}_{n,i}, t \tag{13}$$

$$g^{\text{chp,h}}_{n,i,y,t} - g^{\text{chp,h}}_{n,i,y,t-1} - T_t R^{\text{chp-up}}_y \overline{G}^{\text{chp,h}}_{n,i,y} \leq 0 \ \left(\rho^{\text{chp-up}}_{n,i,y,t}\right), \ \forall n, y \in \mathcal{Y}_{n,i}, t \tag{14}$$

$$g^{\text{chp,h}}_{n,i,y,t-1} - g^{\text{chp,h}}_{n,i,y,t} - T_t R^{\text{chp-down}}_y \overline{G}^{\text{chp,h}}_{n,i,y} \leq 0 \ \left(\rho^{\text{chp-down}}_{n,i,y,t}\right), \ \forall n, y \in \mathcal{Y}_{n,i}, t \tag{15}$$

$$g^{\text{heat}}_{n,i,x,t} - g^{\text{heat}}_{n,i,x,t-1} - T_t R^{\text{heat-up}}_x \overline{G}^{\text{heat}}_{n,i,x} \leq 0 \ \left(\rho^{\text{heat-up}}_{n,i,x,t}\right), \ \forall n, x \in \mathcal{X}_{n,i}, t \tag{16}$$

$$g^{\text{heat}}_{n,i,x,t-1} - g^{\text{heat}}_{n,i,x,t} - T_t R^{\text{heat-down}}_x \overline{G}^{\text{heat}}_{n,i,x} \leq 0 \ \left(\rho^{\text{heat-down}}_{n,i,x,t}\right), \ \forall n, x \in \mathcal{X}_{n,i}, t \tag{17}$$

$$r^{\text{sto}}_{n,i,t} - (1 - E^{\text{dec}})^{T_t} r^{\text{sto}}_{n,i,t-1} - E^{\text{eff}} r^{\text{in}}_{n,i,t} + r^{\text{out}}_{n,i,t} = 0 \ \left(\beta^{\text{bal}}_{n,i,t}\right), \ \forall n, t \tag{18}$$

$$r^{\text{in}}_{n,i,t} - T_t E^{\text{in}} \overline{E}^{\text{max}}_{n,i} \leq 0 \ \left(\beta^{\text{in}}_{n,i,t}\right), \ \forall n, t \tag{19}$$

$$r^{\text{out}}_{n,i,t} - T_t E^{\text{out}} \overline{E}^{\text{max}}_{n,i} \leq 0 \ \left(\beta^{\text{out}}_{n,i,t}\right), \ \forall n, t \tag{20}$$

$$\underline{E}^{\text{min}}_{n,i} \leq r^{\text{sto}}_{n,i,t} \leq \overline{E}^{\text{max}}_{n,i} \ \left(\beta^{\text{lb}}_{n,i,t}, \beta^{\text{ub}}_{n,i,t}\right), \ \forall n, t \tag{21}$$

$$g^{\text{power}}_{n,i,u,t} \geq 0 \ \forall n, u \in \mathcal{U}_{n,i}, t; \ g^{\text{chp,p}}_{n,i,y,t}, g^{\text{chp,h}}_{n,i,y,t} \geq 0 \ \forall n, y \in \mathcal{Y}_{n,i}, t; \ g^{\text{heat}}_{n,i,x,t} \geq 0 \ \forall n, x \in \mathcal{X}_{n,i}, t;$$
$$g^{e}_{n,i,t}, q^{\text{power}}_{n,i,t}, r^{\text{sto}}_{n,i,t}, r^{\text{in}}_{n,i,t}, r^{\text{out}}_{n,i,t} \geq 0 \ \forall n, t \tag{22}$$

$$\Gamma := \{g^{\text{power}}_{n,i,u,t}, g^{\text{chp,p}}_{n,i,y,t}, g^{\text{chp,h}}_{n,i,y,t}, g^{\text{heat}}_{n,i,x,t}, g^{e}_{n,i,t}, q^{\text{power}}_{n,i,t}, r^{\text{sto}}_{n,i,t}, r^{\text{in}}_{n,i,t}, r^{\text{out}}_{n,i,t}\}$$

Firm $i \in \mathcal{I}$

$$g^{\text{power}}_{n,i,u,t} - g^{\text{power}}_{n,i,u,t-1} - T_t R^{\text{power-up}}_u \overline{G}^{\text{power}}_{n,i,u} \leq 0 \quad \left(\rho^{\text{power-up}}_{n,i,u,t}\right), \ \forall n, u \in \mathcal{U}_{n,i}, t \tag{12}$$

$$g^{\text{power}}_{n,i,u,t-1} - g^{\text{power}}_{n,i,u,t} - T_t R^{\text{power-down}}_u \overline{G}^{\text{power}}_{n,i,u} \leq 0 \quad \left(\rho^{\text{power-down}}_{n,i,u,t}\right), \ \forall n, u \in \mathcal{U}_{n,i}, t \tag{13}$$

$$g^{\text{chp,h}}_{n,i,y,t} - g^{\text{chp,h}}_{n,i,y,t-1} - T_t R^{\text{chp-up}}_y \overline{G}^{\text{chp,h}}_{n,i,y} \leq 0 \quad \left(\rho^{\text{chp-up}}_{n,i,y,t}\right), \ \forall n, y \in \mathcal{Y}_{n,i}, t \tag{14}$$

$$g^{\text{chp,h}}_{n,i,y,t-1} - g^{\text{chp,h}}_{n,i,y,t} - T_t R^{\text{chp-down}}_y \overline{G}^{\text{chp,h}}_{n,i,y} \leq 0 \quad \left(\rho^{\text{chp-down}}_{n,i,y,t}\right), \ \forall n, y \in \mathcal{Y}_{n,i}, t \tag{15}$$

$$g^{\text{heat}}_{n,i,x,t} - g^{\text{heat}}_{n,i,x,t-1} - T_t R^{\text{heat-up}}_x \overline{G}^{\text{heat}}_{n,i,x} \leq 0 \quad \left(\rho^{\text{heat-up}}_{n,i,x,t}\right), \ \forall n, x \in \mathcal{X}_{n,i}, t \tag{16}$$

$$g^{\text{heat}}_{n,i,x,t-1} - g^{\text{heat}}_{n,i,x,t} - T_t R^{\text{heat-down}}_x \overline{G}^{\text{heat}}_{n,i,x} \leq 0 \quad \left(\rho^{\text{heat-down}}_{n,i,x,t}\right), \ \forall n, x \in \mathcal{X}_{n,i}, t \tag{17}$$

$$r^{\text{sto}}_{n,i,t} - (1 - E^{\text{dec}})^{T_t} r^{\text{sto}}_{n,i,t-1} - E^{\text{eff}} r^{\text{in}}_{n,i,t} + r^{\text{out}}_{n,i,t} = 0 \quad \left(\beta^{\text{bal}}_{n,i,t}\right), \ \forall n, t \tag{18}$$

$$r^{\text{in}}_{n,i,t} - T_t E^{\text{in}} \overline{E}^{\text{max}}_{n,i} \leq 0 \quad \left(\beta^{\text{in}}_{n,i,t}\right), \ \forall n, t \tag{19}$$

$$r^{\text{out}}_{n,i,t} - T_t E^{\text{out}} \overline{E}^{\text{max}}_{n,i} \leq 0 \quad \left(\beta^{\text{out}}_{n,i,t}\right), \ \forall n, t \tag{20}$$

$$\underline{E}^{\text{min}}_{n,i} \leq r^{\text{sto}}_{n,i,t} \leq \overline{E}^{\text{max}}_{n,i} \quad \left(\beta^{\text{lb}}_{n,i,t}, \beta^{\text{ub}}_{n,i,t}\right), \ \forall n, t \tag{21}$$

$$g^{\text{power}}_{n,i,u,t} \geq 0 \ \forall n, u \in \mathcal{U}_{n,i}, t; \ g^{\text{chp,p}}_{n,i,y,t}, g^{\text{chp,h}}_{n,i,y,t} \geq 0 \ \forall n, y \in \mathcal{Y}_{n,i}, t; \ g^{\text{heat}}_{n,i,x,t} \geq 0 \ \forall n, x \in \mathcal{X}_{n,i}, t;$$
$$g^{e}_{n,i,t}, q^{\text{power}}_{n,i,t}, r^{\text{sto}}_{n,i,t}, r^{\text{in}}_{n,i,t}, r^{\text{out}}_{n,i,t} \geq 0 \ \forall n, t \tag{22}$$

$$\Gamma := \{g^{\text{power}}_{n,i,u,t}, g^{\text{chp,p}}_{n,i,y,t}, g^{\text{chp,h}}_{n,i,y,t}, g^{\text{heat}}_{n,i,x,t}, g^{e}_{n,i,t}, q^{\text{power}}_{n,i,t}, r^{\text{sto}}_{n,i,t}, r^{\text{in}}_{n,i,t}, r^{\text{out}}_{n,i,t}\}$$

Firm $i \in \mathcal{I}$

$$g^{\text{power}}_{n,i,u,t} - g^{\text{power}}_{n,i,u,t-1} - T_t R^{\text{power-up}}_u \overline{G}^{\text{power}}_{n,i,u} \leq 0 \ \left(\rho^{\text{power-up}}_{n,i,u,t}\right), \ \forall n, u \in \mathcal{U}_{n,i}, t \tag{12}$$

$$g^{\text{power}}_{n,i,u,t-1} - g^{\text{power}}_{n,i,u,t} - T_t R^{\text{power-down}}_u \overline{G}^{\text{power}}_{n,i,u} \leq 0 \ \left(\rho^{\text{power-down}}_{n,i,u,t}\right), \ \forall n, u \in \mathcal{U}_{n,i}, t \tag{13}$$

$$g^{\text{chp,h}}_{n,i,y,t} - g^{\text{chp,h}}_{n,i,y,t-1} - T_t R^{\text{chp-up}}_y \overline{G}^{\text{chp,h}}_{n,i,y} \leq 0 \ \left(\rho^{\text{chp-up}}_{n,i,y,t}\right), \ \forall n, y \in \mathcal{Y}_{n,i}, t \tag{14}$$

$$g^{\text{chp,h}}_{n,i,y,t-1} - g^{\text{chp,h}}_{n,i,y,t} - T_t R^{\text{chp-down}}_y \overline{G}^{\text{chp,h}}_{n,i,y} \leq 0 \ \left(\rho^{\text{chp-down}}_{n,i,y,t}\right), \ \forall n, y \in \mathcal{Y}_{n,i}, t \tag{15}$$

$$g^{\text{heat}}_{n,i,x,t} - g^{\text{heat}}_{n,i,x,t-1} - T_t R^{\text{heat-up}}_x \overline{G}^{\text{heat}}_{n,i,x} \leq 0 \ \left(\rho^{\text{heat-up}}_{n,i,x,t}\right), \ \forall n, x \in \mathcal{X}_{n,i}, t \tag{16}$$

$$g^{\text{heat}}_{n,i,x,t-1} - g^{\text{heat}}_{n,i,x,t} - T_t R^{\text{heat-down}}_x \overline{G}^{\text{heat}}_{n,i,x} \leq 0 \ \left(\rho^{\text{heat-down}}_{n,i,x,t}\right), \ \forall n, x \in \mathcal{X}_{n,i}, t \tag{17}$$

$$r^{\text{sto}}_{n,i,t} - (1 - E^{\text{dec}})^{T_t} r^{\text{sto}}_{n,i,t-1} - E^{\text{eff}} r^{\text{in}}_{n,i,t} + r^{\text{out}}_{n,i,t} = 0 \ \left(\beta^{\text{bal}}_{n,i,t}\right), \ \forall n, t \tag{18}$$

$$r^{\text{in}}_{n,i,t} - T_t E^{\text{in}} \overline{E}^{\text{max}}_{n,i} \leq 0 \ \left(\beta^{\text{in}}_{n,i,t}\right), \ \forall n, t \tag{19}$$

$$r^{\text{out}}_{n,i,t} - T_t E^{\text{out}} \overline{E}^{\text{max}}_{n,i} \leq 0 \ \left(\beta^{\text{out}}_{n,i,t}\right), \ \forall n, t \tag{20}$$

$$\underline{E}^{\text{min}}_{n,i} \leq r^{\text{sto}}_{n,i,t} \leq \overline{E}^{\text{max}}_{n,i} \ \left(\beta^{\text{lb}}_{n,i,t}, \beta^{\text{ub}}_{n,i,t}\right), \ \forall n, t \tag{21}$$

$$g^{\text{power}}_{n,i,u,t} \geq 0 \ \forall n, u \in \mathcal{U}_{n,i}, t; \ g^{\text{chp,p}}_{n,i,y,t}, g^{\text{chp,h}}_{n,i,y,t} \geq 0 \ \forall n, y \in \mathcal{Y}_{n,i}, t; \ g^{\text{heat}}_{n,i,x,t} \geq 0 \ \forall n, x \in \mathcal{X}_{n,i}, t;$$
$$g^{e}_{n,i,t}, q^{\text{power}}_{n,i,t}, r^{\text{sto}}_{n,i,t}, r^{\text{in}}_{n,i,t}, r^{\text{out}}_{n,i,t} \geq 0 \ \forall n, t \tag{22}$$

$$\Gamma := \{g^{\text{power}}_{n,i,u,t}, g^{\text{chp,p}}_{n,i,y,t}, g^{\text{chp,h}}_{n,i,y,t}, g^{\text{heat}}_{n,i,x,t}, g^{e}_{n,i,t}, q^{\text{power}}_{n,i,t}, r^{\text{sto}}_{n,i,t}, r^{\text{in}}_{n,i,t}, r^{\text{out}}_{n,i,t}\}$$

Firm $i \in \mathcal{I}$

$$g^{\text{power}}_{n,i,u,t} - g^{\text{power}}_{n,i,u,t-1} - T_t R^{\text{power-up}}_u \overline{G}^{\text{power}}_{n,i,u} \leq 0 \quad \left(\rho^{\text{power-up}}_{n,i,u,t}\right), \ \forall n, u \in \mathcal{U}_{n,i}, t \tag{12}$$

$$g^{\text{power}}_{n,i,u,t-1} - g^{\text{power}}_{n,i,u,t} - T_t R^{\text{power-down}}_u \overline{G}^{\text{power}}_{n,i,u} \leq 0 \quad \left(\rho^{\text{power-down}}_{n,i,u,t}\right), \ \forall n, u \in \mathcal{U}_{n,i}, t \tag{13}$$

$$g^{\text{chp,h}}_{n,i,y,t} - g^{\text{chp,h}}_{n,i,y,t-1} - T_t R^{\text{chp-up}}_y \overline{G}^{\text{chp,h}}_{n,i,y} \leq 0 \quad \left(\rho^{\text{chp-up}}_{n,i,y,t}\right), \ \forall n, y \in \mathcal{Y}_{n,i}, t \tag{14}$$

$$g^{\text{chp,h}}_{n,i,y,t-1} - g^{\text{chp,h}}_{n,i,y,t} - T_t R^{\text{chp-down}}_y \overline{G}^{\text{chp,h}}_{n,i,y} \leq 0 \quad \left(\rho^{\text{chp-down}}_{n,i,y,t}\right), \ \forall n, y \in \mathcal{Y}_{n,i}, t \tag{15}$$

$$g^{\text{heat}}_{n,i,x,t} - g^{\text{heat}}_{n,i,x,t-1} - T_t R^{\text{heat-up}}_x \overline{G}^{\text{heat}}_{n,i,x} \leq 0 \quad \left(\rho^{\text{heat-up}}_{n,i,x,t}\right), \ \forall n, x \in \mathcal{X}_{n,i}, t \tag{16}$$

$$g^{\text{heat}}_{n,i,x,t-1} - g^{\text{heat}}_{n,i,x,t} - T_t R^{\text{heat-down}}_x \overline{G}^{\text{heat}}_{n,i,x} \leq 0 \quad \left(\rho^{\text{heat-down}}_{n,i,x,t}\right), \ \forall n, x \in \mathcal{X}_{n,i}, t \tag{17}$$

$$r^{\text{sto}}_{n,i,t} - (1 - E^{\text{dec}})^{T_t} r^{\text{sto}}_{n,i,t-1} - E^{\text{eff}} r^{\text{in}}_{n,i,t} + r^{\text{out}}_{n,i,t} = 0 \quad \left(\beta^{\text{bal}}_{n,i,t}\right), \ \forall n, t \tag{18}$$

$$r^{\text{in}}_{n,i,t} - T_t E^{\text{in}} \overline{E}^{\text{max}}_{n,i} \leq 0 \quad \left(\beta^{\text{in}}_{n,i,t}\right), \ \forall n, t \tag{19}$$

$$r^{\text{out}}_{n,i,t} - T_t E^{\text{out}} \overline{E}^{\text{max}}_{n,i} \leq 0 \quad \left(\beta^{\text{out}}_{n,i,t}\right), \ \forall n, t \tag{20}$$

$$\underline{E}^{\text{min}}_{n,i} \leq r^{\text{sto}}_{n,i,t} \leq \overline{E}^{\text{max}}_{n,i} \quad \left(\beta^{\text{lb}}_{n,i,t}, \beta^{\text{ub}}_{n,i,t}\right), \ \forall n, t \tag{21}$$

$$g^{\text{power}}_{n,i,u,t} \geq 0 \ \forall n, u \in \mathcal{U}_{n,i}, t; \ g^{\text{chp,p}}_{n,i,y,t}, g^{\text{chp,h}}_{n,i,y,t} \geq 0 \ \forall n, y \in \mathcal{Y}_{n,i}, t; \ g^{\text{heat}}_{n,i,x,t} \geq 0 \ \forall n, x \in \mathcal{X}_{n,i}, t;$$

$$g^{e}_{n,i,t}, q^{\text{power}}_{n,i,t}, r^{\text{sto}}_{n,i,t}, r^{\text{in}}_{n,i,t}, r^{\text{out}}_{n,i,t} \geq 0 \ \forall n, t \tag{22}$$

$$\Gamma := \{g^{\text{power}}_{n,i,u,t}, g^{\text{chp,p}}_{n,i,y,t}, g^{\text{chp,h}}_{n,i,y,t}, g^{\text{heat}}_{n,i,x,t}, g^{e}_{n,i,t}, q^{\text{power}}_{n,i,t}, r^{\text{sto}}_{n,i,t}, r^{\text{in}}_{n,i,t}, r^{\text{out}}_{n,i,t}\}$$

Firm $i \in \mathcal{I}$

$$g^{\text{power}}_{n,i,u,t} - g^{\text{power}}_{n,i,u,t-1} - T_t R^{\text{power-up}}_u \overline{G}^{\text{power}}_{n,i,u} \leq 0 \ \left(\rho^{\text{power-up}}_{n,i,u,t}\right), \ \forall n, u \in \mathcal{U}_{n,i}, t \tag{12}$$

$$g^{\text{power}}_{n,i,u,t-1} - g^{\text{power}}_{n,i,u,t} - T_t R^{\text{power-down}}_u \overline{G}^{\text{power}}_{n,i,u} \leq 0 \ \left(\rho^{\text{power-down}}_{n,i,u,t}\right), \ \forall n, u \in \mathcal{U}_{n,i}, t \tag{13}$$

$$g^{\text{chp,h}}_{n,i,y,t} - g^{\text{chp,h}}_{n,i,y,t-1} - T_t R^{\text{chp-up}}_y \overline{G}^{\text{chp,h}}_{n,i,y} \leq 0 \ \left(\rho^{\text{chp-up}}_{n,i,y,t}\right), \ \forall n, y \in \mathcal{Y}_{n,i}, t \tag{14}$$

$$g^{\text{chp,h}}_{n,i,y,t-1} - g^{\text{chp,h}}_{n,i,y,t} - T_t R^{\text{chp-down}}_y \overline{G}^{\text{chp,h}}_{n,i,y} \leq 0 \ \left(\rho^{\text{chp-down}}_{n,i,y,t}\right), \ \forall n, y \in \mathcal{Y}_{n,i}, t \tag{15}$$

$$g^{\text{heat}}_{n,i,x,t} - g^{\text{heat}}_{n,i,x,t-1} - T_t R^{\text{heat-up}}_x \overline{G}^{\text{heat}}_{n,i,x} \leq 0 \ \left(\rho^{\text{heat-up}}_{n,i,x,t}\right), \ \forall n, x \in \mathcal{X}_{n,i}, t \tag{16}$$

$$g^{\text{heat}}_{n,i,x,t-1} - g^{\text{heat}}_{n,i,x,t} - T_t R^{\text{heat-down}}_x \overline{G}^{\text{heat}}_{n,i,x} \leq 0 \ \left(\rho^{\text{heat-down}}_{n,i,x,t}\right), \ \forall n, x \in \mathcal{X}_{n,i}, t \tag{17}$$

$$r^{\text{sto}}_{n,i,t} - (1 - E^{\text{dec}})^{T_t} r^{\text{sto}}_{n,i,t-1} - E^{\text{eff}} r^{\text{in}}_{n,i,t} + r^{\text{out}}_{n,i,t} = 0 \ \left(\beta^{\text{bal}}_{n,i,t}\right), \ \forall n, t \tag{18}$$

$$r^{\text{in}}_{n,i,t} - T_t E^{\text{in}} \overline{E}^{\text{max}}_{n,i} \leq 0 \ \left(\beta^{\text{in}}_{n,i,t}\right), \ \forall n, t \tag{19}$$

$$r^{\text{out}}_{n,i,t} - T_t E^{\text{out}} \overline{E}^{\text{max}}_{n,i} \leq 0 \ \left(\beta^{\text{out}}_{n,i,t}\right), \ \forall n, t \tag{20}$$

$$\underline{E}^{\text{min}}_{n,i} \leq r^{\text{sto}}_{n,i,t} \leq \overline{E}^{\text{max}}_{n,i} \ \left(\beta^{\text{lb}}_{n,i,t}, \beta^{\text{ub}}_{n,i,t}\right), \ \forall n, t \tag{21}$$

$$g^{\text{power}}_{n,i,u,t} \geq 0 \ \forall n, u \in \mathcal{U}_{n,i}, t; \ g^{\text{chp,p}}_{n,i,y,t}, g^{\text{chp,h}}_{n,i,y,t} \geq 0 \ \forall n, y \in \mathcal{Y}_{n,i}, t; \ g^{\text{heat}}_{n,i,x,t} \geq 0 \ \forall n, x \in \mathcal{X}_{n,i}, t;$$
$$g^{e}_{n,i,t}, q^{\text{power}}_{n,i,t}, r^{\text{sto}}_{n,i,t}, r^{\text{in}}_{n,i,t}, r^{\text{out}}_{n,i,t} \geq 0 \ \forall n, t \tag{22}$$

$$\Gamma := \{g^{\text{power}}_{n,i,u,t}, g^{\text{chp,p}}_{n,i,y,t}, g^{\text{chp,h}}_{n,i,y,t}, g^{\text{heat}}_{n,i,x,t}, g^{e}_{n,i,t}, q^{\text{power}}_{n,i,t}, r^{\text{sto}}_{n,i,t}, r^{\text{in}}_{n,i,t}, r^{\text{out}}_{n,i,t}\}$$

Grid Owner and Market Clearing

$$\min_{\theta_{n^{AC},t}, f_{\ell,t}} - \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{N}} \omega_{n,t} \sum_{\ell \in \mathcal{L}} M_{\ell,n} T_t f_{\ell,t} \tag{23}$$

$$\text{s.t.} \sum_{n^{AC} \in \mathcal{N}^{AC}} T_t H_{\ell^{AC},n^{AC}} \theta_{n^{AC},t} - T_t f_{\ell^{AC},t} = 0 \ \left(\eta_{\ell^{AC},t}\right), \forall \ell^{AC} \in \mathcal{L}^{AC}, t \tag{24}$$

$$T_t f_{\ell,t} - T_t \overline{K}_\ell \leq 0 \ \left(\overline{\mu}_{\ell,t}\right), \forall \ell, t \tag{25}$$

$$- T_t f_{\ell,t} - T_t \underline{K}_\ell \leq 0 \ \left(\underline{\mu}_{\ell,t}\right), \forall \ell, t \tag{26}$$

$$T_t S_{n^{AC}} \theta_{n^{AC},t} = 0 \ \left(\gamma_{n^{AC},t}\right), \forall n^{AC} \in \mathcal{N}^{AC}, t \tag{27}$$

$$\theta_{n^{AC},t} \text{ and } f_{\ell,t} \text{ u.r.s.}, \forall n^{AC} \in \mathcal{N}^{AC}, \ell, t \tag{28}$$

$$\sum_{i \in \mathcal{I}} q^{power}_{n,i,t} - \sum_{i \in \mathcal{I}} \sum_{u \in \mathcal{U}_{n,i}} g^{power}_{n,i,u,t} - \sum_{i \in \mathcal{I}} \sum_{y \in \mathcal{Y}_{n,i}} g^{chp,p}_{n,i,y,t}$$
$$- \sum_{i \in \mathcal{I}} \sum_{e \in \mathcal{E}} g^{e}_{n,i,t} - \sum_{\ell \in \mathcal{L}} T_t M_{\ell,n} f_{\ell,t} = 0 \ \left(\omega_{n,t}, \ \text{u.r.s.}\right), \forall n, t \tag{29}$$

Grid Owner and Market Clearing

$$\min_{\theta_{n^{AC},t},\, f_{\ell,t}} \; -\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} \omega_{n,t} \sum_{\ell\in\mathcal{L}} M_{\ell,n}\, T_t f_{\ell,t} \tag{23}$$

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Grid Owner and Market Clearing

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Grid Owner and Market Clearing

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Grid Owner and Market Clearing

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Grid Owner and Market Clearing

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