Utility Function: Theoretical Underpinnings

Outline

- Utility and Demand
- Axioms of Consumer Choice
- Utility Function
- Indifference Curve
- Examples of Utility Function and Indifference Curves
- Utility Maximization in Energy Modeling
Utility and Demand

Utility

*Utility is a measure of the value which consumers of a product or service place on that product or service*

Demand

*Demand is a reflection of this measure of value, and is represented by price per quantity of output*
Utility and Demand

The Income Effect

*A lower price frees income for additional purchases - and vice versa*

The Substitution Effect

*A lower price relative to other goods attracts new buyers - and vice versa*
Utility and Demand

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<tbody>
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![Graph showing utility function with Tacos consumed per meal on the x-axis and Total Utility (utils) on the y-axis. The graph illustrates the concept of utility and demand.]
Utility Function: Theoretical Underpinnings

Utility and Demand

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The graph illustrates the relationship between the number of tacos consumed per meal and the total and marginal utility. As more tacos are consumed, the total utility increases, but the marginal utility decreases, indicating diminishing returns.
Utility Function: Theoretical Underpinnings

Utility and Demand

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Graph showing total utility and marginal utility as a function of units consumed per meal.
Utility Function: Theoretical Underpinnings

Utility and Demand

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![Graph showing total and marginal utility](image)

- **Total Utility (utils)**
  - Units consumed per meal
  - Values range from 0 to 30

- **Marginal Utility (utils)**
  - Units consumed per meal
  - Values range from 0 to 10
# Utility Function: Theoretical Underpinnings

## Utility and Demand

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![Utility and Demand Graph](image-url)
### Utility Function: Theoretical Underpinnings

#### Utility and Demand

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**Graph: Total Utility and Marginal Utility**

- **Total Utility (utils)**
- **Units consumed per meal**
- **Marginal Utility (utils)**
  - 0 units: 0 utils
  - 1 unit: 10 utils
  - 2 units: 18 utils
  - 3 units: 24 utils
  - 4 units: 28 utils
  - 5 units: 30 utils
Utility and Demand

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Total Utility (utils)

United States consumed per meal

Marginal Utility (utils)
Utility and Demand

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TU

MU

Units consumed per meal

Total Utility (utils)

Marginal Utility (utils)
Utility Function: Theoretical Underpinnings

Utility and Demand

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Observe Diminishing Marginal Utility
Utility and Demand

Utility Maximizing Rule

The consumer’s income should be allocated so that the last unit spent on each product yields the same amount of marginal utility.
Utility Function: Theoretical Underpinnings

Utility and Demand

$10$ income

Unit of product

Product A: 
Price = $1

Product B: 
Price = $2

<table>
<thead>
<tr>
<th>First</th>
<th>Product A: Price = $1</th>
<th>Product B: Price = $2</th>
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<tbody>
<tr>
<td></td>
<td>Marginal utility, utils</td>
<td>Marginal utility per dollar (MU/price)</td>
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How should the $10 income be allocated?
Utility Function: Theoretical Underpinnings

Utility and Demand

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Examine the two marginal utilities
Utility and Demand

Product A: Price = $1

Product B: Price = $2

Marginal utility per dollar (MU/price)

First 10 10 24 12

Examine the two marginal utilities...per dollar
Utility and Demand

$10 income

Unit of product

Marginal utility, utils

Marginal utility per dollar (MU/price)

Product A: Price = $1

Marginal utility per dollar (MU/price)

Product B: Price = $2

First 10 10 24 12

Decision: Buy 1 Product B for $2
Utility Function: Theoretical Underpinnings

Utility and Demand

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What next?

Product A: Price = $1
Product B: Price = $2

$10 income
### Utility and Demand

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**What next?**

**Buy one of each**
Utility Function: Theoretical Underpinnings

Utility and Demand

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Marginal utility, utils
Marginal utility per dollar (MU/price)

and then...
($5 left)
# Utility Function: Theoretical Underpinnings

## Utility and Demand

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*third unit of product B*
# Utility Function: Theoretical Underpinnings

## Utility and Demand

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- **Unit of product**
- **$10 income**
## Utility and Demand

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$3 left...

**Buy both!**
Utility and Demand

**Product A:**
- **Price:** $1
- **Marginal utility per dollar (MU/price):**
  - First: 10 utils, 10 MU/price
  - Second: 8 utils, 8 MU/price
  - Third: 7 utils, 7 MU/price
  - Fourth: 6 utils, 6 MU/price
  - Fifth: 5 utils, 5 MU/price
  - Sixth: 4 utils, 4 MU/price
  - Seventh: 3 utils, 3 MU/price

**Product B:**
- **Price:** $2
- **Marginal utility per dollar (MU/price):**
  - First: 24 utils, 12 MU/price
  - Second: 20 utils, 10 MU/price
  - Third: 18 utils, 9 MU/price
  - Fourth: 16 utils, 8 MU/price
  - Fifth: 12 utils, 6 MU/price
  - Sixth: 8 utils, 4 MU/price
  - Seventh: 6 utils, 3 MU/price

**Income is gone...**
- the last dollar spent on each good gave the same utility (8) per dollar
Utility Function: Theoretical Underpinnings

Utility and Demand

**Algebraic Restatement of the Utility Maximization Rule**

\[
\begin{align*}
\text{MU of product A} & \quad \text{Price of A} \\
8 \text{ Utils} & \quad 1 \text{ $} \\
\hline
\text{MU of product B} & \quad \text{Price of B} \\
16 \text{ Utils} & \quad 2 \text{ $}
\end{align*}
\]
Utility and Demand

**Deriving the Demand Schedule and Curve**

Create a demand schedule from the purchase decisions as the price of the product is varied ...

<table>
<thead>
<tr>
<th>Price per unit of B</th>
<th>Quantity Demanded</th>
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<tr>
<td>$2</td>
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Graphically...
Utility and Demand

Deriving the Demand Schedule and Curve

- **Price per unit of Good B**: $2
- **Quantity Demanded of Good B**: 4
- **Utility Function: Theoretical Underpinnings**
- **Deriving the Demand Schedule and Curve**

Diagram:
- Vertical axis: Price per unit of Good B
- Horizontal axis: Quantity Demanded of Good B
- Graph showing the demand curve $D_B$
Price equals some constant value minus some multiple of the quantity demanded:
\[ p = a - b \cdot D \]

- \( a \) = Y-axis (quantity) intercept, (price at 0 amount demanded);
- \( b \) = slope of the demand function;

\[ D = \frac{a - p}{b} \]
Profit is maximum where Total Revenue exceeds Total Cost by greatest amount.

Maximum Profit

D’1 and D’2 are breakeven points
PROFIT MAXIMIZATION

$D^*$

- Occurs where total revenue exceeds total cost by the greatest amount;
- Occurs where marginal cost = marginal revenue;
- Occurs where $\frac{dTR}{dD} = \frac{dC_t}{dD}$;
- $D^* = \frac{[a - c_v]}{2b}$
BREAKEVEN POINT

D’₁ and D’₂

- Occurs where TR = Cₜ
- aD - bD² = Cᵻ + (cᵥ) D
- - bD² + [ a - cᵥ ] D - Cᵻ = 0
- Using the quadratic formula: D’ =

\[- [ a - cᵥ ] \pm \{ [ a - cᵥ ]^2 - 4 (- b) ( - Cᵻ ) \}^{1/2} \]
\------------------------------------------------------------------------
\[ 2 (- b) \]
Utility and Demand

Consumer Choice

• Rational Behavior
• Clear-Cut Preferences
• Responds to Price Changes
• Subject to a Budget Constraint
Axioms of Consumer Choice

1. Completeness

Consumer choice theory is based on the assumption that the consumer fully understands his/her own preferences, allowing for a simple but accurate comparison between any two bundles of goods presented.

A person can compare any two bundles of goods $A$ and $B$ in such a way that it leads to one of the three following results: he or she (i) prefers $A$ over $B$, or (ii) prefers $B$ over $A$, or (iii) both $A$ and $B$ are the same (they are indifferent)

Three cases:

i. $A$ is preferred to $B$: $A \triangleright B$
ii. $B$ is preferred to $A$: $B \triangleright A$
iii. $A$ and $B$ are the same: $A \sim B$

This axiom of completeness rules out the possibility that the consumer cannot compare between two different baskets of commodities.
Axioms of Consumer Choice

2. Transitivity

Consider any three bundles of goods $A$, $B$, and $C$. If a consumer prefers $A$ to $B$, and also prefers $B$ to $C$, he or she must prefer $A$ to $C$. Similarly, a person who is indifferent between $A$ and $B$, and is also indifferent between $B$ and $C$, must be indifferent between $A$ and $C$.

This is the consistency assumption. This assumption eliminates the possibility of intersecting indifference curves.
Axioms of Consumer Choice

The axiom of \textbf{completeness} and the axiom of \textbf{transitivity} are the two \textbf{most basic assumptions} towards people’s preferences. They are derived \textit{logically} because, without either one of them, an economic analysis cannot be performed.

If someone’s behavior satisfies the axiom of completeness and the axiom of transitivity, we know that he/she can rank any bundles of goods.

\textbf{Proposition: A consumer can consistently rank all bundles of goods in order of preference.}
Further assumptions, however, are still needed to define a typical indifference curve based on a “well-behaved” preference. These are defined in the following. They are based on *empirical observations* towards people’s choices and preferences.
Axioms of Consumer Choice

3. Non-satiation

This is the "more is always better" assumption

If a consumer is offered two almost identical bundles A and B, but where B includes more of one particular good, the consumer will choose B

You may think of a counter-example that after you have eaten 10 ice-creams, you will not want even a single one. But don’t forget that in a market you can always trade those additional ice-creams for money and then purchase other goods. Thus “the more, the better” generally holds.

Non-satiation is not a necessary but a convenient assumption. It precludes circular indifference curves and avoids unnecessary complications in mathematical models.
Axioms of Consumer Choice

4. People’s Preference of Variety

Assume any weight $t$ between 0 and 1, if there are two bundles of commodities $(x_1, y_1)$ and $(x_2, y_2)$ such that $(x_1, y_1) \sim (x_2, y_2)$, we have

$$(tx_1 + (1-t)x_2, ty_1 + (1-t)y_2) \geq (x_1, y_1)$$

This means, given any two bundles that are indifferent to a consumer, the mixture of these two bundles is always as good as any of them.
Utility Function

A utility function is a mathematical expression that shows the relationship between utility values of every possible bundle of goods.

Suppose there are three commodity baskets A, B and C, and the preference ordering towards these three baskets is: A>B>C

When we assign a utility function to this preference ordering, it can simply be considered as a transformation from the above expression: $U(A)>U(B)>U(C)$
Utility Function

Cardinal vs Ordinal

Cardinal Utility Function:
According to this approach $U(A)$ is a cardinal number, that is:
$U : \text{consumption bundle} \rightarrow \mathbb{R}$ measured in "utils"
Assigning numerical values to the amount of satisfaction

Ordinal Utility Function:
More general than cardinal utility function
$U$ provides a "ranking" or "preference ordering" over bundles.
Not assigning numerical values to the amount of satisfaction but indicating the order of preferences
Utility Function

Cardinal vs Ordinal

The problem with cardinal utility functions comes from the difficulty in finding the appropriate measurement index (metric).

Is 1 util for person 1 equivalent to 1 util for person 2? What is the proper metric for comparing U1 vs U2? How can interpersonal comparisons be made?

By being unit-free ordinal utility functions avoid these problems.

All that matters about utility as far as choice behavior is concerned is whether one bundle has a higher utility than another – how much higher doesn’t really matter.

Therefore, **Ordinal Utility Functions** are used in demand/consumer theory.
Utility Function

Since only the ranking of the bundles matters, there can be no unique way to assign utilities to bundles of goods.

If we can find a way to assign utility numbers to bundles of goods, we can find an infinite number of ways to do it – simply by multiplying the utility measure by any positive number.

Any monotonic transformation of $U(x_1,y_1)$ is just as good a way to assign utilities as $U(x_1,y_1)$ itself.

Geometrically, a utility function is a way to label indifference curves in such a way that higher indifference curves get assigned larger numbers.
Indifference Curve

*From Preference to Indifference Curve*

**Definition of Indifference Curve:** An *indifference curve* is a graph showing different bundles of goods between which a consumer is *indifferent*. That is, at each point on the curve, the consumer has no preference for one bundle over another. *Every single indifference curve represents a certain level of utility.* One can equivalently refer to each point on the indifference curve as rendering the same level of utility (satisfaction) for the consumer.
Indifference Curve

1. Characteristics: Negative sloping

As quantity consumed of one good (X) increases, total satisfaction would increase if not offset by a decrease in the quantity consumed of the other good (Y).

If we randomly spot a certain point, D, on a diagram representing two goods, we will see that any combinations of these two goods that lie in the northeast of this point are better; symmetrically, any combinations that lie in the southwest of this point are less desired.
Indifference Curve

1. Characteristics: Negative sloping

If we connect all dots representing the same level of utility as point D and form a curve, the curve should go from northwest to southeast. The curve that connects these points, by definition, is an indifference curve.
Indifference Curve

2. Characteristics: No intersection, parallelity

All points on an indifference curve are ranked equally preferred and ranked either more or less preferred than every other point not on the curve. If this characteristic doesn’t hold, it violates the axiom of transitivity. Suppose two indifference curves intersect each other at point A. We can find two points, one on each indifference curve in the way that they have the same amount of good X but C has more of good Y than B.

Since A and B are on the same indifference curve, we have A~B, and at the same time, we also have A~C. Applying the axiom of transitivity, we immediately see that B~C should hold. But from the axiom of non-satiation, we have C>B. A conflict appears. As a result, indifference curves can never intersect with each other.
Indifference Curve

3. Characteristics: Completeness

There is an infinite number of indifference curves which cover the whole area of the axiom panel. This relates to the axiom of completeness. Since an individual can rank any bundle of goods, and every point on the axiom panel represents one certain basket of goods, then apparently every point should have one and only one indifference curve go through it.
Indifference Curve

4. Characteristics: Convexity

This is the result of the fact that **people prefer variety**. On the graph below, point E represents basket \((x_1, y_1)\) and point F represents basket \((x_2, y_2)\). Any point on the dotted line between E and F can be expressed by

\[
(tx_1 + (1-t)x_2, ty_1 + (1-t)y_2) \geq (x_1, y_1)
\]

by applying different value of \(t\). Because people prefer variety, any point on the dotted line that between E and F has a higher utility than either E or F, and thus points that have the same utility as E or F (which means they are on the same indifference curve as E and F) lie on the southwest of the dotted line.
Indifference Curve

5. Characteristics: Diminishing Marginal Utility

As more of a good is consumed total utility increases at a decreasing rate - additions to utility per unit consumption are successively smaller. Thus as you move down the indifference curve you are trading consumption of units of Y for additional units of X.

The absolute value of the slope of the indifference curve represents the Marginal Rate of Substitution.
Indifference Curve

5. Characteristics: Diminishing Marginal Utility

**Definition of Marginal Rate of Substitution (MRS):** The measurement of how many units of good Y a consumer would be willing to give up to get one additional unit of good X while the consumer keeps his or her level of satisfaction (utility).

Mathematically, we can calculate MRS with the help of Marginal Utility

\[
MRS_{xy} = \frac{MU_x}{MU_y}
\]
Indifference Curve

5. Characteristics: Diminishing Marginal Utility

An important property of MRS is called “Diminishing Marginal Rate of Substitution”.

This captures the fact that while you are getting more and more of a certain good, say X, you are less likely to give up the other good, Y. For example, if you are very hungry, you may be willing to pay 12 $ for a hamburger (Y). But when you are almost full, you don’t even want to pay 4 $ for the same hamburger.

This graph also shows another way to calculate the MRS: \( MRS_{xy} = \frac{\Delta Y}{\Delta X} \)
Indifference Curve

5. Characteristics: Diminishing Marginal Utility

In this graph, $\Delta X$ always equals to 1 because the indifference curve is partitioned in such a way that the consumer is getting only one more unit of $X$ each time. Thus $MRS_{XY}$ can be directly read from the change in $Y$. If the consumer currently has 5 units of $X$, his/her MRS is 5 (The consumer is willing to give 5 units of $Y$ in order to get one more unit of $X$). If the consumer currently has 7 units of $X$, his or her MRS is 1.
Examples of Utility Function and Indifference Curves

Perfect Substitution
The utility function for someone towards two perfect substitutes \((X \text{ and } Y)\) is given by:

\[ u(X, Y) = aX + bY \]

That is, for this individual, only the total amount of these two goods matters. Graphically, the indifference curves look like

The slope of all these indifference curves are the same, \(-a/b\). We do not have diminishing MRS in the case of perfect substitution because MRS is also constant and equals to \(a/b\).
Perfect complement
The utility function for someone towards two perfect complements is given by:
\[ u(X, Y) = \min\{ax, by\} \]

That is, for this individual, only the smallest amount of these two goods matters. A widely cited example of perfect complement is right foot shoes and left foot shoes. A hundred right foot shoes and one left foot shoe only makes one pair of shoes and only works for one normal person. Graphically speaking, the indifference curves look like:
Examples of Utility Function and Indifference Curves

The slope of the indifference curves, as well as MRS, can be calculated according to the following rules:

(1) At the turning point of each indifference curve, slope of MRS does not exist;

(2) Above the turning point, the value of slope and MRS are infinity;

(3) On the right of the turning point, the value of slope and MRS are zero.

Mathematically speaking, you cannot calculate the slope of the indifference curve (thus MRS) at the turning point because at that point the (utility) function is discontinued.
Examples of Utility Function and Indifference Curves

**Cobb-Douglas Utility Function**

Charles Cobb was a mathematician at Amherst College, and Paul Douglas was an economist at the University of Chicago. In 1928, they published a paper titled “A Theory of Production” on *American Economic Review*, proposed the following functional form that can be used to capture the relationship between two different inputs of production:

\[ \text{Output} = X^aY^b \]

Normally, the exponents \( a \) and \( b \) satisfied

\[
\begin{align*}
  &a > 0 \\
  &b > 0 \\
  &a + b = 1
\end{align*}
\]

In terms of utility function, a Cobb-Douglas utility function is written as

\[ u(X,Y) = X^aY^b \]
Examples of Utility Function and Indifference Curves

*Cobb-Douglas Utility Function*

The indifference curves given by the Cobb-Douglas utility function satisfy all the four characteristics introduced in this lecture. Its wide use is partly because of its goodness in fitting the theory and data. The well-behaved indifference curve we’ve seen is from a Cobb-Douglas utility function.
Utility Function: Theoretical Underpinnings

Utility Maximization in Energy Modeling

ETA-MACRO
MARKAL-MACRO
TIMES-MACRO
BUEMS-MACRO
……..-MACRO
Edgeworth Box

- The prices of substitutes and complements for a good will influence the demand for it, and, the prices of goods that people sell will affect the amount of income they have and thereby influence much of other goods they will be able to buy.

- How do demand and supply conditions interact in several markets?

- Francis Edgeworth developed this method of analysis in the last portion of the 19th century.

- Provides a powerful way of graphically studying exchange and the role of markets.
Edgeworth Box

- Simple economy: two goods, two consumers: Liu and Pan
- Rotate one of the graphs onto the other one until it forms a box.
Here the axes for Liu have been rotated
Move axes for Liu to close box
The Edgeworth Box

A

Pan

$y_1$

$x_1$

$y_2$

$x_2$

Liu

Total Fixed Supply of $y$

Total Fixed Supply of $x$
Consider Liu and Pan’s Indifference Curves for two products.
The Edgeworth Box

Utilitarian Function: Theoretical Underpinnings
The Edgeworth Box
Utility Function: Theoretical Underpinnings

The Edgeworth Box
The Edgeworth Box
The Edgeworth Box
Utility Function: Theoretical Underpinnings

The Edgeworth Box

- Initial endowment A
- Trading area?

Pan

y_1

\( x_1 \)

y_2

x_2
Utility Function: Theoretical Underpinnings

The Edgeworth Box

What about A here?
**Utility Function: Theoretical Underpinnings**

The Edgeworth Box

When no change can make one better off without making the other worse off.

PARETO OPTIMAL

[Diagram showing the Edgeworth Box with labeled axes and points]
The Edgeworth Box

Is the locus of Pareto optimal points

Contract line
Utility Function: Theoretical Underpinnings

The Edgeworth Box

Pan

\( x_1 \)

\( y_1 \)

Liu

\( x_2 \)

\( y_2 \)

IV_2 \ III_2 \ II_2 \ I_2

I_1 \ II_1 \ III_1 \ IV_1
Utility Function: Theoretical Underpinnings

The Edgeworth Box

Pareto improving--from B or C to A or A’
Any point in the Edgeworth box indicates a particular distribution of the two goods among Pan and Liu.

Each individual has an indifference curve going through that point.

If the distribution is Pareto optimal, those two indifference curves are tangent at that point.

At that tangency of the two indifference curves, the slope of the tangency line—the straight line drawn through the point of tangency—represents the relative prices for the two goods. Hence, there are relative prices that will be consistent with the Pareto optimum.
Utility Function: Theoretical Underpinnings

Pan

Price or budget line

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Edgeworth Box

Tangent line is really a budget line for both individuals

- If one extends the tangent line to each axis, we now have a budget line.
- For example, the budget line for Liu is

\[ I_{Liu} = P_x x_{Liu} + P_y y_{Liu} \]

where \( I \) is the income Liu could get from selling the \( X \) and \( Y \) she holds at the Pareto optimum point.
Utility Function: Theoretical Underpinnings

\[ I_{Liu} = P_x x_{Liu} + P_y y_{Liu} \]