Coupling Transmission and Energy Markets Reduces Market Power

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Abstract

Transmission constraints fragment electricity markets and enhance regional market power of electricity generators. In continental Europe rights to access transmission between countries are auctioned to traders, which arbitrage *separate* energy spot markets of these countries. In Scandinavia the system operator *integrates* these markets and simultaneously clears energy spot markets of several countries and decides on optimal energy transmission. In any unconstrained or partially constrained network, integration mitigates market power of strategic generators and avoids inefficient production decisions. A testable prediction for both effects is applied to the Dutch-German and Norway-Sweden interconnection and supports the theory. In meshed networks, integration also mitigates market power when constraints are permanently binding. Le Chatelier's principle extends to electricity networks in the presence of market power. Demand is more responsive to price changes and aggregate output increases if markets are integrated.

1 Introduction

In electricity markets transmission constraints in networks act as quotas on trade between regions. Two basic designs can be used to internalise the constraints in the market. Previous analysis showed that both designs are equivalent in competitive markets if transaction costs and liquidity

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are ignored. This paper shows that even under these favourable conditions a clear ranking between the designs exist if electricity generation companies exercise market power.

The introduction and the paper start with a description of the market, the design options, compare them in an analytic model, present a test and apply the test to different European markets.

Vertically-integrated electricity companies are frequently unbundled into generation, transmission, distribution and supply companies. Generation companies compete in the wholesale electricity market to sell to larger customers and supply companies. Limited transmission capacity can fragment the wholesale electricity market into smaller regional markets. Using an example from some European countries[1], Figure 1 shows that at times of peak demand, with the exception of Switzerland, the largest generator's production is a multiple of spare national generation capacity. Even if all other generators supply at full capacity, the largest generator can create scarcity and extreme price spikes by withholding a fraction of his output, although a greater portion of his capacity must be withheld to create scarcity at off-peak times. Imports, or the threat of imports, reduce but do not eliminate the dominance of the largest generator, as Figure 1 illustrates. Effective allocation of scarce transmission capacity is therefore crucial to mitigate market power in European countries.

The first approach to addressing transmission constraints in the wholesale electricity market I will refer to as market *coupling.* It has evolved from a pricing mechanism introduced by Bohn, Caramanis and Schweppe (1984) to deal with transmission constraints between different nodes of a network. Generation companies, large electricity customers, and supply companies submit bids and offers to a system operator, specifying the price, location and quantity they want to buy or sell at. The system operator determines a separate price for each node at which accepted bids pay and offers must be paid for. This system is often referred to as nodal pricing. If all bids are competitive, nodal pricing implements the welfare-maximising dispatch, subject to the transmission constraints of the system.[2]

An alternative approach, which I will refer to as *separate* transmission and energy markets, is frequently advocated because it seems not to require centralised institutions. Consumers,

[1]Source: UCTE Power Balance Forecasts 2002-2004, ETSO's NTC publications 2001/2002, ICF Consulting, annual reports and presentations.

[2]Zonal pricing and market splitting simplify nodal pricing by aggregating several nodes into one zone, at the cost of reduced efficiency and increased potential for the exercise of market power (Harvey and Hogan, 2000). This paper assumes nodal pricing and does not address inefficiencies due to zonal aggregation, but provides insights into zonal pricing as long as intrazonal constraints are limited. Hogan (1992) supplemented nodal pricing with tradable congestion contracts (TCC), auctioned by the transmission operator to allow hedging and provide long-term information to guide investment decisions.

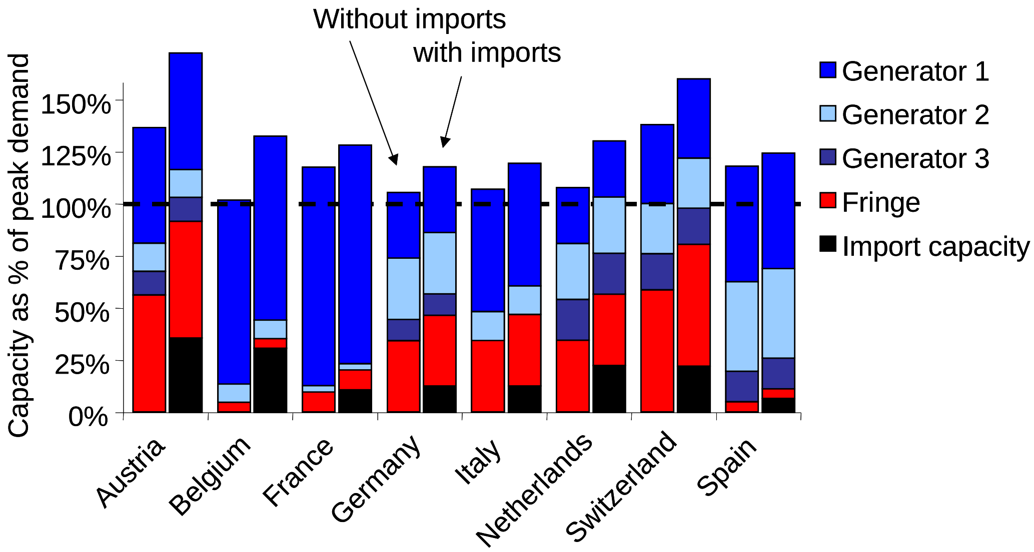

Figure 1: Capacity of generator as % of peak demand during winter peak. In most countries, demand is not satisfied if the largest generator withholds a fraction of output during peak demand (left columns), even if available import capacity is utilised (right columns).[1]

traders and generators trade energy bilaterally or in regional spot markets. If they want to trade and schedule flows between regions, then they have to obtain transmission contracts for the appropriate time, quantity and origin-destination relation. Transmission contracts are effectively property rights for scarce transmission capacity and can be initially either grandfathered or auctioned and subsequently retraded. Chao and Peck (1996) prove that the concept achieves a social optimum and therefore coincides with market coupling in the presence of complete and competitive markets with no uncertainty and complete information.

In reality, not all of these requirements are satisfied, and several difficulties of a separation between energy and transmission markets have been identified. The transaction costs of trading physical transmission contracts can be high, because transmission contracts have to be traded separately for each period of the day, origin and destination of transmission resulting in low liquidity. Bushnell (1999) shows that generators can exercise market power by buying but not using physical transmission contracts. However 'use-it-or-lose-it' provisions are now frequently implemented and can at least partially prevent withholding, as Joskow and Tirole (2000) argue. Smeers and Jing-Yuan (1997) show that if only a limited number of traders interact in the transmission market to arbitrage prices between the nodes, they exercise market power and distort the dispatch. Harvey, Hogan and Pope (1996) argue that competitive generators and traders face uncertainty about the prices in the energy market when deciding on their bids for transmission markets, and might therefore buy an inappropriate amount of transmission rights. In section 5, I support this claim with empirical evidence from the Netherlands-Germany interconnector.

However, the main aim of this paper is to show that *coupling* of energy markets has the benefit of mitigating market power of strategic generators in cases of unconstrained, partially and constrained transmission lines. These cases are introduced and explained in the following three subsectoins.

1.1 Unconstrained transmission

A mechanism to deal with transmission constraints still has to be in place even in times of no binding transmission constraints because volatile demand and supply can suddenly result in constraints. At unconstrained times, the system operator implementing market coupling calculates one market-clearing price for all regions and determines the amount of energy to be transmitted. In the case of separate transmission and energy markets the process is more complex, as can be illustrated in the Netherlands-German example. Traders buy transmission rights from Germany to the Netherlands in the day ahead auction. If transmission rights are defined as options, then all potentially beneficial contracts are allocated because, without scarcity and uncertainty, the price drops to zero. Traders then decide how many of these contracts to use to arbitrage the German day ahead electricity spot market, LPX, and the Netherlands day ahead electricity spot market, APX.[3] Traders cannot condition their trade volume on spot prices in these markets because bids must be submitted before results in either market are announced. Traders price their buy bid in one market very high and their offer in the other market very low. This ensures that both bids are accepted simultaneously. Were only one bid accepted, then traders would be in imbalance. They would have to find a last minute bilateral trading partner to make up for this imbalance or else be exposed to a costly imbalance fee imposed by the system operator to induce market participants to contribute to a balanced system. Since the bids placed by traders are virtually price inelastic, transmitted energy will be independent of price and therefore independent of the realised output choice of generators. Generators only compete against local generators and face the local demand slope. Generators therefore exercise more market power if energy and transmission markets are separate than under market coupling.

Are there other approaches to resolve inefficiencies of separate markets? First, decentralized

[3] If transmission rights are defined as obligations then traders already have to decide how many transmission rights to buy and use at the time of the transmission auctions. In the analysis of this paper transmission rights defined as options and obligations provide for the same results. This is because the auction for transmission rights does not reveal additional information, therefore it does not matter whether traders have to commit themselves to a certain amount of energy transmission between the markets at the time before the transmission or before the energy auction.

trading in energy and an iterative market for transmission could integrate the demand of both markets. This would decrease the demand slope and reduce market power. In practice, electricity and transmission rights seem to be too complex to ensure liquidity in short-term markets for each half-hour and for each location. Second, transmission constraints could be ignored, as in England and Wales, and generators and traders would only contract for energy and then submit a dispatch schedule, leaving the system operator to resolve constraints. If constraints are significant, this approach results in inefficient dispatch and perverse incentives for the location of new generators (Kamat and Oren, 2000, Neuhoff, 2002), and creates additional opportunities for generators to exercise market power (Harvey and Hogan, 2000). Third, traders could sign option contracts for energy in one energy market, allowing them to submit bids conditional on moderate prices in the energy market at the other node. Traders would benefit from a share of the savings, if inefficient transmissions are avoided, and would reduce the demand slope, thereby reducing the market power of generators.

1.2 Transmission is constrained as a function of the output choice

In this case market coupling provides an incentive for strategic generators located at the importing region to increase their output and resolve the constraint. If transmission and energy markets are separated, then the amount of imported energy is determined by traders in their price independent bids to the energy spot market and can therefore no longer be influenced by the strategic generator in the spot market. This implies that the strategic generator faces no incentive to increase output to resolve the constraint. Not only does this example show that with partially constrained transmission lines market coupling reduces market power, it also indicates a means of an indirect test of the proposition. Importing generators facing coupled markets are more inclined to resolve the import constraint, the bigger the market at the other end of the constraint. For example, strategic generators in Northern Norway face the relative big Swedish market when they increase output, and therefore import constraints into Northern Norway are rapidly resolved. In contrast, if transmission is constrained from Northern Norway towards Sweden, then Swedish generators could increase output to serve part of the Northern Norwegian market and resolve the constraint. But the smaller market is less attractive and therefore the constraint is retained over longer periods of time. The Netherlands and Germany provide a second example of asymmetric market sizes. However, their markets are not coupled and as predicted by theory, they do not exhibit the same pattern.

1.2.1 Transmission lines are permanently constrained

In the two-node model, the transmitted energy is constant and therefore separate markets and market coupling have the same impact. This is not the case in networks with more than one link, of which a subset is constrained. If energy markets are coupled then transmission capacity will be allocated as a function of the energy bids submitted by generators. Strategic generators will anticipate the reaction of flow patterns on their output decisions when choosing their bids in a nodal pricing regime (Hogan, 1997) If energy and transmission markets are separated, traders determine, by their bids in the transmission auction, how scarce transmission capacity is to be allocated, e.g. how scarce capacity on a link that is used for exports from several regions should be split such that the optimal amount of exports can be executed from each region. [4] This split is fixed at the subsequent energy spot markets, which is an artificial constraint created by the market design. I show that Le Chatelier's principle is applicable even in imperfectly competitive electricity markets. Adding an additional constraint slightly reduces responsiveness of choice variables. One analytical results is, that strategic generators located at one node of any meshed network face a less responsive net demand curve with separate transmission and energy markets and will exercise more market power than under market coupling.

Metzler, Hobbs and Pang (2003) also compare the separated market and market coupling and conclude "Cournot competition among producers yields the same outcomes for two distinct market designs". However, the result requires the assumption that strategic generators ignore the effect of their output choices on transmission prices. If generators own generation assets at more than one node of the network, as in Cardell, Hitt and Hogan (1997), then they face the incentive to change output at one location to influence prices at other locations (Oren, 1997, Chao and Peck, 1996). Numerical methods can be used to simulate the market designs with generators owning assets at several nodes. Ehrenmann and Neuhoff showed via an example of the meshed Benelux network with strategic generators at several locations that market coupling reduces prices (2003).

1.3 Literature Review

Strategic output choices by generators have been represented looking at discrete bidding strategies (von der Fehr and Harbord, 1993), continuous supply functions (Green and Newbery, 1992).or their linear approximation (Day et.al., 2002). However, the most common approach is a Cournot representation of bidding behavior (e.g. by Borenstein and Bushnell (1999) to analyse the potential for market power in California). It is typically difficult to prove the exer-

[4] In a three-node model, this problem could be solved with a flow-gate design. Difficulties arise in complex networks (Ruff, 2001).

cise of market power, as demonstrated by the discussion between Joskow and Kahn (2002) in their simulation for the California's summer of 2000. Their simulated competitive prices were below observed market prices, but Harvey and Hogan (2002) claim the results were sensitive to parameter choices. However, the application of market power mitigation procedures and price auditing indicates that market power is a major concern to electricity regulators. Extending the market power analysis from the simplified spaceless electricity market to electricity networks, Borenstein, Bushnell and Stoft (2000) show that it can be profitable for generators to withhold output in order to constrain a transmission line that would not have been constrained under perfect competition. Borenstein et al. (1996) cite empirical evidence from Northern California to this effect. Oren (1997) presents another scenario with the transmission constraint located between two strategic generators in a three-node network. Stoft (1998) solves the corresponding Cournot game and Joskow and Tirole (2000) give the interpretation: the transmission configuration can turn output of generators at two different nodes into 'local complements', thereby increasing the incentive for a generator to withhold output, as this constrains the output of the other generator and increases price levels. Cardell, Hitt and Hogan (1997) show that, if strategic generators own generation assets at node A and B of a three-node network, they might increase output at node A relative to a competitive scenario if this reduces the total energy delivered to node B due to loop flows and therefore increases prices at node B.

Transmission contracts change their value if the price differences between regions change. If strategic generators own transmission contracts, then they will consider the impact of their output decisions on the value of these contracts (Hogan, 1997). Joskow and Tirole (2000) show that physical and financial transmission contracts provide similar incentives on strategic generators to alter their output decisions. They show that monopoly generators will buy market power enhancing transmission contracts in a discriminatory price auction or inherit them. Gilbert et.al. (2002) extend the analysis to oligopolies and show, for complete information, that uniform price auctions only allow strategic generators to obtain transmission contracts that weakly mitigate market power. However, with asymmetric information and uncertainty this unambiguouse result no longer holds. Furthermore, generators do not sell market power enhancing contracts in secondary markets. Therefore, guidelines are suggested to exclude generators from purchase of contracts that enhance market power.

2 Integrated energy and transmission markets

Assume that the electricity network can be represented by two nodes which are connected with an electric interconnector with limited transmission capacity (Figure 2). This simplified representation captures some features observed between England and Wales, Germany and the

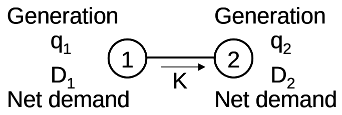

Figure 2: Two nodes interconnected by a transmission line of capacity K.

Netherlands or Sweden and Northern Norway. While throughout the section several restrictive assumptions will be made to simplify the illustration, theorem 1 at the end of the section is general. One strategic generator is located at each node. Both generators have symmetric and constant marginal costs, which are normalized to zero. Capacity constraints of strategic generators are assumed not to be binding.[5]

The remaining (fringe) generators are assumed not to bid strategically. They sell all output that has marginal costs at or below the market clearing price. Fringe generation is therefore increasing with price and can be directly subtracted from demand such that the model only requires residual demand. Residual demand facing strategic generators is price responsive even if demand is rather price inelastic. Residual demand at the exporting node one, D_1, and importing node two, D_2, are assumed to be linear in price p with intercepts A and $A + D$:

$$D_1(p) = A - p, \qquad D_2(p) = A + D - p. \quad D \geq 0 \tag{1}$$

The timing of the game is as follows. First, all generators and demand simultaneously submit one or multiple bids to the energy spot market. A bid is binding and consists of a price and a maximum quantity of electricity offered or requested at a specified node. The demand bid functions (1) are the aggregation of the individual bids of different demands and fringe generators. The two strategic generators are assumed to be Cournot players and offer a specified quantity q_i at price 0 (which equals the normalised marginal costs) at their respective home node i. [6] Given the limited alternatives, the Cournot approximation seems to provide some insights into the difficulties of separate energy markets. It might seem questionable to trust the quantitative results, but the qualitative outcomes seem reasonable as long as demand elasticity is finite at all nodes.

In the second period the system operator (SO) determines market clearing prices for all nodes at which bids and offers are paid. The SO accepts all bids above and offers below the

[5] The working paper proves the results for two strategic generators located at each node with increasing marginal costs (CMI EP16, available at www.econ.cam.ac.uk/electricity).

[6] Generators are allowed to submit multiple bids, such that a generator could also submit a supply function to the spot market. With supply functions multiple equilibria can be calculated, but in the case of no uncertainty and pure strategy equilibria, the equilibrium which is most profitable for both generators is characterized by a infinitely steep supply function - the Cournot equilibrium.

market clearing price of a node (uniform price auction). In order to match supply and demand the SO may have to accept only a fraction of bids or offers that equal to the market clearing price. Traders are not required to arbitrage the spot markets and are therefore not present in this model. Under market coupling they have to focus their activity on the longer-term markets which are not represented in this model, but where their risk hedging activities can provide real value.

How does the SO determine the market clearing prices? She has to ensure that global demand equals global supply and all transmission constraints are respected. These conditions alone do not determine a unique solution. Therefore the SO includes the objective to allocate transmission capacity such that total welfare is maximised. In the process she has to assume that bid functions are competitive and hence represent marginal costs and benefits (Bohn et.al., 1984). In the two node network it is sufficient to assume that the system operator first ignores the transmission constraint and calculates one global market clearing price. The SO calculates how much energy would be transmitted between the nodes at the global market clearing price. If available transmission capacity K is sufficient then the SO implements the global market clearing price p_u as the market clearing price at both nodes. This will be referred to as the unconstrained case.

If the transmission limit is violated, then the SO restricts exports to K and calculates a market clearing price p_1 for node one at which strategic generation q_1 equals residual demand $D_1(p_1)$ and exports K. Likewise the SO calculates p_2 such that strategic generation q_2 and imports K satisfy residual demand $D_2(p_2)$. This will be referred to as the constrained case. (2) summarises the price determination of the SO.

$$\begin{aligned} D_1(p_u) + D_2(p_u) &= q_1 + q_2 & & \text{if } D_2(p_u) - q_2 \leq K \\ D_1(p_1) + K &= q_1 \qquad D_2(p_2) = q_2 + K & & \text{otherwise.} \end{aligned} \tag{2}$$

In what follows, the Nash equilibrium will be calculated for three cases. First generators hold consistent beliefs that the line is unconstrained, then that the line is constrained and finally that it is partially constrained.

2.1 Unconstrained case

Assume both generators anticipate that the link will not be constrained. They expect that the system operator calculates one global market clearing price p_u and therefore the generators behave as a duopoly in the global market (2). Each generator chooses output $q_{i,u}$ to maximise his profits π_u taking the output choice of the other generators $q_{j,u}$, $j \neq i$ as given:

$$\pi_u\left(q_{i,u}\right) = \frac{2A + D - q_{i,u} - q_{j,u}}{2} q_{i,u} \quad i \neq j \tag{3}$$

The optimal response function is $q_{i,u} = \frac{2A+D-q_{j,u}}{2}$ and the optimal output quantity, price and profits are:

$$q_{i,u} = \frac{2A+D}{3} \quad p_u = \frac{2A+D}{6} \quad \pi_{i,u} = q_{i,u}p_u = \frac{1}{2}\left(\frac{2A+D}{3}\right)^2 \tag{4}$$

To ensure that beliefs are consistent, the output choices $q_{i,u}$ have to allow for one global market clearing price p_u. Substituting (4) in the condition of (2) shows that an unconstrained output choice is only feasible if the available transmission capacity K is greater than the average demand difference D between the nodes: $D \leq 2K$.

For an output choice to be an equilibrium, deviations from the choice may not be profitable. With linear demand the first order condition of (3) ensures that infinitesimal deviations are unprofitable. Finite deviations can be profitable for the strategic generator at the importing node because he faces a convex net demand. If he reduces his bid by $\delta q_{2\,u}$ then initially the system operator will increase market clearing price by δp_u such that the reduction of residual demand at both nodes compensates for the output reduction:

$$\delta q_{2,u} = \delta p_u \frac{\partial D_1(p_u) + D_2(p_u)}{\partial p_u}. \tag{5}$$

Increasing p_u while keeping q_1 constant would result in excess supply at node one, therefore the SO also increases exports from node one by $\delta p_u \frac{\partial D_1(p_u)}{\partial p_u}$. If generator two reduces output sufficiently then transmission would exceeds the available capacity K. The SO then has to issue separate market prices for both nodes. Imports to node two stay constant at K and the SO has to increase prices δp_2 at node two to compensate for output reductions.:

$$\delta q_{2,c} = \delta p_2 \frac{\partial D_2(p_2)}{\partial p_u}. \tag{6}$$

The SO can no longer reduce output at both nodes and hence prices are more price responsive to output changes. This is the motivation for strategic generators to induce transmission constraints (Borenstein et.al., 2000). The initial output reductions $\delta q_{2\,u}$ is not profitable, because lost revenue on $\delta q_{2\,u}$ is not compensated for by increased price δp_u charged on the remaining output. However, if δq_2 is reduced sufficiently to create the constraint, then any additional reduction results in larger price increases δp_2 and can make the deviation profitable.

If the potentially import constrained generator chooses output $q_{2,c}$ such that the transmission is constrained his profits and therefore optimal output choice are:

$$\pi_{2,c} = (A + D - K - q_{2,c})\, q_{2,c}, \quad q_{2,c} = \frac{A + D - K}{2}. \tag{7}$$

Such a deviation is profitable if $\pi_{i,u} < \pi_{2,c}$, and substituting from (3), (4) and (7):

$$\frac{3K - \left(3 - 2\sqrt{2}\right) A}{3 - \sqrt{2}} < D. \tag{8}$$

(7) assumes the constraint is binding if generator one bids $q_{1,u}$ and generator two deviates to $q_{2,c}$. The assumption requries $\frac{3K-A}{2} < D$, which is satisfied in all the cases in which (8) holds.

Therefore we can conclude that the unconstrained case (4) is a Nash equilibrium whenever the demand difference D is small enough such that a deviation (8) is not profitable.

2.2 Constrained case

Assume both generators anticipate that the transmission will be constrained. They anticipate that the SO sets separate prices for both nodes and schedules a transmission at full capacity K. They will determine their output as monopolists in their respective markets:

$$\begin{aligned} q_{1,c} &= \frac{A+K}{2} \qquad p_{1,c} = \frac{A+K}{2} \qquad \pi_{1,c} = \left(\frac{A+K}{2}\right)^2 \\ q_{2,c} &= s\frac{A+D-K}{2} \quad p_{2,c} = \frac{A+D-K}{2} \quad \pi_{2,c} = \left(\frac{A+D-K}{2}\right)^2 \end{aligned} \tag{9}$$

The SO can only schedule flows at full capacity if the price at node two is equal to or above price at node one, which is the case for $2K < D$. As before the output choice is robust to local deviations, but it has to be checked whether finite deviations made by generator two are profitable and feasible. If generator two increases output then initially the SO reduces $p_{2,c}$ to increase residual demand at node two. If the output increase $\delta q_{2,c}$ is large enough, the prices at both nodes equalize. With the additional output increase $\delta q_{2,u}$ generator two also replaces some imports and therefore faces the joint demand response of both nodes. Price falls less with additional output (6). If the revenue on additional output exceeds the losses due to the price reduction, then the finite deviation is profitable. Generator two makes profits:

$$\pi_{2,c,d}(q_{2c,d}) = \frac{2A+D-q_{1,c}-q_{2,c,d}}{2} q_{2,c,d}.$$

and therefore optimal output choice for the deviation is:

$$q_{2c,d} = \frac{3}{4}A + \frac{1}{2}D - \frac{1}{4}K \qquad p_{2,c,d} = \frac{2A+D-q_{1,c}}{4}. \tag{10}$$

The deviation is profitable, if $\pi_{2,c} < \pi_{2,c,d}$:

$$D < \frac{\left(1/2-\sqrt{2}\right)K - \left(3/2-\sqrt{2}\right)A}{1-\sqrt{2}}. \tag{11}$$

The implicit assumption in (10) of an unconstrained transmission is satisfied if exports from node one after the deviation are smaller than available transmission capacity $q_{1,c} + p_{2,c,d} - A < K$. This condition is always satisfied if (11) is satisfied.

The constrained case (9) is a Nash equilibrium, if D is large enough such that deviation is not profitable (11).

2.3 Partially constrained case

The previous sections could identify neither an equilibrium with constrained (11) nor with unconstrained transmission (8) for a non-empty set of D :

$$D\epsilon \left] \frac{3K - \left(3 - 2\sqrt{2}\right) A}{3 - \sqrt{2}}, \frac{\left(1/2 - \sqrt{2}\right) K - \left(3/2 - \sqrt{2}\right) A}{1 - \sqrt{2}} \right[. \tag{12}$$

For a D from this set only mixed strategy equilibria are possible. Stoft (1998) gives a different channel which can make a pure strategy equilibrium infeasible.[7] The policy implications still follow and the empirical test is still applicable.

One possible mixed strategy Nash equilibrium is the following: Generator two located at the importing node chooses, with probability ρ a low output quantity $q_{2,m,c}$ such that the constraint will be binding and with probability $(1-\rho)$ a high output quantity $q_{2,m,u}$ such that the constraint will be relaxed. Generator one cannot observe the bid of generator two and can therefore not submit a contingent bid. He always bids $q_{1,m}$. These four variables $\rho, q_{2,m,c}, q_{2,m,u}$ and $q_{1,m}$ are uniquely defined by the following four conditions.

First, generator two chooses $q_{2,m,c} = q_{2,c}$ to maximise profits in the constrained case (9). Second, generator two chooses $q_{2,m,u}$ to maximise profits in the unconstrained case:

$$\pi_{2,m,u} = \frac{2A + D - q_{1,m} - q_{2,m,u}}{2} q_{2,m,u}, \qquad q_{2,m,u} = \frac{2A + D - q_{1,m}}{2}. \tag{13}$$

Third, generator one chooses the profit maximising output $q_{1,m}$. The expected price is the weighted average of constrained and unconstrained price and therefore expected profits are:

$$\pi_{1,m}\left(q_{1,m}\right) = \left(\rho\left(A + K - q_{1,m}\right) + (1-\rho)\frac{2A + D - q_{1,m} - q_{2,m,u}}{2}\right) q_{1,m}$$

The response function is therefore:

$$q_{1,m} = \frac{\rho\left(A + K\right) + \frac{1-\rho}{2}\left(2A + D - q_{2,m,u}\right)}{1 + \rho}. \tag{14}$$

Fourth, generator two choosing ρ to make generator one choose $q_{1,m}$ such that generator two makes the same profits for both of his output choices $\pi_{2,m,u} = \pi_{2,c}$ and can credibly mix strategies.

[7] In his three-node network, the output of two generators is complementary, because flows of electric energy can be superimposed and therefore cancel each other. The high-cost generator's output relieves the transmission constraint which the low-cost generator faces. This allows the high-cost generator to limit the low-cost generator's output and, by creating a constraint, receive higher prices for his output under a nodal pricing design. The high-cost generator reduces output to keep the constraint binding and to obtain high prices, while the low-cost generator would like to reduce output to relax the constraint and obtain high prices. The low-cost generator can only do so by mixing output strategies to ensure that the high-cost generator does not constrain his output all of the time.

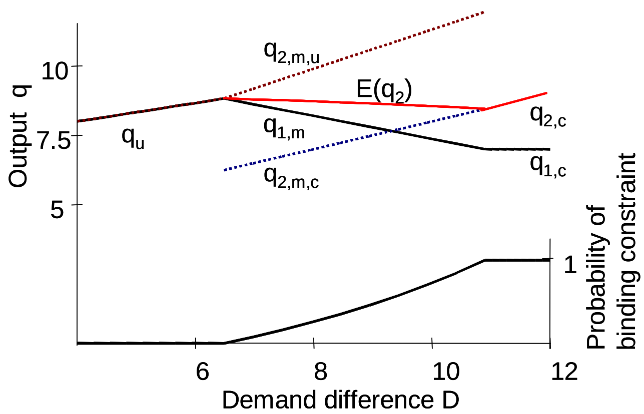

Figure 3: Output of generators as function of demand difference (A=10, K=4)

Substituting from (9) and (13) gives:

$$q_{1,m} = 2A + D - q_{2,m,u} - 2\frac{q_{2,m,c}}{q_{2,m,u}}\left(A + D - K - q_{2,m,c}\right). \tag{15}$$

Solving (9), (13), (14) and (15) gives:

$$\begin{aligned} q_{1,m} &= 2A + D - \sqrt{2}\left(A + D - K\right) & q_{2,m,c} &= \frac{A+D-K}{2} \\ \rho &= \frac{2A + D - \sqrt{2}\left(A + D - K\right)}{2K - 4A - 3D + 3\sqrt{2}\left(A + D - K\right)} & q_{2,m,u} &= \frac{A+D-K}{\sqrt{2}} \end{aligned} \tag{16}$$

As in the previous sections we have to check that the constraint is binding when generator two plays $q_{2,m,c}$. Substituting from above shows that $p_{1,m,c} < p_{2,m,c}$ is always satisfied. Likewise the constraint has to be relaxed if two chooses $q_{2,m,u}$. It can easily be shown that this is the case for $A + D > K$, a condition always satisfied. Generator two already chooses outputs for the constrained and unconstrained case, therefore we do not have to check for possible deviations from his side.

Figure 3 illustrates the output of individual generators for various levels of demand differences. For demand differences $D \leq 6.49$ generators chose output q_u in the unconstrained scenario. For $6.5 < D < 10.9$ generator two plays a mixed strategy. For example, for $D = 8$ generator two chooses $q_{2,m,u}$ with probability $\rho = 0.26$ and otherwise chooses $q_{2,m,c}$.

The probability ρ of a binding constraint increases with D and therefore the expected demand slope faced by generator one increases. Hence generator one decreases $q_{1,m}$ with D. Because ρ increases, the expected output of generator two $E(q_2) = \rho q_{2,m,c} + (1 - \rho)\, q_{2,m,u}$ decreases with increasing D. Only after the constraint is permanently binding for $10.9 \leq D$ will the output of generator two increase again with D.

The mixed strategy equilibrium (16) describes a Nash equilibrium for strategic generators in the domain when neither the constrained nor unconstrained output choice represent an equilibrium.

3 Separate energy and transmission markets

A potential benefit of electricity liberalisation could be closer coordination of electricity dispatch between neighbouring regions to exchange flexible generation capacity, smooth out volatility of intermittent generation and to balance hydro and demand cycles. However, electricity markets are still separated in different regions or European countries, mainly because liberalisation inherited existing structures. These regional markets are then arbitraged by traders buying and selling electricity and transmission rights. In the simplified model, the historic evolution which explains and potentially justifies such a market design is obviously not represented. In comparison to market coupling the chief difference therefore is that it is the traders, rather than the centralised system operator, that arbitrage the markets. I will now assess the implications for competition among generators.

To simplify the model it is assumed that traders expect zero profits, either because the market is contestable or because the number of traders is large. Only traders, and not generators, buy transmission contracts. If generators buy transmission contracts, they experience financial incentives that influence their dispatch decisions, as shown by Joskow and Tirole (2000). This paper therefore complements Gilbert et.al. (2002), which evaluates policy guidelines for generators' access to transmission contracts, while ignoring the effects resulting from separate energy and transmission markets.

As illustrated in Figure 4, the system operator first auctions transmission contracts to traders. Assuming complete information and perfect arbitrage, traders pay the marginal value p_k for transmission contracts t, in both a uniform price or a discriminatory price auction. If they paid less, additional traders would find it profitable to enter the market. If they paid more, then they would make losses and some traders would exit the market. The marginal value will be zero if transmission supply exceeds demand.

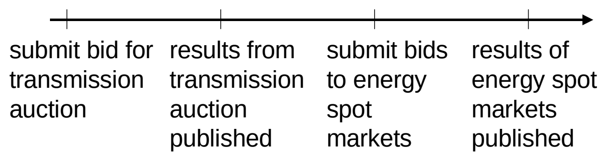

Figure 4: Timeline of day ahead market for separate energy and transmission markets.

In step three, generators and traders submit their bids to the energy spot market. Following the Cournot assumption, strategic generators at node i submit a quantity bid $q_{i,s}$ to the energy spot market at their node. Likewise, each trader must decide what fraction of his transmission rights t to use for energy transmission. In the model with full information the transmission auction does not reveal additional information, hence traders will only buy the quantity of transmission rights they will subsequently require to arbitrage the markets. They will submit a quantity bid to buy t units of energy at the exporting node and sell t units at the importing node. The assumption that traders submit quantity bids which are price independent is based on the model of continental power exchanges as currently implemented in Germany and the Netherlands. In these power exchanges, all bids must pay the market clearing price. Traders will submit a very high-priced buy bid for t in the exporting country and a very low-priced sell bid for t in the importing country. This ensures that both bids will be accepted and corresponds to the Cournot model. It is important that either both bids or neither bid are accepted, otherwise traders have an open energy position and are exposed to high imbalance fees.[8]

The equilibrium is calculated backward, using the previous demand and generation assumptions. The market clearing prices at both nodes are:

$$p_1 = A + nt - q_{1,s} \qquad p_2 = A + D - q_{2,s} - nt \tag{17}$$

The optimal response function for generators at both nodes is therefore:

$$q_{1,s} = \frac{A + nt}{2} \qquad q_{2,s} = \frac{A + D - nt}{2}. \tag{18}$$

If sufficient transmission capacity is available, then competitive traders schedule flows such that the prices at both nodes will be arbitraged. Substituting (17) and (18) in $p_1 = p_2$ gives:

$$nt = \frac{D}{2} \qquad q_{1,s} = q_{2,s} = \frac{A + D/2}{2}.$$

Transmission capacity is not scarce, and therefore traders will obtain $k = t = \frac{D}{2n}$ units of transmission capacity in the transmission auction at price 0.

If $\frac{D}{2} > K$ then each trader can only buy $t = K/n$ transmission rights and bid accordingly in the energy markets. Equilibrium prices and quantities in the energy markets are then identical to (9). Transmission capacity is scarce and therefore competitive traders will pay a positive price for rights in the transmission auctions which corresponds to the subsequent revenue from arbitraging the energy markets (9):

$$p_k = p_{2,c} - p_{1,c} = \frac{D}{2}.$$

[8] A trader with energy contracts in only one zone might still try to use bilateral negotiations to provide for the second energy contract. However, he is under significant time-pressure and therefore in a bad bargaining position, and is likely to obtain an unsatisfactory price.

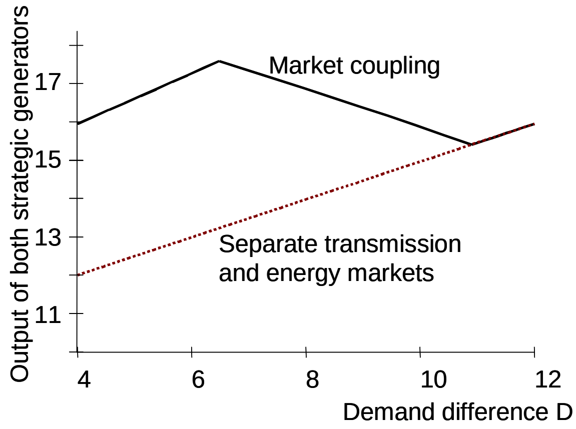

Figure 5: Total production of strategic generators as a function of demand difference between nodes.

Figure 5 compares total expected production under market coupling with separate transmission and energy markets. The separation enhances generators' market power and results in lower output quantities. Only if demand difference between the two nodes exceeds $D > 10.9$ and the transmission constraint is permanently binding, will the output choices not differ in the two market designs. In the separate markets, the entire transmission capacity will already be used for $D > 8$; this does not show up in total output, because separation of energy spot markets has the same effect as if the link were always constrained.

The model is built on the assumption that generators cannot condition their bids at one node on the market clearing price at the other node. This assumption could be questioned in three different ways. First, simultaneously clearing markets at both nodes could allow traders to submit bids conditional on the market clearing price in the neighboring node. However, if such close cooperation is feasible, why not go for an integrated market which ensures that arbitrage is perfect? Second, energy spot markets can close sequentially. Indeed, the German spot markets opens later than the Netherlands spot market. If Netherlands results are, as is frequently the case, announced before closure of the German spot market, traders could submit bids to the Netherlands auction and condition their participation in the German auction on success in the Netherlands market. Such a conditioning might improve the situation in the two-node case; however, if the auctions are expanded to several countries, sequential energy spot markets are difficult to implement. The third approach towards conditional bids is continuous trading. In theory, continuous trading allows traders to continuously adapt their positions. However, liquidity is typically low in very short-term energy markets and therefore, at least according to UK reports, traders are reluctant to go with a big position into this market.[9]

The following theorem proves the benefit of integrating energy and transmission markets in the unconstrained case. The example in Figure 5 shows integration also has a positive effect if the transmission is partially constrained in a mixed strategy equilibrium. ($6.5 < D < 10.9$). Integration has no effect for $D > 10.9$.

Theorem 1 *If the transmission line between two nodes is unconstrained, integration of nodal energy markets, which are only arbitraged by traders, increases output and decreases prices (assuming variable costs are convex and demand is elastic at both nodes).*

Proof. *Traders determine the amount of transmission, so that prices at both nodes coincide. $P_i(Q_i)$ is the inverse net demand function faced by generators at each node and Q_i equilibrium production by generators at node $i = 1, 2$. We are now assessing output at node one, but the argument equally applies to node two.*

If energy and transmission markets are separated, output quantity q_m of generator m at node one equals the choice q that maximises profits $\pi_m(q) = P_1(Q_1 - q_m + q)q - C_m(q)$; therefore, the FOC and $q = q_m$ give the result that marginal revenue equals marginal costs:

$$P_1(Q_1)' q_m + P_1(Q_1) = C_m'(q_m).$$

Assume generators choose the same output quantities in the integrated market, with total inverse net demand $P_{int}(Q)$. As demand and supply are identical so are the prices of the arbitraged markets $P_1(Q_1) = P_2(Q_2) = P_{int}(Q_1 + Q_2)$. Aggregated demand respond exceeds individual demand responses: $P_{int}'(Q_1 + Q_2) > P_1'(Q_1)$. Therefore, marginal revenue exceeds marginal costs:

$$P_{int}(Q_1 + Q_2)' q_m + P_{int}(Q_1) > P_1(Q_1)' q_m + P_1(Q_1) = C_m'(q_m),$$

and generators with convex costs increase output q_m to maximise profits. This reduces prices in the integrated transmission and energy market. ■

4 A test for the theory

The theoretical model showed that strategic generators submit different bids under market coupling and separate energy and transmission markets. The results are only useful if the model captures the bidding strategies of real generators despite the simplifications, e.g. ignoring that energy spot markets are repeated daily.

In this section a test of the model is proposed: If generators bid according to the model, then electricity spot prices should exhibit a specific asymmetry under market coupling which

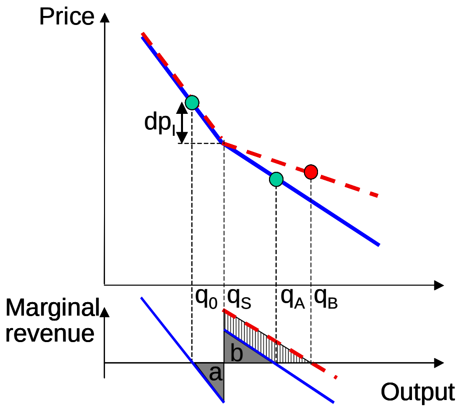

Figure 6: Effective demand generator faces at importing node with market coupling

is not present in separate energy and transmission markets. Figure 6 illustrates a situation where residual demand at two nodes is such that a strategic generator at the importing node is indifferent between choosing output quantity q_0, which results in the import constraint, and output quantity q_A for an unconstrained case. Let demand at the importing node be D_2, demand elasticity be ε, output choice be q and price be p. Setting marginal profits of changing output in the constrained case to zero $\frac{\partial \pi_{const}}{\partial q} = 0$ defines the constrained output choice q_0:

$$\frac{\partial \pi_{const}}{\partial q} = p\left(q\right) - q\frac{dp\left(q\right)}{dq} - c(q) = p + \frac{pq}{D_2}\varepsilon - c(q) \qquad \frac{\partial \pi_{const}}{\partial q}|_{q0} = 0. \tag{19}$$

Assume for simplicity that demand elasticity is equal at both nodes and demand at the exporting node is D_1. Then in the unconstrained case the marginal profits and optimal output choice are:

$$\frac{\partial \pi_{unconst}}{\partial q} = p + \frac{pq}{D_1 + D_2}\varepsilon - c(q) \qquad \frac{\partial \pi_{unconst}}{\partial q}|_{qA} = 0. \tag{20}$$

The strategic generator not only serves the home market, but his output increase also replaces imports. The strategic generator effectively serves the joint market. The price response of the larger market to output changes is smaller, as expressed by the sum of D_1 and D_2 in the denominator, therefore for the same, hypothetical, pair of q, p it would follow that $\frac{\partial \pi_{unconst}}{\partial q} > \frac{\partial \pi_{const}}{\partial q}$ and therefore the unconstrained equilibrium is achieved for a larger $q_A > q_0$ allowing under some circumstances for two profit maximising output choices.

The output choice preferred by a strategic generator will depend on which provides for larger profits. Assume that demand is such that the strategic generator is indifferent between either output choice and $\pi_{qA} - \pi_{q0} = 0$:

$$\pi_{qA} - \pi_{q0} = \int_{q0}^{qS} \frac{\partial \pi_{const}}{\partial q} dq + \int_{qS}^{qA} \frac{\partial \pi_{unconst}}{\partial q} dq = 0. \tag{21}$$

[9] Stated at Regulation Initiative Workshop, "How well is NETA doing?", LBS, October 17, 2001

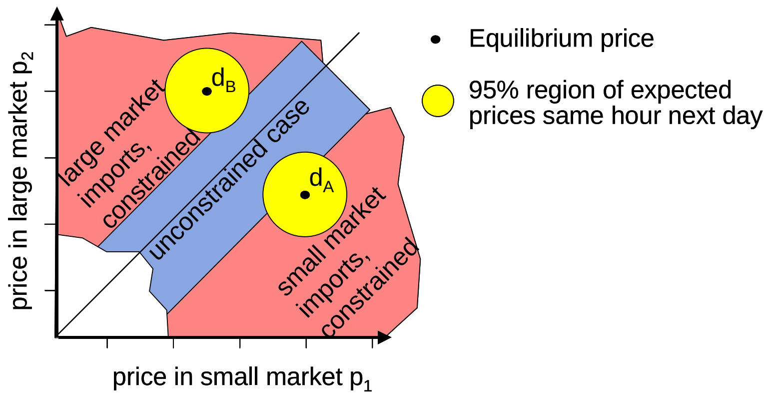

Figure 7: Assymetric incentive to resolve constraint results in assymetric probability of import and export constraints to be resolved.

Now assume the exporting market with all generators and demand is scaled by the factor $r > 1$. The marginal profits (20) change to:

$$\frac{\partial \pi_{unconst,r}}{\partial q} = p + \frac{pq}{rD_1 + D_2}\varepsilon - c(q) > \frac{\partial \pi_{unconst}}{\partial q} \quad r > 1. \tag{22}$$

This also implies, that for q_A with $\frac{\partial \pi_{unconst}}{\partial q}|_{q_A} = 0$ it follows that $\frac{\partial \pi_{unconst,r}}{\partial q} > 0$. At the profit maximising unconstrained case with $\frac{\partial \pi_{unconst,r}}{\partial q}|_{q_B} = 0$ it follows that $q_B > q_A$.

Therefore a strategic generator facing the scaled exporting market r changes his profits as follows when shifting from the constrained to the unconstrained case, using inequality (22) and profit change (21):

$$\begin{aligned}\pi_{qB} - \pi_{q0} &= \int_{q_0}^{q_S} \frac{\partial \pi_{const}}{\partial q} dq + \int_{q_S}^{q_A} \frac{\partial \pi_{unconst,r}}{\partial q} dq + \int_{q_A}^{q_B} \frac{\partial \pi_{unconst,r}}{\partial q} dq \\ &> \pi_{qB} - \pi_{q0} + \int_{q_A}^{q_B} \frac{\partial \pi_{unconst,r}}{\partial q} dq = \int_{q_A}^{q_B} \frac{\partial \pi_{unconst,r}}{\partial q} dq > 0.\end{aligned}$$

If the exporting node is larger, then it is more profitable to increase output for a strategic generator at the importing node. This implies that a larger price difference dp_I between the import constrained node and the exporting node is required to ensure that a deviation from the constrained case is not profitable. This is depicted in Figure 7. The region where the import constraint into the larger market is binding is bigger.[10]

To assess an asymmetry that can be empirically tested, compare the two price pairs d_A and d_B for two hourly day ahead prices. In both cases the transmission constraint is binding and

[10] This would imply that, in an integrated market, prices between nodes are either equal or differ significantly, while small price difference between the markets should not exist. This interval should differ for import and export constraints. In contrast, price differences between separated energy markets should not exhibit such a gap. I did not explore this route, but used a method derived from this initial idea.

the price difference between the markets is equal in size but with inverted direction. How will demand and supply evolve towards the next day? Since the transmission constraint is binding, therefore only changes in local demand and supply will affect the price changes. Between the same hour of consecutive days these changes in local supply and demand are typically small and therefore the local price changes are likely to be small as well. As illustrated by the circles around d_A and d_B in Figure 7, the price pair for the same hour of the following day is likely to stay in the vicinity. If the larger market is importing and the smaller market exporting, then it is likely that the price difference between the markets will remain sufficiently large to ensure that strategic generators in the importing market do not deviate towards an unconstrained scenario. It is very likely that the price difference remains positive and the transmission constraint binding the following day. In contrast, if the small market is importing, then in a significant number of cases, the price difference between the markets can frequently drop below the level at which strategic generators at the importing node will find it profitable to increase output to face the larger, combined market. Then the probability is higher that prices will be equalized and transmission unconstrained.

Summarising, we can say for market coupling: If the smaller market is importing with price difference Δp_I between the markets, then the price difference is more likely to be annulled the same hour next day than if the small market is exporting with price difference $-\Delta p_I$.

This asymmetry does not appear if transmission and energy markets are separated. If the strategic generator varies his output the amount of imports stays constant, the non-convexity in the net demand function of Figure (6) vanishes, (19) and (4) coincide and the demand response and therefore size of the exporting market does not impact the stability of the import constraint.

5 Empirical evidence

Data from the Netherlands-Germany interconnector show that traders arbitraging separated transmission and energy markets face uncertainty and can not condition their bids on all information that is revealed in the markets hence the arbitrage is inefficient. In 5.2 the theoretical predictions are tested by comparing the Netherlands-Germany interconnector with the interconnection between Sweden and Northern Norway. The results do not contradict the hypothesis that market coupling reduce market power of generation companies in comparison to separate transmission and energy markets.

5.1 Incomplete arbitrage at the Netherlands-Germany interconnector

Transmission rights to the interconnector are auctioned in annual and monthly auctions for the entire time span and in day-ahead auctions for each hour separately. I focus on the day-ahead auction, where traders must submit their bids by 8.30am and receive confirmation of the results by 9am.[11] Traders then submit bids to the Netherlands power exchange APX by 10.30am and to the German power exchange LPX, which now includes the EEX, by 12 noon. The Netherlands power exchange commits itself to publishing the results by 12 noon, implying that in effect the markets clear simultaneously.[12]

In Figure 8, the spot price difference between the Netherlands and Germany is depicted as a function of the (positive) day-ahead auction prices for each hour in the period January 2001 to June 2002. For all the observations left of the dashed line, the price paid in the transmission auction exceeded the revenues subsequently obtained in the energy markets.

The large variation of the spot price difference for any one price paid for transmission rights shows that arbitrage is only based on the expected prices. If traders could anticipate the real price difference, transmission prices would never exceed the price difference between the two markets.

The figure only represents prices below 25 Euro/MWh, falsely creating the impression that traders lose more than profit from trading. Including all observations with positive transmission prices shows that traders' average profits from the combined interaction in transmission and energy market equal 1.56 Euro/MWh plus 0.5 times the price paid in the transmission auction. This indicates insufficient competition among traders, allowing them to bid low in the day-ahead auction to secure capacity at below its arbitrage value, thereby increasing trading profits. In 2001, a very unsophisticated strategy of using all transmission contracts bought for a positive price in the auction to transmit energy from the German spot market to the Netherlands market

[11] Transmission rights to and from the Netherlands can be obtained in two separate auctions, starting in the grid of two neighbouring German utilities RWE or E.ON. The analysis is based on the average of both, because so far both rights are perfect substitutes, as traders are not exposed to transmission constraints within Germany. Rights are auctioned separately for both directions in monthly and daily auctions. To avoid abuse of transmission rights, they must be used or returned to the auction to allow for re-use.

[12] However, traders report that the Netherlands power exchange frequently clears earlier. If traders anticipate obtaining the APX results before 12, they could condition their bids to the German power exchange on the Netherlands results. This represents a potential integration of transmission and energy markets and would therefore result in higher probability of the import constraint into the Netherlands being resolved, which is not observed.

The continuous trading platform Xetra of LPX is not included in the analysis, first, because trading volume is only 10-15% of total day-ahead trading volume at LPX and, second, because trading closes at 12 noon. Additional trading opportunities would only improve the situation in the period 9am to 10.30am because bids to APX must be submitted after that time.

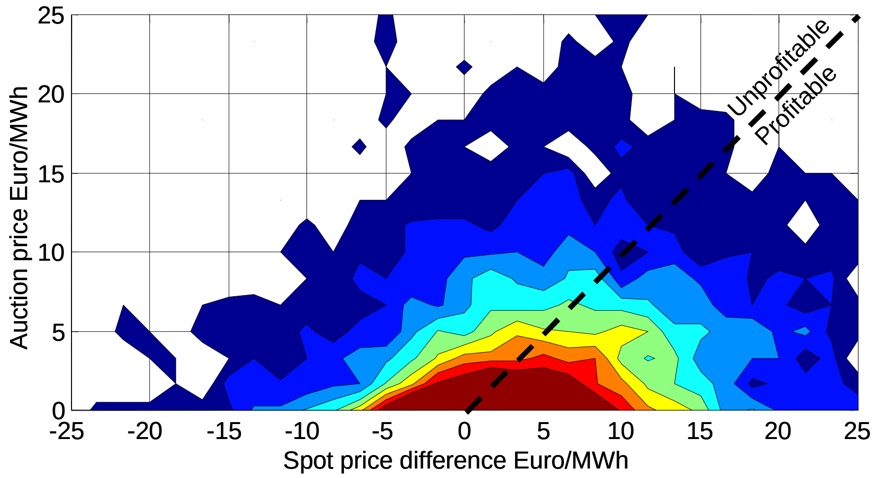

Figure 8: Spot price difference (Netherlands-Germany) observed after realisation of positive day-ahead auction prices in the period January 2001 to June 2002. Colour coding corresponds to the number of hourly observations in 1 Euro/MWh.

created arbitrage profits of 30.6m Euros. These high profits must have attracted additional traders and increased competition, reducing profits to 1.2m Euros for the first six months of 2002. This is a lower limit for the transfers from consumers and generators to traders, and could be higher if traders used more sophisticated trading strategies. Borenstein et.al. (2001) observe a similar delay of "no more than a couple of months", during which price differences between the (day-ahead) future energy market and the spot energy market persisted, until traders learnt how to deal with a rule change.

Even if the markets are arbitraged on expectation, the main disadvantage of the separation of transmission and energy markets still remains. In all the hours which are represented on the left hand half of Figure 8, traders paid a positive price in the transmission auction at 9am and therefore probably bid later in the morning on the energy spot markets to trade energy from Germany to the Netherlands. However, in these cases the spot price in the Netherlands turned out to be lower than in Germany. Assuming the spot markets are efficient and represent variable costs of the marginal generator, this implies that low-cost generators in the Netherlands are replaced by higher-cost generators in Germany. This effect did not change with improved arbitrage; Figure 8 does not differ from a separate plot of 2001 or 2002.

The reason for this inefficiency is that traders cannot predict the spot prices because of uncertainty and because private information is only aggregated in the spot market. Usually, spot markets are specifically introduced to reveal private information; it is therefore inconsistent to introduce a decentralized mechanism for decisions on energy transmission, which can only work efficiently if traders correctly predict spot market prices.

Output generated by wind, solar and combined heat and power generators is not predictable long-term, and information is only aggregated in the spot market. Therefore, a higher contribution by these energy sources will increase the inefficiency of the separate energy and transmission market. The separation is also biased against intermittent generation. Imagine that traders anticipate low generation in the Netherlands and therefore schedule imports. If the spot market reveals high (renewable) generation, the price will fall below the German price and renewables will receive low revenues. If transmission and energy markets were integrated, exports would be scheduled instead of imports and Netherlands renewables would receive the higher German electricity price (assuming transmission is not constrained).

5.2 Comparison with Nordpool

I test the theoretical claim that integrating energy and transmission markets reduces market power, using hourly data from January 2000 to November 2001. If true, in an integrated energy and transmission market such as Nordpool, the probability that an import constraint into the small country with a larger demand slope will be resolved by the same hour of the next day should be higher than the probability that an export constraint will be resolved. Under separate markets, e.g. the Netherlands-Germany interconnector, both probabilities should be identical.

Figure 9 shows the member countries of Nordpool. Sweden and Finland each constitute one zone in the initial market splitting, while Norway and Denmark are split up into several zones to address internal transmission constraints. Discussion of market power in Sweden goes back to Anderson and Bergman (1995). Johnsen, Verma and Wolfram (1999) identify market power in Norway if transmission constraints are binding. I will focus on Northern Norway NO2 because it represents a two-node model with the major interconnection to Sweden by transmission links (1) with capacity of more than 1000MW. Interconnection (3) towards Southern Norway NO1 is comparatively small, at only 300MW, and exhibits almost identical behavior because Southern Norway is well-integrated with the Swedish market. Northern Norway is sometimes split up in two separately priced zones, Tromsø and Trondheim, but prices in both zones behave almost identically; therefore, only results for Trondheim are presented. Concentration in Northern Norway is high, with Statkraft owning 3002MW of 6287MW installed capacity.[13]

Figure 10 shows that in Northern Norway there is a higher probability that an import constraint is resolved by the same hour of the next day than an export constraint for all price differences $|\Delta p|$. The observation confirms the predictions of Section 4.

[13] Norwegian Competition Authority 2002, published in context of enquiry into acquisition of Trondheim Energiverk (TEV) by Statkraft.

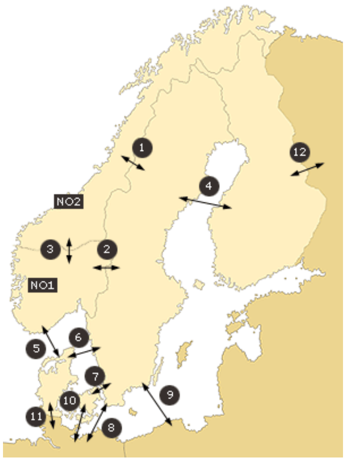

Figure 9: Different zones of Norpool to which market splitting is applied. Connection 1 between Northern Norway NO2 and Sweden represents the two-node model and is frequently constrained.

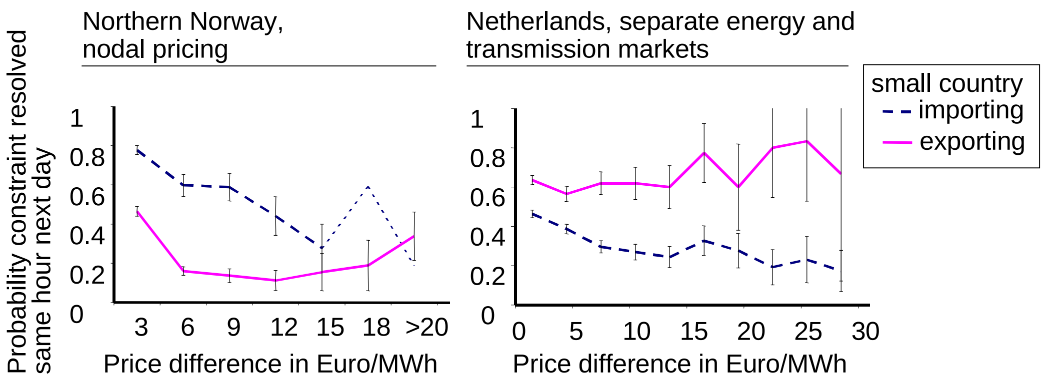

Figure 10: Observed probabilities that a price difference $|\Delta p|$ will disappear by the same hour of the next day, for the small market importing and exporting. (Northern Norway and Netherlands)

Figure 10, right, shows that in the Netherlands there is a lower probability that an import constraint will be resolved than that an export constraint will be resolved. This is in contrast to the previous Section, which suggests equal probabilities for both events in the case of separate markets, while it certainly rejects the predictions for integrated markets. The deviation from the predicted result is because the assumption that price changes are independent from the price level is not justified.

Figure (11) shows that prices in all countries are mean-reverting. In the case of the integrated markets, the Swedish price is slightly more mean-reverting; therefore, high Swedish prices should drop faster, and therefore constraints when the small country is exporting should be resolved with higher probability. Mean reversion would have resulted in the opposite from the observed effect and only reinforces the results of the integrated market. The prices in the Netherlands are more

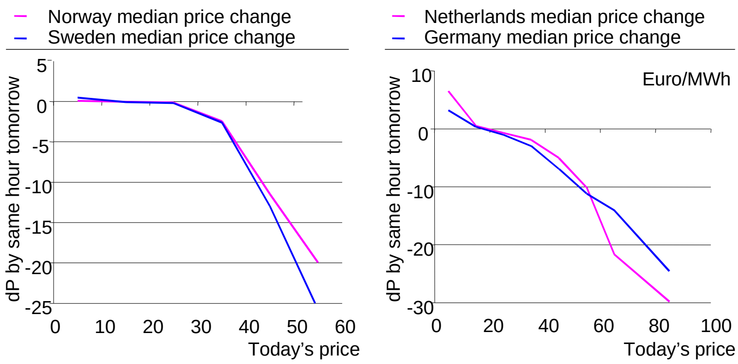

Figure 11: Mean reversion of day ahead spot prices

strongly mean-reverting than those in Germany; we would therefore expect a higher probability that the import constraint into the small country will be resolved. Once again, mean reversion on its own does not explain our result. However, a second difference is that the average price in the Netherlands in the observation period was 30.53 Euro/MWh, while the German average price was 23.41 Euro/MWh. If the Netherlands were exporting to Germany, the price in Germany must have been far above its average price. Therefore the probability that the price in Germany would drop and the constraint would be resolved was quite high, due to mean reversion. This could explain why export constraints from the Netherlands are resolved with higher probability than import constraints. In the Scandinavian case, the average prices in Northern Norway and Sweden were very similar, 17.57 and 18.17 Euro/MWh and in opposite direction to the observed effect. Therefore, mean reversion in combination with average price levels explains the deviations from equal probabilities expected with separate markets in the Netherlands-Germany case while it reinforces the results for the integrated markets in the Northern Norway-Sweden case.

I use the asymmetry between import- and export-constrained situations to eliminate other effects that might distort the results. These other effects can be observed when comparing deviations from the export-constrained scenario in Northern Norway with deviations in the Netherlands. Significantly, the higher probability of all deviations in the Netherlands can be explained by higher price volatility in the Netherlands, illustrated in Figure 12 (For a systematic comparison, see Bower, 2002). Nordpool prices are generally more stable, because their main determinant is the water level in hydro storage, which evolves slowly, as Johnsen, Verma and Wolfram (1999) argue.

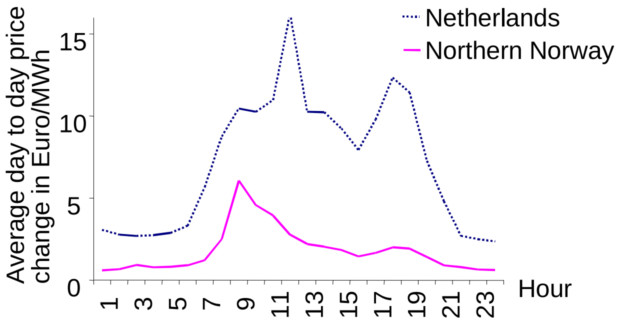

Figure 12: Price volatility in the Netherlands and Northern Norway

One effect that one might expect to distort the analysis is the different generation patterns. Northern Norway generates electricity exclusively from hydro power. Production is sometimes constrained by generation capacity and sometimes by the energy stored in the dams. The Netherlands mainly use coal and gas. However, the analysis compares the same hour of consecutive days and should therefore not 'pick up' differences between peak and off-peak price determination. Using the asymmetry between deviation from imports and exports finally ensures that generation technology and demand patterns that are independent from power flows on the interconnector are filtered.

6 Network effects

In a simple two-node network with a single link, all power from one node must flow along the single link to the other node. In a meshed network with more than one possible path from one node to another, electricity will flow over all links, distributed according to Kirchoff's Laws (Bohn et. al. 1984). Thus in Figure 13, a generator at node two may sign a contract to deliver power to a consumer at node three, and then seek to sign a contract with the transmission operator of the most direct link, $\overline{23}$, but only some of the power will actually flow along this link, with the balance creating 'loop flows' along all other paths connecting the source (the generator) to the sink (final consumer), in this case along $\overline{21}$ and $\overline{13}$. These loop flows bedevil the management of interconnected transmission systems, in which various sub-grids of the interconnected system are under the jurisdiction of separate Transmission System Operators (TSOs). One direct consequence of these loop flows is that a transmission constraint on one link impacts on the flows that are possible on every electricity transmission link in the network. Two different approaches have been proposed to explicitly address transmission constraints and to allocate scarce transmission capacity in a liberalised electricity market: property rights for separate transmission markets and nodal prices.

First, property rights allocate physical transmission capacity either on a constrained link (flow-gate rights), for transmission between two locations (point-to-point contracts) or insertion and withdrawal at specified locations (entry/exit rights). Flow-gate rights require that any energy trade be matched with individual property rights for each transmission constraint in the network, and therefore do not seem feasible in most real networks (Hogan, 2000). A discussion of flow-gate rights is nevertheless helpful to understand alternative designs that are theoretically more difficult but in practice easier to handle. Point-to-point contracts and entry/exit rights aggregate the underlying information to increase liquidity and facilitate trading, but require a central system operator to define the aggregated rights based on the fundamental flow-gate rights.

In the flow-gate design, the system operator calculates proportionality factors γ_{ij}^k to determine

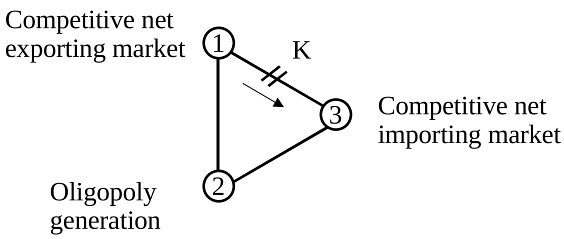

Figure 13: Symmetric 3-node network with a single constraint

what proportion of energy flow between injection node i and offtake node j will pass over link k. The proportionality factor γ_{ij}^k is negative if the energy flow goes in the opposite direction to the defined orientation of the link.[14] A trader m multiplies the power volumes q_{ij}^m (positive amount of MW)[15] he wants to transmit between different nodes with the corresponding proportionality factor γ_{ij}^k. This determines how many flow-gate rights he must obtain for each link $f^{k,m} = \gamma_{ij}^k q_{ij}^m$. The system operator can issue or auction (O'Neill et. al. 2000) a net amount of flow-gate rights $\sum_m f^{k,m}$ up to the capacity K^k of the link, and market participants can subsequently trade

[14] Orientations are determined arbitrarily, and these will determine the signs of the factors γ_{ij}^k and hence the consistency of the flow analysis.

[15] measured in MW. We define a unit of time during which flows are constant, and the energy is then MW multiplied by the time interval, taken here as 1 unit.

these flow-gate rights.

In the second approach, nodal pricing, generators, traders and consumers submit energy bids to a system operator, who is the central auctioneer. Each bid specifies a location in the network, a quantity of energy to be offered or requested and a price. The system operator determines the market clearing price at each node, by effectively simulating a market for flow-gate rights.[16] Generators receive the nodal price of their injection point while consumers pay the nodal price at the off-take point. Nodal pricing can be interpreted as an interface to simplify the underlying market structure and reduce transaction costs to match physical transmission contracts to energy delivery.

What is the effect of separation of transmission and energy markets in the presence of market power? Separate markets imply that the configuration of transmission rights determines which energy flows traders must schedule. Flows are therefore no longer a function of changing bids of generators in the energy spot market.[17] Integrating the markets therefore reduces the slope of the demand curve (7). Figure 14, left, illustrates that integration with changing demand slopes does not change outcome of competitive markets because output is only determined by intercept of demand and marginal costs, not the slope. By contrast, generators with market power determine their output based on demand slope, and a decrease in demand slope results in higher output (**??**). Figure 14, right, illustrates that generators could continue to produce at their previous output level, but, in fact, increase output towards the competitive choice because $\pi_I > \pi_S$.

The analysis assumes that the same transmission constraints are always binding, whereas the effects described in the first part required generators to relax transmission constraints on their outputs.

7 Combining E&T markets increases demand elasticity

Le Chatelier observed in physics, and Samuelson translated the following principle to economics:

"*While the change in an x with respect to its own parameter is always negative, regardless of the number of constraints, it is most negative if there are no constraints, only less so when there is a single constraint, and so forth ...*" Samuelson (1947).

[16]The price determination is based on the assumption that bids are cost reflective. If sufficient information about generators with market power and their location is available, the algorithm determining nodal prices can be changed to mitigate market power (See DAE Mimeo Gilbert, Neuhoff, Newbery 2001 for an example in a three-node network).

[17]Flowgate rights would, in theory, allow bilateral trading to allow for reconfiguration of energy flows to match changing output decisions of generators. In practice, with complex congestion patterns flowgate rights are frequently considered to be too complex for implementation.

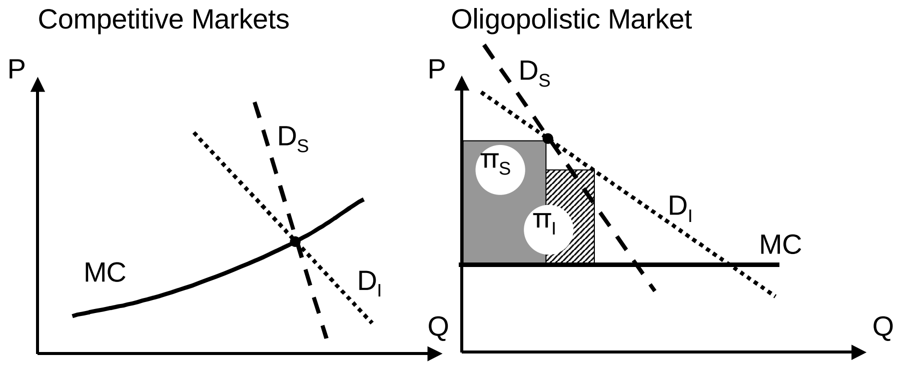

Figure 14: If separate (S) markets are integrated (I), demand slope decreases, with no effect in competitive markets, but increased output in monopolistic markets.

I show that the Le Chatelier Samuelson principle is also applicable for to market design for transmission access in electricity networks. Separation of energy and transmission markets imposes an additional constraint on the system with the result that net-demand is less responsive to price changes.

Kusumoto (1976) gives a broader description of the principle. Fujimoto (1980) provides a general proof for a system of non-linear equations and Milgrom and Roberts (1996) give conditions for applicability of Le Chatelier even with non-local deviations. However, these proofs do not apply to the situation of electricity networks, because of the twofold appearance of prices in transmission trading. First, prices clear local markets and therefore determine net exports from every node, and second, price differences between nodes are linked to price differences between other nodes by the scarcity value of constrained transmission links.

The allocation of transmission capacity is based on supply and demand bids by generators, either to the system operator or to energy spot markets. Bids can be either quantity bids, as in a Cournot game when the market clearing price determined by the intersection with demand, or bids can be supply functions, as in a supply function equilibrium (Green and Newbery, 1992). The system operator does not differentiate between competitive and any kind of strategic bids, and always applies the same transmission allocation mechanism. Neither do competitive traders differentiate between the bids when arbitraging the markets. The calculation of network flows and prices according to Bohn, Caramanis and Schweppe (1984) is therefore applicable both to competitive and strategic bids. The algorithm to define nodal pricing can be summarised as follows. The system operator allocates transmission capacity as if energy bids were competitive and he wanted to maximise welfare. The calculation is based on a DC approximation. This allows for linear treatment of all constraints, while retaining a sufficiently accurate representation of the underlying physical reality. In appendix 9.1 the effect of relaxing a constraint set by separation of energy and transmission markets is determined and the following result is calculated:

Theorem 2 *For local deviations generators at any node of a meshed network face a weakly flatter effective demand under nodal pricing than under separate transmission and energy markets.*

Integration of energy and transmission markets for meshed networks increases demand elasticity which generators face at each node. If strategic generators are located at one node, output will be increased and welfare improved. The results suggest that market coupling reduces market power in meshed networks.

8 Conclusion

Does market coupling (nodal pricing, market splitting) reduce market power of generation companies relative to a market design relying on physical transmission contracts, with subsequent separate energy markets? The question was first asked for the case with a transmission line between two nodes that is sometimes constrained, and then for the case of one or several permanently constrained transmission lines in a meshed network where the dispatch of individual generators does not change the selection of lines that is constrained, although it can change flow patterns.

Output choice of strategic generators in a two-node network has been calculated as a function of demand difference between the nodes. If strategic generators are located at both nodes, small demand differences result in an integrated market. If demand difference increases, a mixed strategy equilibrium with partially-binding transmission constraint exists. For large demand differences, both markets are separated. By comparison, output of strategic generators under separate transmission and energy markets is lower, which can be interpreted as reduced welfare. Empirical evidence from the Netherlands-Germany interconnector shows that the separation of energy and transmission markets prevents traders from arbitraging the interconnector in realisation, and allows them at most to arbitrage in expectation. In the second step, the Netherlands-Germany interconnector is compared with the interconnection between Northern Norway and Sweden. Theory suggests, and empirical evidence confirms, that with market coupling it is more likely that the import constraint into the small zone of Northern Norway will be resolved by the next day than that the export constraint out of Northern Norway will be resolved. This is because it is more profitable for generators in the small country to deviate from an import constraint towards an unconstrained equilibrium in order to face the large market than it is for generators in the large market to deviate from an import constraint situation to obtain a small, additional benefit in the small market. In contrast, theory suggests that with separate energy and transmission markets there is equal probability that a constraint into and out of the 'small' country, the Netherlands, will be resolved, because deviation does not change flows and does therefore not change the constraint. Empirical evidence even shows that the probabilities are not only equal, but even inverted, which can be explained by differences in average prices in both regions.

In a simple network with only one link, market design was relevant if the line is unconstrained or partially constrained. In meshed networks, market design even matters if transmission lines are permanently constrained. This is because integration allows flexible allocation of transmission capacity. I prove that Le Chatelier Samuelson's principle is also applicable in the specific circumstances of electricity networks, where transmission prices and local energy prices are linked. Integration of transmission and energy markets increases the demand elasticity in meshed networks if the same transmission constraints continue to be binding. If generators with market power are located at one node of any meshed network, this increases welfare.

The empirical evidence furthermore supports Hogan (1997), that separate energy and transmission markets are inefficient in the presence of uncertainty. Usually, spot markets are specifically introduced to reveal private information. It is therefore inconsistent to introduce a sequential mechanism for decisions on energy transmission, which can only work efficiently if traders correctly predict spot market prices. Generation from wind, solar and CHP have output which is not predictable over shorter time periods, and information is only aggregated in the spot market. Therefore, a higher contribution by these energy sources will increase the inefficiency of the separate energy and transmission market. The separation is also biased against intermittent generation because prices will be excessively low at times of unexpectedly high generation.

The analysis should be of relevance for enhancing competition and integrating European electricity markets. It provides an argument against the current proposals for a coordinated auction of physical transmission rights to govern electricity trade between continental European countries.[18] This paper assumes that all transmission contracts are acquired by traders and cannot provide financial incentives for generators to alter their energy bids, as in Joskow and Tirole (2000). The paper therefore complements the policy suggestions for the allocation of transmission contracts in Gilbert e.a. (2002).

A further application might be in strategic trade theory, using the parallel between transmission constraints and quotas on trade flows. Strategic trade theory suggests that quotas are usually dominated by tariffs because the latter can be made to replicate the effect of quotas, while also providing both a means of dealing with uncertainty (Newbery and Stiglitz, 1981) and an incentive for competition (Bhagwati, 1965).[19] Yet the use of quotas remains widespread,

[18] European Transmission System Operators, Coordinated use of PX for Congestion Management, 5/03/01

[19] Weitzman (1974) assess the efficiency of tarrifs vs. quantity constraints on output to implement policy goals e.g on emissions. Uncertainty creates a second order effect and the ranking of both options depends on the curvature

chiefly because they help push forward political agendas, such as protecting local industries or determining $C0_2$ emission targets. The analysis suggests that quotas covering several product categories or countries increase the elasticity of demand or supply relative to narrowly-defined quotas. This should increase competitiveness of markets in the presence of quotas.

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9 Appendix

9.1 Proof that Le Chatelier Samuelson is Applicable

How does local price level vary with the bid of a strategic generator y? Power flows on the network are determined entirely by physical characteristics like the resistance of the lines. While power electronics can reroute flows on a network it is expensive and difficult to find agreement upon a mode of operation if it does not benefit all users. Therefore flows on all links are only a function of inflows at all nodes. In the DC approximation the relationship is linear and described by the transfer admittance matrix $\mathbf{H}$ (see Bohn et.al. 1984).

Let $d_i(p_i)$ be at each node n the net demand function given by demand minus competitive generation and strategic generation by all but one generator. The remaining strategic generator has output y at node one. Net demand is assumed to be a function of local prices p_i. So output increase at higher prices is equivalent to demand decrease at higher prices:

$$\frac{\partial d_i}{\partial p_i} = -\mathbf{B}_{ii}(p_i), \quad \frac{\partial d_i}{\partial p_j} = \mathbf{B}_{ij} = 0 \text{ for } i \neq j. \tag{23}$$

Using the transfer matrix, the vector $\overrightarrow{z}$ of power flows on all r links is given by:

$$\overrightarrow{z} = \mathbf{H}_{rn} \begin{pmatrix} d_1(p_1) - y \\ d_2(p_2) \\ \dots \end{pmatrix}. \tag{24}$$

The law of energy conservation implies that the difference between inserted energy and withdrawn energy equals network losses L which are assumed to be zero to simplify subsequent calculations.

$$\sum d_i(p_i) - y = L(\overrightarrow{p}) \equiv 0. \tag{25}$$

The system represented by (24) and (25) is overdetermined and one equation can be dropped by rewriting the transfer matrix $\mathbf{H}$, such that row one only contains 0. The corresponding node one is called the swing bus. Changes of net-demand $d_1(p_1)$ or generation y at the swing bus will not directly influence $\overrightarrow{z}$ in (24), but according to (25), these changes induce changes of p_j with $j \neq 1$, and thereby 'indirectly' influence $\overrightarrow{z}$ in (24). Using the notation of Bohn et.al. (1984) the prices and net demand at node one can be excluded from the vector notation: $N = n - 1$, $\overrightarrow{P} = (p_2, .., p_n)$, $P_s = p_1$, $\overrightarrow{D} = (d_2, .., d_n)$, and $D_s = d_1$.

The subsequent proof only assesses local deviations. The set of constraint links does not change. Assume the first R of r links are constrained and these flows are given by $\overrightarrow{Z}$. Redefine $\mathbf{H}_{R,N}$ to represent only the first R rows and columns 2 to n of the full matrix $\mathbf{H}_{r,n}$. Flows on constrained links are:

$$\overrightarrow{Z} = \mathbf{H}_{R,N}\overrightarrow{D}. \tag{26}$$

Using the information on transmission constraints the system operator determines nodal prices as if all energy bids were competitively priced and he were to maximise social surplus. This is implemented by maximising the sum of short run value added functions of net demand $V_i(D_i)$[20] at all nodes, while satisfying energy conservation and capacity constraints of transmission links $\overline{-Z_k} < Z_k < \overline{Z_k}$. Effectively the system operator finds a saddle point of the Lagrangian:

$$\pounds(D_i, y_{i,j}) = \sum_{i=s,1..N} V_i(D_i) + P_s(\sum_{i=s,1..N} D_i - y) - \sum_{k=1..R} \eta_{k,+}\left(Z_k - \overline{Z_k}\right) + \sum_{k=1..R} \eta_{k,-}\left(Z_k - \overline{-Z_k}\right). \tag{27}$$

[20] The system operator subsequently only requires information about the marginal utility $V_i'() = P_i()$ as provided by bids and offers.

The Lagrange parameters can be interpreted as energy prices at the swing bus, P_s, and scarcity rent of transmission lines in either direction, $\eta_{k,+}$ and $\eta_{k,-}$. A transmission line can only be constrained in one direction, therefore define $\eta_k = \eta_{k,+} - \eta_{k,-}$. Lines $i = R+1, .., r$ are unconstrained and therefore $\eta_i = 0$ and therefore not listed in the sum of (27).

Substituting $\overrightarrow{Z}$ from (26) in (27) and differentiating with respect to D_i gives the optimal allocation of net demand (in vector notation):

$$\overrightarrow{P} = \nabla \overrightarrow{V}(D_i) = P_s \begin{pmatrix} 1 \\ 1 \end{pmatrix}_N + \mathbf{H}'_{R,N} \overrightarrow{\eta}. \tag{28}$$

The local prices $\overrightarrow{P}$ equal the energy price at the swing bus P_S plus the number of transmission rights required times their price $\overrightarrow{\eta}$. Define the first R rows of $\mathbf{H}_{R,N}$ as $\mathbf{H}_{R,R}$, and the first R components of the price vector $\overrightarrow{P}$ as $\overrightarrow{P_R}$ to obtain:

$$\mathbf{H}_{RR} \overrightarrow{\eta_R} = \overrightarrow{P_R} - \begin{pmatrix} 1 \\ 1 \end{pmatrix}_R P_s. \tag{29}$$

$\mathbf{H}_{r,n}$ has the form $\mathbf{\Omega A} \left(A'\Omega A\right)^{-1}$ with Ω a $r * r$ diagonal matrix with admittances of links and $\mathbf{A}$ the $[r * (n-1)]$ network incidence matrix consisting of $-1, 0, 1$ for network interconnections (See appendix of Bohn et.al.). Let A' be the inverse of $\mathbf{H}_{r,n}$ for multiplication from the left. Existence of A' can be easily understood from the law of local energy conservation. Given the flows on all links, the residual of inflows and outflows of links towards a node is the net energy demand at the node $\overrightarrow{P} = \mathbf{A}'\overrightarrow{Z}$.

Invertability of $\mathbf{H}_{r,n}$ does not imply invertability of a sub-matrix $\mathbf{H}_{R,R}$. However, usually R can be chosen out of the $(n-1)$ nodes, such that shadow prices on R constraint links $\overrightarrow{\eta}$ follow from the prices at the R nodes and at the swing bus. Therefore, subsequently assume that $\mathbf{H}_{R,R}^{-1}$ exists and calculate $\overrightarrow{\eta} = \mathbf{H}_{RR}^{-1} \left(\overrightarrow{P_R} - \binom{1}{1}_R P_s \right)$ to obtain from (28) an expression for $\overrightarrow{P}$ as function of P_s and $\overrightarrow{P_R}$:

$$\overrightarrow{P} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}_N P_s + \mathbf{H}_{N,R} \mathbf{H}_{RR}^{-1} \left(\overrightarrow{P_R} - \begin{pmatrix} 1 \\ 1 \end{pmatrix}_R P_s \right). \tag{30}$$

The set of constraint transmission will not change with local deviations $d\overrightarrow{Z}/dy = 0$ and therefore differentiating (26) with respect to y and using the slopes of net demand functions $\mathbf{B}_{NN}$ from (23) gives:

$$\mathbf{H}_{R,N} \mathbf{B} \frac{d\overrightarrow{P}}{dy} = \overrightarrow{0}_R. \tag{31}$$

Energy conservation will also continue to be satisfied with local deviations, and differentiating (25) with respect to y gives:

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}'_N \mathbf{B} \frac{d\overrightarrow{P}}{dy} + B_s \frac{dP_s}{dy} = -1. \tag{32}$$

Finally, differentiating (30) with respect to y gives an equation for the nodal price changes. Differences between nodal prices represent the transportation charges.[21]

$$\begin{aligned}\frac{d\overrightarrow{P}}{dy} &= \binom{1}{1}_N \frac{dP_s}{dy} + \mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\left(\frac{d\overrightarrow{P_R}}{dy} - \binom{1}{1}_R \frac{dP_s}{dy}\right), \\ &= \left(\binom{1}{1}_N - \mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\binom{1}{1}_R\right)\frac{dP_s}{dy} + \mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\frac{d\overrightarrow{P_R}}{dy}. \end{aligned} \tag{33}$$

Inserting (33) in (31) gives:

$$\frac{d\overrightarrow{P_R}}{dy} = -\left(\mathbf{H}_{R,N}\mathbf{B}\mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\right)^{-1}\mathbf{H}_{R,N}\mathbf{B}\left(\binom{1}{1}_N - \mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\binom{1}{1}_R\right)\frac{dP_s}{dy}. \tag{34}$$

Inserting (33) in (32) gives:

$$\left(\binom{1}{1}_N' \mathbf{B}\left(\binom{1}{1}_N - \mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\binom{1}{1}_R\right) + B_s\right)\frac{dP_s}{dy} + \binom{1}{1}_N' \mathbf{B}\mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\frac{d\overrightarrow{P_R}}{dy} = -1. \tag{35}$$

Inserting (34) into (35) gives:

$$\begin{aligned}\frac{-1}{\frac{dP_s}{dy}} &= \binom{1}{1}_N' \mathbf{B}\left(\binom{1}{1}_N - \mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\binom{1}{1}_R\right) \\ &\quad -\binom{1}{1}_N' \mathbf{B}\mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\left(\mathbf{H}_{R,N}\mathbf{B}\mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\right)^{-1}\mathbf{H}_{R,N}\mathbf{B}\left(\binom{1}{1}_N - \mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\binom{1}{1}_R\right) + B_s, \\ &= \binom{1}{1}_N' \left(\mathbf{B} - \mathbf{B}\mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\left(\mathbf{H}_{R,N}\mathbf{B}\mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\right)^{-1}\mathbf{H}_{R,N}\mathbf{B}\right)\binom{1}{1}_N + B_s, \\ &= \binom{1}{1}_N' \left(\mathbf{B} - \mathbf{B}\mathbf{H}_{N,R}\left(\mathbf{H}_{R,N}\mathbf{B}\mathbf{H}_{N,R}\right)^{-1}\mathbf{H}_{R,N}\mathbf{B}\right)\binom{1}{1}_N + B_s.\end{aligned}$$

$\mathbf{B}$ is positive, semi-definite and diagonal, therefore define $\mathbf{C}$ such that $\mathbf{CC} = \mathbf{B}$ to obtain:

$$\frac{-1}{\frac{dP_s}{dy}} = \binom{1}{1}_N' \mathbf{C}\left(\mathbf{1}_{NN} - \mathbf{C}\mathbf{H}_{N,R}\left(\mathbf{H}_{R,N}\mathbf{CC}\mathbf{H}_{N,R}\right)^{-1}\mathbf{H}_{R,N}\mathbf{C}\right)\mathbf{C}\binom{1}{1}_N + B_s.$$

Defining $\mathbf{X} = \mathbf{C}\mathbf{H}_{N,R}$ gives:

$$\frac{-1}{\frac{dP_s}{dy}} = \binom{1}{1}_N' \mathbf{C}\left(\mathbf{1}_{NN} - \mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\right)\mathbf{C}\binom{1}{1}_N + B_s.$$

$\mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'$ projects $\mathbf{B}\binom{1}{1}_N$ to a subspace of R^N, therefore $\mathbf{1}_{NN} - \mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'$ gives the components orthogonal to this subspace. $\binom{1}{1}_N'\mathbf{B}\left(\mathbf{1}_{NN} - \mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\right)\mathbf{B}\binom{1}{1}_N$ gives the length of the component of $\mathbf{A}\binom{1}{1}_N$ orthogonal to the space spanned by $\mathbf{X} = \mathbf{C}\mathbf{H}_{N,R}$, a semi-positive number. The result is that the slope of net demand facing the generator $-1/\frac{dP_s}{dy}$ is weakly larger than B_s. If allocation of transmission capacity is adjusted in response to output choices of the strategic generator, then the strategic generator faces a weakly larger net demand response.

[21] Output changes dY therefore change the transportation charges. This corresponds to Hogan's observation that "transportation prices are both endogenous and not taken as given by the Cournot participants" (1997).