Efficient Tariffs for Cost Revelation in a Price and Quantity Constrained Electric Power Market.

Serhiy Kotsan and Stratford Douglas *

Department of Economics

West Virginia University

June, 2003

Abstract. We analyze a market in which an auctioneer or system operator must satisfy a fixed demand at a fixed price, and attempts to minimize cost as indicated by sellers' bids. This situation may arise in wholesale electricity markets, e.g. when generation price caps are in effect. We show that no equilibrium exists in this situation, and the operator can not identify low-cost generators from bids. As a remedy, we propose a simple tariff on generation, proportional to the spread between market price and bid. With a tariff imposed, generators' Nash equilibrium bids diverge from marginal cost but contain information sufficient to identify the least-cost bidders. In a simple case, generators make the same profit as they would in a second-price Vickrey auction, regardless of the tariff rate or number of generators.

*Both are at WVU Department of Economics, PO Box 6025, Morgantown, WV, 26506-6025. Serhiy Kotsan can be reached at sokotsan@yahoo.com. Stratford Douglas phone number is (304)-293-7863, e-mail douglas@wvu.edu

1 Introduction

The ongoing vertical disintegration of electric utilities creates fundamental problems for the electrical System Operator (SO). Under traditional regulation, the SO ordered the dispatch of captive generators whose costs were known. In restructured markets, in contrast, the SO takes bids from independent generators whose true costs may be unknown. Generating firms generally possess accurate information about the structure of the market and the algorithm that the SO uses to determine when and how much each generator will produce (the "dispatch algorithm"). Generators can be expected to use this information to adjust their bids strategically in order to maximize profit, and evidence from California and other markets indicates that they do. Thus, modern system designers and operators must possess training in principles of market economics and applied game theory as well as electrical engineering.

A growing academic literature has developed recently concerning the design of electricity markets (see e.g. Wilson 2002 and Stoft 2002 for surveys and outlines of the major issues). Regulatory authorities have taken a keen practical interest in this problem as well (FERC 2002). There has been some research about the use of bids to improve the performance of the system in the very short run (e.g. Cardell 1997), but most work has concentrated on day-ahead and real-time (five to 15-minute-ahead) energy markets, which have sufficient time to clear. In contrast, the present research focuses on cost-minimizing in markets that do not clear and require the intervention of a system operator to stay in balance.

Improving the efficiency of system operator interventions is of special interest for electricity markets. There is currently considerable diversity in market design (Wilson 2002; FERC 2002), but most systems take bids from generators in both day-ahead and real-time markets. Market-clearing prices are then adjusted by location according to transmission losses and constraints. Though increasingly sophisticated, electricity markets are not yet able to respond quickly enough to prevent instability in real time, and they may never be. Therefore, the SO generally obtains generation resources from ancillary markets for regulation and reserve capacity to allow it to intervene to maintain balance. However, the efficiency of the SO's intervention is circumscribed by its ability to determine each generator's cost. It must rely on bids rather than direct knowledge of generation costs, and those bids are often influenced by the desire of generators to exert market power.

Empirical evidences concerning strategic bidding of the generators were found by Wolfram 1998 on the power market of England and Wales. Theoretical aspect of generators' market power were examined by Joskow 2000 and Borenstein 2000. Joskow examines effects of transmission rights on the market power of the generators and Borenstein looks at the transmission capacity of the line.

Electricity markets are prone to failure and manipulation in times of stress, as in California in 2000. Electrical energy can not be stored economically, and the system will quickly become unstable if there is either a surplus or shortage. Demand for electricity is volatile and unresponsive to short-term price changes. Supply is also unresponsive to price changes when the system is producing near capacity, and in extreme (but realistic) cases supply and demand may not even intersect. Also production and delivery may be constrained by physical limitations of the transmission system, and these constraints vary from moment to moment and location to location across the transmission grid. It is not uncommon for certain portions of the grid to become "islanded" or isolated from the rest of the system. The SO must therefore continually adjust production to follow shifts in demand, both systemwide and within the transmission-constrained islands. Incremental demand may have to be satisfied by just a few generators who may have considerable market power and who will not wish to reveal their production costs. Price caps are commonly used to reduce price volatility, but they have the side effect of reducing the efficiency of market price signals.

Many electrical systems therefore experience significant periods during which price adjustments are either too slow or constrained by a regulatory cap, and quantity is fixed by the inelasticity of demand. The dispatch choices of the SO during these periods have important long-run implications, because the high prices experienced during these periods are required to finance crucial high-cost, quick-response generation facilities. Therefore, it is crucial that the SO have a mechanism to identify and dispatch the most efficient generation resources, at the very times when simple bidding systems may be most failure prone.

In this paper we analyze an electricity market in which a SO who wishes to minimize cost must satisfy an exogenously determined electricity load at a fixed price. The SO is assumed to know only the generators' bids, rather than their true costs. Generators are assumed to know their own costs, each others' bids, and the SO's dispatch algorithm. We find that no set of Nash equilibrium bids exist for such a market, and that generators' bids will diverge widely from actual marginal costs. We propose a simple tariff mechanism that is incentive-compatible in the sense that it induces bidding that will allow the SO to minimize the total cost of production, even in the presence of transmission constraints. Interestingly, these bids, though efficient, diverge from true marginal cost.

Throughout the paper, for purposes of exposition we focus on a two-generator market with quadratic cost functions, but we also show that the major results hold for any number of generators, with general convex cost functions. We focus on the problem of overcapacity (i.e., the SO must choose among generators who wish to produce more in aggregate than the load requires), but a symmetric result occurs if insufficient voluntary generation is available at the fixed price.

The paper's structure is as follows. In the next section we will introduce the market model, including market participants and rules of the game. In the following section, we show that the generators' bids will depart from true costs, and demonstrate the nonexistence of a Nash equilibrium in the model without tariffs. Next, we introduce the tariff mechanism and show that it will create a Nash equilibrium in the market, which we demonstrate to be efficient. Finally, we briefly discuss implementation issues.

2 The Model

Some number n independent **generators** compete on the energy market. (For expositional simplicity, we discuss the case where there are only two generators, designated G1 and G2.) We assume that the generators do not cooperate with each other when they make decisions. Each generator is paid a price, P, for output, determined exogenously, and not affected by either the bid or the quantity that the generator produces. Consumers create an exogenous **load** Q on the system, which must be satisfied in real time. The **System Operator** determines the production level of each generator, using an algorithm that minimizes the total "bidded" cost of serving the load, treating the generators' bids as though they were the actual costs of production. The SO knows the market demand of energy, and the bids and production capacity of each generator, and it takes into account the rational response of generators to its dispatch algorithm.

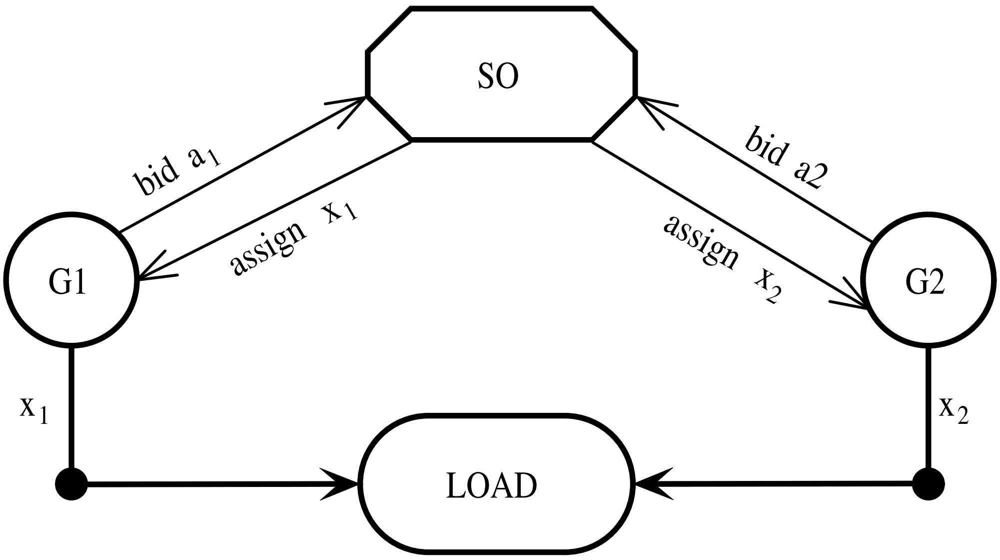

Figure 1: Power Market system.

Thus, although generators take market price and quantity as fixed, their bids determine the quantity of energy that the SO allows them to produce. This approximates the situation faced in real time by providers of capacity reserve services, or the situation faced by generators in an isolated or islanded system subject to a binding price cap.

Generator's problem. Each generator wants to produce at the level x_i that maximizes its profit, $\pi(x_i) = Px_i - C(x_i)$, where $C(\cdot)$ is the cost function of the generator. For expositional purposes we assume a simple quadratic cost function[1]

$C_i(x_i) = \alpha_i x_i^2 + \beta_i$.

The generators' bids are determined strategically, and may differ from actual costs. Each generator's bid is a function $B_i(x_i)$, assumed again for expositional purposes to be quadratic:

$B_i(x_i) = a_i x_i^2 + b_i$

From the SO's operational point of view, the difference between α_i and a_i determines the effective difference between a generator's true and reported cost. By manipulating a_i the generator can influence the SO's dispatch decision. The ultimate purpose of the successful bid for a generator is to induce the SO to assign him the level of production that maximizes his profit. For the simple, two-generator quadratic case, the solution of the generator's problem is

$$x_i^*(P, \alpha_i) = \frac{P}{2\alpha_i} \quad i = 1, 2$$

where $x_i^*(\cdot)$ is generator i's profit-maximizing output level. Therefore, a generator will attempt to craft a bid a_i such that the SO will assign him to production level x_i^*.

The System Operator's Problem.

The SO monitors the load Q, and determines how much power should be supplied by each generator, based on their bid functions $B(\cdot)$. Formally, the SO minimizes the overall bidded cost of production, while satisfying market demand.

[1]Quadratic cost functions are commonly assumed by electrical engineers in practice, though the engineering cost function typically also contains a term linear in output. Inclusion of a linear term would not affect the outcome of the analysis, though it would clutter the mathematical expressions.

$$\min_{\{x_1,x_2\}} \quad B_1(x_1) + B_2(x_2)$$
$$\text{s.t.} \quad x_1 + x_2 = Q$$

The solution to this optimization problem provides an algorithm for the SO to use in allocating the market load Q among generators. For the two-generator case, the minimization problem is as follows:

$\text{Cost}(x_1) = a_1 x_1^2 + b_1 + a_2(Q - x_1)^2 + b_2$

Setting the first derivative of bidded cost with respect to x_1 equal to zero and solving for x_1,

$$\hat{x}_1(a_1, a_2) = \frac{a_2 Q}{a_1 + a_2}$$
$$\hat{x}_2(a_1, a_2) = \frac{a_1 Q}{a_1 + a_2}$$

where $\hat{x}_i$ is the amount of power that the SO will assign generator i to produce.

3 Divergence of Bids from Actual Cost, and Market Instability

It is clear that if both generators were to bid their true marginal costs α, the SO will achieve a socially optimal (cost-minimizing) allocation of output. However, each generator will seek to bid strategically to maximize his own profit, regardless of society's welfare. In general, the sum of the profit-maximizing levels of output for all generators will virtually never exactly satisfy the load, i.e., $Q \neq \sum_i \hat{x}_i$. We will therefore examine the case where $Q < \hat{x}_1 + \hat{x}_2$, i.e., where the local system has a surplus of capacity. (Analogous results are obtained when there is insufficient capacity, $Q > \hat{x}_1 + \hat{x}_2$). Whenever there is excess capacity, the SO will force at least one generator to produce less than it would like. In response that generator will reduce its bid in order to increase his production share and consequently profit.

In case of two generators (G1 and G2) it is easy to demonstrate that the generators' bids will diverge from actual cost. Suppose that G2 bids its actual costs, $a_2 = \alpha_2$. Anticipating this bid, and knowing the SO's allocation algorithm, G1 will bid a_1 such that the SO will assign G1 to produce its own-profit-maximizing quantity. Formally, G1 will determine his bid parameter a_1 by solving the SO's system bidded cost minimization problem conditional on $a_2 = \alpha_2$. Setting $\hat{x}_i = x_i^*$,

$$\frac{P}{2\alpha_1} = \frac{\alpha_2 Q}{a_1 + \alpha_2}$$

and solving for G1's bid a_1, yields

$$a_1^* = \frac{2\alpha_1\alpha_2 Q}{P} - \alpha_2$$

where a_1^* is the profit-maximizing bid for G1, conditional on G2 bidding α_2. The bids will be collected by the SO, who will assign G1 to produce its profit maximizing amount x_1^*, leaving $Q - x_1^*$ for G2. Thus, the profit-maximizing bid for G1 will diverge from its true cost. Unfortunately for G1, G2 will also bid strategically. In fact there will be no stable market equilibrium, as each generator seeks to underbid the other *ad infinitum.* (In the case of insufficient capacity, the bids will increase *ad infinitum.*) This situation arises because there is no penalty for bidded cost deviating from actual cost. Formally, the system of simultaneous bid response equations

$$\begin{cases} \dfrac{P}{2\alpha_1} = \dfrac{a_2 Q}{a_1 + a_2} \\ \\ \dfrac{P}{2\alpha_2} = \dfrac{a_1 Q}{a_1 + a_2} \end{cases}$$

does not have a solution unless $x_1^* + x_2^* = Q$, and therefore the market will fail to provide useful cost information to the SO. Apart from the bids, the SO has no independent knowledge of actual costs, and so it will be unable to achieve an efficient allocation of production. To remedy this situation, the SO will need to introduce some additional incentive to induce bidders to provide better cost information.

4 Tariffs

Without knowledge of actual costs, how can the SO detect, and penalize, strategic bidding that deviates from actual costs? It turns out to be sufficient to introduce a tariff that applies to the difference between the bids and the market price. In the case of overcapacity, this tariff can be thought of as a tax on reported profit. Each generator will face a tradeoff. On the one hand, by bidding low the generator can induce the SO to increase its production share. On the other hand, a lower bid implies lower reported costs and consequently higher reported profit and a higher tariff payment. In our model we assume that all generators pay the same flat tariff rate t.

The Generator's Problem

For expositional purposes, assume that there is system overcapacity at the current price. If all generators in this simple market pay the same fixed tariff rate t, then $t[Px_i(a_i) - B_i(x(a_i), a_i)]$ is Gi's tariff payment, and the net (after-tax) profit for G_i is

$\pi_i(a_i) = Px_i(a_i, a_{-i}) - C_i(x_i(a_i, a_{-i})) - t[Px_i(a_i, a_{-i}) - B_i(x_i(a_i, a_{-i}), a_i)]$

where a_{-i} is a vector of other generators' bids.

To find the profit-maximizing bid, take the first derivative with respect to a_i and set it equal to zero:

$\pi_i'(a_i) \equiv P\frac{\partial x_i(\cdot)}{\partial a_i} - \frac{\partial C_i(\cdot)}{\partial x_i(\cdot)}\frac{\partial x_i(\cdot)}{\partial a_i} - t[P\frac{\partial x_i(\cdot)}{\partial a_i} - \frac{\partial B_i(\cdot)}{\partial x_i(\cdot)}\frac{\partial x_i(\cdot)}{\partial a_i} - \frac{\partial B_i(\cdot)}{\partial a_i}] = 0.$

Dividing by $\frac{\partial x_i(\cdot)}{\partial a_i} \neq 0$

$$\frac{\pi_i'(a_i)}{x_i'(a_i)} \equiv P - \frac{\partial C_i(\cdot)}{\partial x_i(a_i)} - t[P - \frac{\partial B_i(\cdot)}{\partial x_i} - \frac{\partial B_i(\cdot)}{\partial a_i}\frac{1}{\partial x_i/\partial a_i}] = 0 \tag{1}$$

In the two-generator case with quadratic costs and bids, the profit-maximizing level of output is

The System Operator's Problem.

The SO seeks to minimize the bidded cost of satisfying exogenous market demand Q by allocating the required output among the bidders, taking into account the bidders' strategic response to the allocation algorithm.

$$\min_{\{x_1 \dots x_N\}} \quad \sum_{i=1}^{n} B_i(a_i(x_i), x_i)$$
$$\text{s.t.} \quad \sum_{j=1}^{n} x_j = Q$$

Solving the first-order conditions for bidded-cost minimizing yields a system of differential equations

$$\frac{\partial B_i(a_i(x_i), x_i)}{\partial a_i(x_i)} \frac{\partial a_i(x_i)}{\partial x_i} + \frac{\partial B_i(a_i(x_i), x_i)}{\partial x_i} = \lambda \qquad i = 1, 2 \dots N \tag{2}$$
$$\sum_{j=1}^{n} x_j = Q$$

where $\frac{\partial a_i(x_i)}{\partial x_i}$ is the change in the generator i's bid when the SO increases its output by 1 unit, and λ is a Lagrange multiplier. Because λ is a constant, the condition is equivalent to setting the marginal bidded costs equal to each other across all generators.

$$\frac{\partial B_i/\partial a_i}{\partial x_i/\partial a_i} + \frac{\partial B_i}{\partial x_i} = \frac{\partial B_j/\partial a_j}{\partial x_j/\partial a_j} + \frac{\partial B_j}{\partial x_j} \qquad i, j = 1, 2 \dots N$$
$$\text{subject to} \quad \sum_{j=1}^{n} x_j = Q.$$

The solution of this system of differential equations yields the dispatch function $x_i(a)$ for each generator. Dispatch functions depend on bids rather then true costs and are common information for all generators. Bearing in mind all the dispatch functions, each generator adjusts its bid so as to maximize its own profit function. The set of bids $\{a_1, a_2, \dots a_N\}$ at which no generator can improve its profit by changing its own bid is called the Nash Equilibrium. The following section demonstrates that the Nash Equilibrium allocation is efficient [2].

[2]Existence of the solution for the well-behaved bid functions is shown in Mathematical Appendix

5 The Efficiency of Nash Equilibrium

The simple tariff induces a stable equilibrium set of bids in the market, but a stable NE is not necessarily efficient. The system operator does not know the generators' costs, so it is not obvious that minimizing the bidded "cost" of serving the load will also minimize the actual cost of doing so. However, in this case it can be shown that the NE does produce the same efficient allocation of production among the generators as would occur if the true marginal costs were known.

Rearranging the generator's profit-maximization condition from (1) yields the following profit-maximizing equilibrium relationship between bids and true marginal costs for all generators:

$$[P - \frac{\partial C_i(\cdot)}{\partial x_i}]/t - P = \frac{\partial B_i(\cdot)}{\partial x_i} + \frac{\partial B_i(\cdot)/\partial a_i}{\partial x_i/\partial a_i} \quad i = 1, 2 \ldots N \qquad t \neq 0 \tag{3}$$

On the right-hand side of the above equation is the marginal effect on bidded cost of a unit-change in the output of generator i. (The first term is the anticipated strategic change in a_i to the dispatch level x_i, while the second term is the change in the bid function due to the change in x_i, holding a_i constant.) Note that when the SO's cost minimization condition (2) is satisfied (i.e., in Nash Equilibrium), this right-hand-side marginal effect will be the same across all bidders, implying that

$$[P - \frac{\partial C_i(\cdot)}{\partial x_i}]/t - P = [P - \frac{\partial C_j(\cdot)}{\partial x_j}]/t - P \qquad i, j = 1, 2 \ldots N \qquad t \neq 0$$

which implies that in Nash Equilibrium true marginal costs will be equal across all generators:

$$\frac{\partial C_i(\cdot)}{\partial x_i} = \frac{\partial C_j(\cdot)}{\partial x_j} \qquad i, j = 1, 2 \ldots N$$

$$\sum_{j=1}^{n} x_j = Q$$

Equality of true marginal costs of the generators subject to the production constraint solves following true cost minimization problem.

$$\min_{\{x_1 \dots x_N\}} \quad \sum_{i=1}^{n} C_i(x_i)$$
$$\text{s.t.} \quad \sum_{j=1}^{n} x_j = Q$$

which is efficient.

Mathematically, efficiency condition is achieved when cost and bid functions are continuous differentiable and $t \neq 0$. These restrictions are innocuous for the quadratic cost functions (or any other well-behaved cost function) and $t \in (0; 1]$.

6 Examples of Cost functions for two generators

Linear Costs

A market with two generators with linear cost and bid functions provides a simple example that illustrates the function of the efficient tariff. Suppose that the cost and bid functions of generator i (G_i) are, respectively,

$C_i = \alpha_i x_i$

and

$B_i = a_i x_i$.

For expositional simplicity, assume that either of the generators could supply the entire market on its own, and either would profit from doing so (i.e., $P > a_i$). (This is a reasonable assumption, as price caps are generally set above marginal production costs.) Therefore, the SO's task is simply to use the bid parameters a_i to determine which generator has the lower cost parameter α_i, and allocate all production to that generator. If there is no tariff, the profit function of G_i is

$\pi_i = Px_i - \alpha_i x_i$

and there will be no Nash equilibrium because each generator will attempt to underbid the other. Because the payment is fixed by the market price, the bids will decrease without bound.

If, on the other hand, the SO institutes a tariff rate t on the winner's apparent profit, then the winner's actual profit will be

$\pi_i = Px_i - \alpha_i x_i - t(Px_i - a_i x_i)$

Suppose that $\alpha_1 < \alpha_2$, and that therefore G1 is the efficient producer. In Nash equilibrium, each player bids a_i that maximizes its own profit, given the other generators' actual bids. If the price is fixed, then G1's true profit for any production level will always exceed G2's profit, because G1's costs are lower. From the profit function, it is apparent that G_i obtains positive economic profit if and only if

$a_i > (1/t)[\alpha_i - (1-t)P]$.

Because $\alpha_1 > \alpha_2$, there will always exist bids that would yield a positive profit for G1, but a negative or zero profit for G2. There can be no Nash equilibrium, therefore, where the higher-cost generator G2 wins the bid. Instead, Nash equilibrium in this simple market occurs where

$a_2 = (1/t)[\alpha_2 - (1-t)P]$

and

$a_1 = (1/t)[\alpha_2 - (1-t)P] - \varepsilon$

where ε is the smallest increment allowed between bids. G1 is satisfied with this outcome because it is allowed to produce the amount that maximizes its profit, and a higher bid would put it at risk of losing to G2. G2 is also satisfied, because it is breaking even, and if it lowered its bid below G1's bid it would be producing at a loss.

Note that the market result is efficient, and therefore the SO is satisfied, regardless of the regardless of the level of t. In fact, changes in t will not even affect the tariff collected in the linear case. Substituting the bid into the profit function (and neglecting ε), the tariff collected in this simple case will be $(P - \alpha_2)Q$, regardless of the tariff rate t. Also, G1's profit will be the same as in a second-price or Vickrey auction: $(\alpha_2 - \alpha_1)Q$. Changes in t will, however, affect the NE levels of the bids. Bids will even become negative if $tP < (P - \alpha_2)$, but the tariff will create a lower limit to their decline.

Quadratic Costs

A more realistic but complicated case is that of two generators with quadratic costs and bids

$C_i(x_i) = \alpha_i x_i^2 + \beta_i \quad i = 1, 2$

where α_i is a marginal cost parameter and β_i represents fixed costs.

$B_i(x_i) = a_i x_i^2 + b_i$

If Generator 2 (G2) bids a_2, Generator 1 (G1) will adjust its bid a_1 to maximize its own profit.

$$a_1 = \arg\max[\pi_1(a_1, a_2)] \tag{4a}$$

Similarly, G2 will adjust its own bid a_2, so as to maximize its own net profit conditional on G1's bid of a_1 and SO assigned production level $x_2(a_1, a_2)$. Thus,

$$a_2 = \arg\max[\pi_2(a_1, a_2)] \tag{4b}$$

In Nash equilibrium, neither generator will be able to improve its net profit by changing its bid. Operationally, we may substitute the expressions for $x_1(a_1, a_2)$ and $x_2(a_1, a_2)$ derived from SO's problem (2) into (4a) and (4b) to define conditions under which Nash equilibrium is obtained.

The Nash Equilibrium (NE) for this market occurs where bids a_1 and a_2 solve the following system of equations:

$$\begin{cases} P - 2\alpha_1 x_1(a_1, a_2) - t[P - 2a_1 x_1(a_1, a_2) - \dfrac{x_1^2(a_1, a_2)}{\partial x_1(a_1, a_2)/\partial a_1}] = 0 \\ \\ P - 2\alpha_2 x_2(a_1, a_2) - t[P - 2a_2 x_2(a_1, a_2) - \dfrac{x_2^2(a_1, a_2)}{\partial x_2(a_1, a_2)/\partial a_2}] = 0 \end{cases}$$

where $x_1(\cdot)$ and $x_2(\cdot)$ are found endogenously by SO as a solution for overall bid minimization problem.

$$\begin{array}{cl} \min\limits_{\{x_1 \dots x_N\}} & \sum_{i=1}^{n} B_i(a(x_i), x_i) \\ \text{s.t.} & \sum_{j=1}^{n} x_j = Q \end{array}$$

The SO's problem reduces to the system of non-linear differential equations described in the above section where $x_1(\cdot)$ and $x_2(\cdot)$ are unknown functions.

It is useful to map the NE solution in terms of response functions in a_1, a_2 space, as shown in Figure 2. The response function $R_1(a_2)$ gives the bid, a_1, that would maximize Generator 1's net profit if Generator 2 were to bid a_2. Correspondingly, $R_2(a_1)$ is the best response of the second generator given the bid a_1 of the first generator. The point of intersection represents the NE solution as defined above. At this point, each generator is getting as much profit as possible, given the other's bid. In other words, at this point any increased revenue that either generator would get by decreasing its bid is just balanced by the additional tariff it would have to pay.

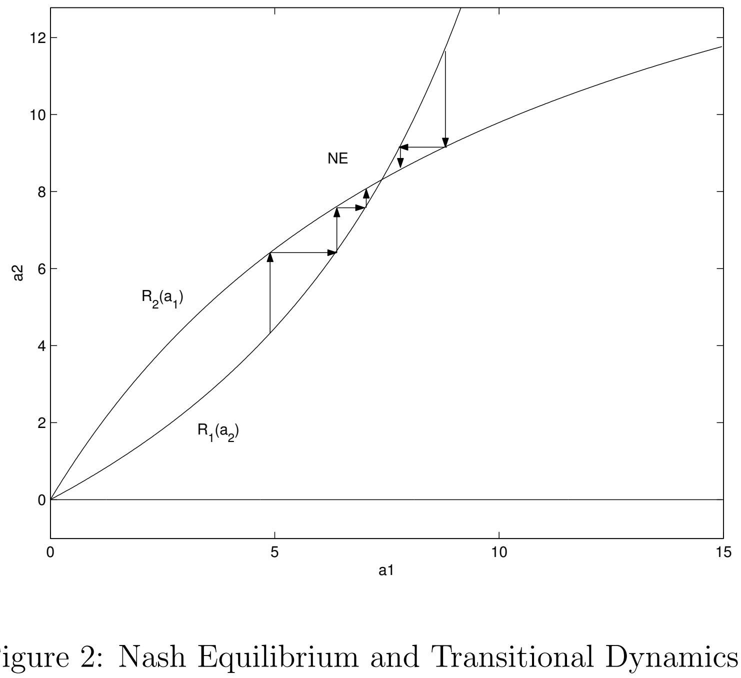

Figure 2: Nash Equilibrium and Transitional Dynamics.

Dynamic analysis shows that if cost functions are convex then the NE is stable; i.e., that any movement away from NE will automatically create incentives that move the generators back to NE. In Figure 2, the arrows show these dynamics. For example, if G1 sets $a_1 = 5$, then G2 will respond by increasing a_2 to approximately 6.5. Because of G2's higher bid, the SO will assign G1 to produce more (and pay more tariff) than G1 had hoped, which will induce it to raise a_1 some more, which will reduce its tariff. This higher bid by G1 will cause the SO to assign more output (and therefore a higher tariff payment) to G2 than it had planned on, which will induce G2 to raise its bid, and so on.

This result is also efficient. To see the efficiency of the NE substitute the SO bid minimization condition (2) in the profit function of both generators

$$\begin{cases} P - 2\alpha_1 x_1(a_1, a_2) - t[P - \lambda] = 0 \\ P - 2\alpha_2 x_2(a_1, a_2) - t[P - \lambda] = 0 \\ x_1 + x_2 = Q \end{cases} \Rightarrow \begin{cases} 2\alpha_1 x_1(\cdot) = 2\alpha_2 x_2(\cdot) \\ x_1 + x_2 = Q \end{cases} \Leftrightarrow \begin{cases} \frac{\partial C_1(x_1)}{\partial x_1} = \frac{\partial C_2(x_2)}{\partial x_2} \\ x_1 + x_2 = Q \end{cases}$$

NE produces an efficient production level

$$x_1(Q, a_1^{NE}, a_2^{NE}) = \frac{\alpha_2 Q}{\alpha_1 + \alpha_2}$$

$$x_2(Q, a_1^{NE}, a_2^{NE}) = \frac{\alpha_1 Q}{\alpha_1 + \alpha_2}$$

as in the cost minimization problem when both G's bid true marginal costs.

7 Conclusions

Information flow in electricity markets is increasingly decentralized, yet the importance of efficient operation is undiminished. Often the system operator must make a dispatch decision based on bids rather than costs, and will need to serve a fixed load at a price fixed by regulatory price caps. The tariff mechanism we propose is simple to implement, because the system operator does not need to know the true parameters of the generators' cost functions. Instead, the SO needs to know only the system load and the generators' bids.

Despite this simplicity, the tariff has very strong short-run efficiency implications. It allows the operator to identify the least-cost producers, and thereby minimize the total cost of production. Generators obtain a profit equal to the difference between price and the cost of the marginal producer. Remarkably, efficiency does not require a large number of competitive bidders; the equilibrium bids produce a fully efficient outcome even in a duopoly.

This paper has introduced the tariff system and demonstrated its efficiency, but it has not explored interesting questions related to its long-term implications. Imposing an efficient tax has the side benefit of generating revenues, which could be used to finance transmission system improvements, pay startup costs of generators, or defray other costs of system managers, in much the way that system charges and transmission tariffs do in current systems. The extraction of tariff revenues will have obvious implications for the profitability of the industry, and for the level of investment that it can attract. Price and quantity constraints typically occur at peak loads and times of system stress, which are precisely the times that are crucial for collecting the revenues needed to finance large-scale capital investment. The tariff mechanism may be an effective tool for balancing efficiency and equity while maintaining system stability when the system is both stressed and constrained.

8 Mathematical Appendix

Existence and Uniqueness of Nash Equilibrium

The solution to the SO's cost-minimization problem can be represented by the system

$$\frac{\partial B_i(a_i(x_i), x_i)}{\partial a_i(x_i)} \frac{\partial a_i(x_i)}{\partial x_i} + \frac{\partial B_i(a_i(x_i), x_i)}{\partial x_i} = \lambda \qquad i = 1, 2 \ldots N \tag{5}$$

represents N *independent* non-linear differential equations of the first order. Because the bid depends on the bid parameter, i.e.,

$$\frac{\partial B_i(\cdot)}{\partial a_i} \neq 0$$

We can, without loss of generality, rewrite the equation in the form:

$$\frac{\partial a_i(x_i)}{\partial x_i} = a_i' = g(a_i, x_i) \qquad i = 1, 2 \ldots N$$

where

$$g(a_i, x_i) \equiv \frac{\lambda - \frac{\partial B_i(a_i(x_i), x_i)}{\partial x_i}}{\frac{\partial B_i(a_i(x_i), x_i)}{\partial a_i(x_i)}}$$

Assuming that all functions and their partial derivatives are continuous in the rectangle $(\underline{a}_i \leq a_i \leq \overline{a}_i \ ; \ \underline{x}_i \leq x_i \leq \overline{x}_i)$, there exists a unique solution $a_i = \phi(x_i)$ of the initial value $a_i(x_{i0}) = a_{i0}$. (See Boyce, 1996).

Assuming that vector $(\underline{a}_i \ ; \ \underline{x}_i) \geq 0 \ \forall \ i$ there exists a positive vector of bids $(a_1, a_2, ...a_N) \geq 0$ and a positive production vector $(x_1, x_2, ...x_N) \geq 0$ that solves the SO's problem.

9 References

Boyce W.E., Diprima R.C., (1996), “Elementary Differential Equations”, Wiley & Sons Inc., New York, Chichester, Brisbane, Toronto, Singapore, 6th edition, 1996, p.41

Borenstein S., Bushnell J, Stoft S, (2000) “ The competitive aspects of transmission capacity in a deregulated electricity industry”. *RAND Journal of Economics* Vol. 31, No. 2 Summer 2000 pp.294-325

Cardell, Judith B (1997), “Market Power and Strategic Interaction in Electricity Networks”. *Resource and Energy Economics* v19, n1-2 (March 1997): 109-37

Federal Energy Regulatory Commission (FERC) (2002) Notice of Proposed Rulemaking, “Remedying Undue Discrimination through Open Access Transmission Service and Standard Market Design.” Docket No. RM01-12-000, July 31, 2002

Joskow P, Tirole J, “Transmission rights and market power on electric power networks”, *RAND Journal of Economics* Vol. 31, No. 3 Autumn 2000 pp.450-487

Oren, Shmuel (1997) “Economic Inefficiency of Passive Transmission Rights in Congested Electricity Systems with Competitive Generation”. *The Energy Journal*, Vol. 18, NO. 1-1997

Stoft, S (2002): *Power System Economics: Designing Markets for Electricity.* NY: Wiley-IEEE Press

Wilson Robert (2002), “Architecture of Power Markets” *Econometrica*, Vol. 70 NO.4- July.

Roth, Alvin (2002) “The Economist as Engineer: Game Theory, Experimentation, and Computation as Tools for Design Economics” Econometrica, Vol. 70 NO.4- July,

Hogan, William (1997) “A Market Power Model with Strategic Interaction in Electricity Networks” *The Energy Journal*, Vol. 18, NO.4-

Hogan, William (1992) “Contract Networks for electric power transmission”. *The Energy Journal*,

Stoft Steven (1992) “Financial Transmission Rights Meet Cournot: How TCC Curb Market

Power" *The Energy Journal*.

Willems Bert (2002) "Modelling Cournot Competition in an Electricity Market with Transmission Constraints". *The Energy Journal*, Vol. 23 No. 3

Wolfram Catherine, "Strategic bidding in a multiunit auction: an empirical analysis of bids to supply electricity in England and Wales". *RAND Journal of Economics* Vol. 29, No. 4, Winter 1998 pp.703-725