# STOCHASTIC APPROACH FOR AN ENDOGENOUS CAPACITY EXPANSION LONG-TERM WORLD COAL MARKET MODEL

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## **OVERVIEW**

The goal of this paper is to present a long-term (from 2009 to 2035) stochastic world coal model with endogenous investment capacity. A first deterministic coal model with endogenous investment capacity had been developed [1]. This first model was based on a bottom-up approach. The economic concept chosen in our modelling of the global steam coal trade was production (extraction + transport) organized to response to the demand at a least-cost. Two economical formulations had been treated: a rational price expectation [2] and an adaptive price expectation [3]. From this first model, we propose to handle uncertainty on demand in a framework in between rational price expectations and adaptive price expectations by using an original tree scenario approach. Then, like we have done for the deterministic approach, we propose by performing some mathematical analysis, a breakdown of the long-run marginal cost into a marginal production cost and a marginal equivalent investment cost.

### **METHODS**

There are many different methods to model uncertainty in optimization: dynamical programming [4], robust linear programming [5], statistic approaches [6]. Our stochastic approach is based on a well-known tree scenario method [7]. The tree is built from demand scenarios. For instance, we consider three kinds of scenarios from 2009 to 2035: a low demand, a medium demand and a high demand scenario. Then, we simulate each scenario in the following way (Figure 1). On the short-term period, we consider only one scenario (the scenario we are simulating), while on the medium-term period and on the long-term period, we define branches with the three scenarios available (low, medium and high). Once, the tree is built, branches with low probability of realization are not kept after measuring the distance between the simulated scenario and the other scenarios. If the distance is bigger than a predefined number, the branch is not kept. For every branch kept we assign at every node of branch a weight between 0 and 1 reflecting the probability of realization. A good image is to think to a capacity of a mine. A capacity on a branch will be available with a weight between 0 and 1 reflecting that the capacity could be available between 0% and 100%. Once, the tree is built a minimization cost in expectation is calculated under constraints living on the nodes of the tree.

The originality of our approach is to move the tree during time iteration in order to offer both deterministic and stochastic view (none deterministic view of the future) to the investor. Time iterations go from 2009 to 2035. We rebuild for each time-iteration a new tree and we solve a linear programming problem with a cost function in expectation (taking into account the weight of each possible future). In this way, the root of the tree is always in the short-term period, while the medium-term and the long-term period constitute the branches of the tree. After one time-iteration, we keep only the investment decision in the root of the tree.

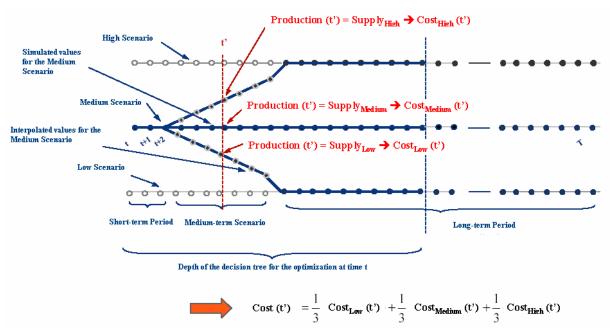


Fig.1. Short-Term WCM and Long-Term CEM model interaction

### RESULTS

From the macro-economical point of view, the slipping tree scenario approach is equivalent to the situation, where the investor at time t has perfect view of the near future (the short-term period) and some uncertainties on the medium-term and long-term periods to decide new projects. The model is able to give us some insights about the sensitivity of the investment decisions at time t to the length of the short-term, medium-term and the long-term periods. The investment depends also on the weight assigned on each branch. For instance, if a weight is very close to 1 on the high-scenario-branch and we simulate the low scenario, the investor is going to develop at time t some investments that could be not profitable in the future because he is living (we are simulating the low scenario) on the low scenario. In other words, if the investor takes decision considering wrong views of the future (he considers the high demand with a too high probability of realization), he will invest too much with in the future a low demand. In this work, another result has been obtained by performing some mathematical results. As we have done for the deterministic model by calculating the lagrangian and evaluating its stationarity, we have obtained analytical solutions with only one investor. In this situation, we demonstrate that the long-run marginal costs are just a function of production cost, when lead times are taking into account. In a deterministic model, the longrun marginal cost breakdown shows us that long-run marginal costs are function of both production and investment costs, when lead times are taken into account.

### CONCLUSIONS

We present in this paper a long-term (from 2009 to 2035) stochastic world coal model with endogenous investment capacity. The uncertainty is handled with an original approach based on a tree scenario method. The originality is based on the dynamic of the tree over time. The dynamic allows us to solve linear programming problems iteratively and to mix deterministic and stochastic views. This model is also a mix between rational price expectation and adaptive price expectation. At each time-iteration a tree is defined and the "investor" has in front of him all the possible futures trend with a weight attached to each possible future trend. Then, a linear programming method is used to find the optimal solution. The investor has not moved yet and one can say that he is in the framework of the rational price expectations. He

has a perfect view of the probabilities. But by the next iterations, the "investor" moves and then we build a new tree. The probabilities change. In this way, one can say that the investor has a new view of the future and is not any more in the framework of the rational price expectations. In conclusion, by moving the tree, we encapsulate the rational price expectation framework in an adaptive price expectation framework.

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