ON THE INFLUENCE OF INFLOW UNCERTAINTY ON PRICING OF ELECTRICITY FROM HYDROPOWER

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(1) **Overview**

A very basic feature of hydropower electricity operation is that inflows to the reservoirs are stochastic variables. A decision about use of water, i.e., production in the current period and transferring water to the next period, has to be made in the current period while the inflows of the future periods up to the horizon are known only by their predictions. The need for both space heating and cooling depends on the outside temperature that is stochastic. Further real-life stochastic events in the case of a complete electricity system with transmission lines and thermal capacities, are transmission capacity being reduced due to transformer accidents, storms blowing down trees on lines, breaking of lines due to icing, etc., and thermal capacities going down due to accidents. Considering windmills the output depends crucially on the wind speed that is stochastic. The best the social planner can do in the current period is to formulate an optimal plan by maximising the *expectation* of the sum of consumer plus producer surpluses for future periods by applying backwards induction.

The problem for finding optimal solutions of the hydro management problem created by uncertainty was recognised early in the literature (Little, 1955; Koopmans, 1957; Gessford and Karlin, 1958; Massé, 1946; Morlat, 1964). In Norway a special solution strategy termed the expected water value approach was introduced in Hveding (1968) based on Stage and Larsson (1961). In the more specialised engineering literature an early contribution was Pereira (1989). In the engineering literature stochastic problems are solved numerically using discrete-time stochastic dynamic programming formulations. Solution algorithms have been developed over the last decades in the engineering literature approximating optimal solutions However, a discussion of the qualitative nature of the solution to such problems understandable for economists is hard to find. The purpose of the paper is to provide such qualitative understanding of the impact of uncertainty on hydropower electricity pricing.

(2) Methods

The analysis is based on the assumption that inflows only are assumed to be stochastic. Furthermore, only a reservoir constraint will be introduced, within an aggregated system consisting formally of one plant and one reservoir. Simplifying further by considering two periods only, assuming that the inflow for the current period is known, the decision problem evaluated in period 1 under uncertainty is to determine how much water to be used in period 1 and how much to be left in the reservoir for period 2. The expected price for period 2 will be a function of the amount of water transferred from period 1. Such a function may be termed the *expected water value table* corresponding to a concept used in the literature (Hveding, 1968). The information given by such a "table" may be utilised determining the actual quantities and prices as time evolves from the start of the planning period.

(3) Results

The methodological approach is illustrated in Figure 1 with three situations for period 1; a normal period wrt inflows (N), a wet period (W) and a dry period (D). The inflows are measured by AC^i (i= N, W, D), and the storage capacity by B^iC^i. The demand curve for period 1 is for simplicity drawn as a linear curve, and likewise for the expected price for period 2. The expected water values start at the maximal transfer of water from period 1 to period 2, and end at the minimal transfer of zero. For the normal situation (N) we get the rule that the price in period 1 should be set equal to the expected price in period 2. For a wet situation in period 1 we get a corner solution of the price in period 1 set lower than the expected price in period 2 and maximal transfer of water, and for a dry situation we get a corner solution of zero transfer of water and a higher price in period 1 than the expected price in period 2.

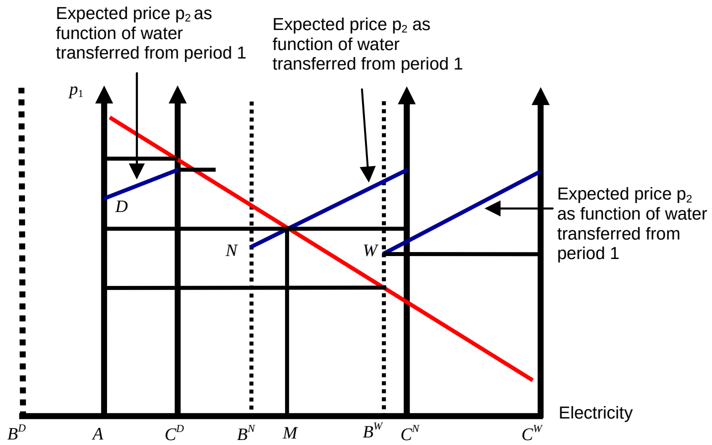

Figure 1. Optimal prices and transfer of water from period 1 to period 2

(4) Conclusions

The general qualitative rule is to set the price in the current deterministic period equal to the expected price in the next period. This expectation is a function of the amount of water transferred form the current period to the next and the problem has to be solved using backwards induction. Possible corner solutions leading to deviations from the pricing rule should be noted. Assuming the distribution for inflows to be known no new information will be gotten as time goes by. Introducing many periods it will be arbitrary that expectations will be met. The realised price in the consecutive next period will therefore deviate systematically from the expectation of the price held in the previous period. The actual prices will fluctuate, and will be lower than expected if inflows are higher than inflows corresponding to the expected price, and higher if inflows turn out to be lower.

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