# ACCOUNTABILITY FOR UNCERTAINTY IN ELECTRICITY MARKETS

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## Overview

Large scale integration of stochastic renewable energy sources (RES) has brought important economic and environmental benefits but also challenges in electricity markets given they were designed under the dominance of dispatchable and fully predictable sources of energy. In this context, the uncertain and intermittent nature of RES, has prompted many countries to adopt favourable regulatory frameworks and support mechanisms that safeguard RES producers from the price volatility that they introduce in electricity markets. However, further increase in shares of RES tends to make such support mechanisms inefficient as they transfer the costs of power imbalances to the society.

In this work we hold RES producers accountable for the uncertainty they introduce, through market based support mechanisms that allocate premiums proportionally to the aforementioned uncertainty. We use the probabilistic electricity market introduced by Papakonstantinou and Pinson (2016) that models producers' offers as probabilistic estimates, instead of deterministic offers (e.g. point forecasts). We design a side-payment based on an information-divergence metric and through a numerical example illustrate that the proposed mechanism creates incentives for precise offers, while keeping the favourite properties of the two-stage settlement.

## Methods

The probabilistic electricity market is based on the two-price balancing settlement and can be derived analytically by following an intuitive process that replaces balancing costs with a probabilistic loss function: an affine transformation of the Brier Score, a strictly proper scoring rule. Although, strictly proper scoring rules are used to elicit accurate forecasts, with their application in balancing having similar effects, support mechanisms such as feed-in tariffs or price premiums can alleviate such favourable properties. Still, the decomposition of the Brier Score (Hersbach, 2000) allows the design of *market based support mechanisms* that create incentives for precise estimates by quantifying the value of probabilistic information using the selectivity index introduced by Matheron (1984). Nevertheless, scoring rules depend on the realisation of the stochastic outcome and hence are not suitable for use in the day-ahead market.

To address this, we propose the use of *Jensen-Shannon divergence* (JSD), an information-theoretic measure of the divergence of two probability distributions P and Q (Lin, 1991). JSD is based on the *Kullback-Leibler divergence* (KLD) which measures the inefficiency of assuming that the actual distribution is Q, while it is infact P. KLD is in turn based on the *Differential Entropy*, the continuous counterpart of the *Shannon Entropy*. JSD is an improvement of the KLD, as it is bounded and symmetric, which given that JSD(P,Q) $\geq$ 0 with JSD=0 iff P=Q, shows that JSD defines a *distance space*, but not an actual *metric space*. For a distance to become metric it must also satisfy the triangle inequality. Endres and Schnidelin (2003) prove that the square root of JSD can satisfy the three mentioned properties and thus is considered a metric space. Their metric, denoted here as D<sub>JSD</sub> and referred as *JSD metric*, is bounded in [0,1] if it is divided by  $\sqrt{2}$ , assuming the use of logarithms with base 2 when calculating the differential entropies.

In this paper we use the JSD metric to create incentives for RES producers to provide precise probabilistic offers in an electricity market that can accept them. Using such offers allows us to allocate a part of the price premium that the RES producer would have fully received in a conventional market. The allocation is based on the divergence between a producer's reported probabilistic estimate and a very sharp distribution of the same mean (converges to a Dirac distribution with a same centre). Naturally, a producer providing a very sharp distribution will receive the full tariff as  $(1-D_{JSD}) \rightarrow 1$ . As we illustrate through an example below, the use of a side-payment based on the D<sub>JSD</sub> complements the scoring rule based probabilistic market by guarantying that the producers will report *accurate* and *precise* distributions despite the use of a support mechanism.

## Results

In this section we numerically evaluate the sensitivity of an information divergence metric and illustrate its impact on support mechanisms. Specifically, we compute  $D_{JSD}(P,Q)$  where Q is the reported Beta distribution with  $\mu$ =0.55 and  $\sigma^2$ =0.025 and P is a numerical approximation of a Dirac distribution centred at  $\mu$ =0.55, with

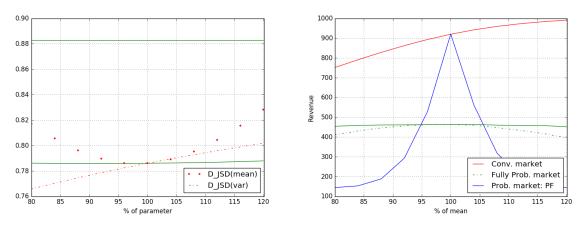


Figure 1: Calculations of entropy between distributions and application in a decision theoretic framework.

0.55 being the mean of the actual distribution of production. Inaccuracies in the reported distribution Q are modelled by multiplying the mean and the variance of Q with a parameter  $\varepsilon$  taking values from 0.80 to 1.20.

In the first graph in Fig. 1, the curvature of D\_JSD(mean) shows that the JSD metric is minimised at truthful reporting of Q, while the shape of D\_JSD(var) shows that the metric increases as the variance increases. The green lines denote the relative bounds: the lower bound is the distance between the Dirac approximations centred at  $\varepsilon_*\mu$  and Q with mean  $\varepsilon_*\mu$ , and the upper bound is the distance between the Dirac approximations and U(0,1).

In the second graph in Fig. 1 we evaluate the support mechanisms by comparing the total revenues of a RES producer under various market frameworks. The red line represents the conventional market, while the blue line the probabilistic one using point forecasts and the JSD based metric (1-  $D_{JSD}$ ). In both cases the producer receives an additional payment for its scheduled production at a price 4 times the day-ahead clearing price. It is shown that the premium distorts the conventional market, while the JSD metric creates an incentive for the producer to report accurate point forecasts. Moreover, we eliminate the dependence on the scheduled quantity by using the unit's capacity. The dotted green line is the JSD metric between Qs with varying means and the Dirac approximation with  $\mu$ =0.55, while the green line is the relative upper bound. It can be seen that the JSD metric reduces the premium while it maintains the incentives for generating accurate estimates. However, this hypothesis cannot be tested without a solid theoretical analysis of the used information-divergence measures, followed by extensive simulations in realistic case studies.

#### Conclusions

There are several obstacles en route to power systems with 100% renewables. Operational issues connected with renewables' unpredictability and intermittency are closely followed by the regulatory challenges of existing support schemes. In this paper we address both challenges by proposing a market based support mechanism that rewards producers for accurate and precise probabilistic offers, by measuring the information value of the distributions drawn from the probabilistic offers submitted by RES producers.

#### References

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