Reconciling Hotelling Resource Models with Hotelling's Accounting Method

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In recent decades, economists have taken increased interest in measuring (accounting for) the many capital inputs to economic activity based on economic theory. Innumerable studies have striven to make the national accounts more comprehensive by including assets, such as natural and environmental assets, for which market prices do not exist or are not adequate. Among the first studied were nonrenewable resources, which had assumed great importance during and after the oil crises of the 1970s.

Taking center stage has been a quest to determine theoretically supported measures of net product, net income, net investment and net depreciation, which directly affect human well-being, as opposed to gross measures. With few exceptions, the theory has utilized optimal-control methods applied to mathematical expressions for the path of an economy through time.

A finding of these economic investigations is that only the monetary values of "real" variables should be reported in accounting statistics. Because they are *pure price effects*, capital gains should be excluded. This finding is of practical importance for reported income and product in resource-producing regions. According to the theory of nonrenewable resources enunciated by Harold Hotelling in 1931 and significantly examined since, the prices of oil and gas, uranium, noble and base metals, etc. will (at least eventually) increase through time as they are depleted, so that the values of in-ground deposits will increase commensurately, producing capital gains.

In this paper, we revisit this issue. A general investigation demonstrates that the neglect of capital gains is inconsistent with the fundamental property of depreciation that an asset's depreciation over time must sum to its original value, as enunciated by Hotelling in a 1925 paper. A modification of the underlying mathematical relationships that is consistent with the property lends theoretical support to including capital gains. Capital gains exist if time plays a direct role in mathematical expressions, that is to say, if the model *non-autonomous*. In a non-autonomous model, capital gains should be included. We argue that, in practice, time plays a direct role.

To grasp the implications more fully, we study six canonical models from the economics of nonrenewable resources. Even in the simplest models, of a competitive market or a planner's solution, non-autonomy raises fine distinctions. In cases where aggregation is subtle, namely, the stock effect or durability of the resource, mathematical representation can mask the effects of non-autonomy. Moreover, if a firm faces u-shaped average extraction cost, an unpriced, unobserved asset can affect accounting. The economic and accounting implications of such assets are pursued in a separate section. We are able to clarify how certain assets may be theoretically discernable but not given accounting prices. Once we adjust for unobservable values, we obtain expressions for net income, product, investment and depreciation that take a familiar form but include capital gains.

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