A Primer on Estimating Short and Long-Run Demand Elasticities: Energy-Sector Applications

John T. Cuddington

William J. Coulter Professor of Mineral Economics, Colorado School of Mines

Leila Dagher

Assistant Professor of Economics, American University of Beirut

Many empirical exercises estimating demand functions, whether in energy economics or other fields, are concerned with estimating dynamic effects of price and income changes over time. A review of the literature on energy demand reveals two common deficiencies: (i) the use of restricted models without testing the relevant restrictions and (ii) the omission of standard errors when reporting short-run and especially long-run elasticities. Although our paper focuses on demand estimation in the energy sector, the issues raised are also relevant when estimating dynamic supply equations. Moreover, they apply in a wide variety of contexts beyond the energy sector where estimating short (SR) and long-run (LR) elasticities is a recurring topic of interest.

This paper first reviews a number of commonly-used dynamic demand specifications to highlight the implausible *a priori* restrictions that they place on SR and LR elasticities. These models include a log-linear model with an autoregressive error process, the partial adjustment model (PAM), and a simple error-correction model (ECM) with no lagged differences. All of these specifications impose very implausible restrictions on the demand equations.

In both the PAM and the simple ECM with no lagged differences, the LR to SR *ratios* for all demand elasticities are forced to be the same. Besides this unreasonable LR/SR ratio restriction, these specifications also restrict all short-run elasticities (price, cross-price, and income) to be less than their LR counterparts. Again this is not a reasonable *a priori* restriction to impose. Rather it is a testable restriction if one begins with a more general specification.

The first-order autoregressive distributed-lag model [ADL(1, 0-1, 0-1, 0-1)] is more general than the other specifications and nests them as special cases. Each determinant of demand (e.g. price, cross-price, and income) enters with at least two time subscripts (lags zero and one, say) and no non-linear restrictions are placed on their coefficients.

In general, to avoid the unreasonable restrictions imposed by various demand specifications, the researcher can estimate an ADL model adopting a general-to-specific modeling methodology -- beginning with sufficient lags of each variable to insure that serial correlation has been expunged from the error process. The number of lags can then be selected based on a standard lag selection criterion (e.g., the Schwarz or Akaike information criterion).

General ECMs also eliminate the implausible *a priori* restrictions! Indeed, every ADL – whether it includes contemporaneous regressors or not – can be always rewritten as an equivalent ECM. The two are isomorphic, but the ECM parameterization is especially useful when cointegrated I(1) variables are involved. Moreover, the ECM variant of the ADL allows us to identify the SR and LR elasticities directly, i.e., without auxiliary 'hand' calculations.

The second part of the paper focuses on the ADL model and discusses estimation issues for getting point estimates and associated standard errors for both short and long-run elasticities – key information that is missing from many published studies. In this section we examine the following relevant cases: (i) only *lags* of the independent variables (not contemporaneous values) appear in the ADL and all variables are stationary, and (ii) the ADL contains contemporaneous as well as lagged regressors; all variables are stationary, and (iii) some variables are nonstationary, but are not cointegrated, and (iv) variables are non-stationary and cointegrated.

The last section begins with a vector-error correction model, but shows that it can be reduced to a single-equation conditional error-correction model. This specification is used to estimate short-run and long-run elasticities for residential electricity demand in Xcel Energy's Minnesota service area. The ECM dominates the AR(1), PAM, and simple ECM, and produces sensible short and long-run price, cross-price, and income elasticities. The elasticities have been of longstanding interest to analysts and policymakers working on the energy sector.