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Online appendix for

Specifying an Efficient Renewable Energy Feed-in Tariff

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Appendix A: Deriving the partial differential equation governing the discounted expected FiT payoff

The Kolmogrov backward equation governing the probability $\rho(S_\tau|S_t)$ of getting a VWAP S at time τ, given a VWAP S_t at time t ($\tau \le t$) is:

$$-\frac{\partial \rho}{\partial \tau} = \mu S \frac{\partial \rho}{\partial S} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 \rho}{\partial S^2}. \tag{1}$$

with final condition $\rho(S_t|S_t) = \delta(S - S_t)$, where $\delta(.)$ represents the Dirac delta function (Wilmott et al., 1993; Wilmott, 2000). The discounted expected value of the FiT at time τ, given S_τ and expiry at time t may be defined as[1]

$$P(S_\tau, \tau, t) = e^{-r(t-\tau)} \int_0^\infty \rho(S_\tau|S_t) V_t dS_t. \tag{2}$$

Now consider the following derivatives of equation 2:

$$\frac{\partial P}{\partial \tau} = rP + e^{-r(t-\tau)} \int_0^\infty \frac{\partial \rho}{\partial \tau} V_t dS_t \tag{3}$$

$$\frac{\partial P}{\partial S} = e^{-r(t-\tau)} \int_0^\infty \frac{\partial \rho}{\partial S} V_t dS_t \tag{4}$$

$$\frac{\partial^2 P}{\partial S^2} = e^{-r(t-\tau)} \int_0^\infty \frac{\partial \rho}{\partial S^2} V_t dS_t \tag{5}$$

Multiplying equation (1) by $e^{-r(t-\tau)} V_t$ and integrating over all possible values of V_t leads to

$$-e^{-r(t-\tau)} \int_0^\infty \frac{\partial \rho}{\partial \tau} V_t dS_t = \mu S e^{-r(t-\tau)} \int_0^\infty \frac{\partial \rho}{\partial S} V_t dS_t + \frac{\sigma^2 S^2}{2} e^{-r(t-\tau)} \int_0^\infty \frac{\partial^2 \rho}{\partial S^2} V_t dS_t \tag{6}$$

When equations (3) - (5) are substituted into equation (6) the following PDE is obtained:

$$\frac{\partial P}{\partial \tau} + \mu S \frac{\partial P}{\partial S} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 P}{\partial S^2} - rP = 0, \tag{7}$$

which has the terminal condition:

$$P(S_t, t, t) = \int_0^\infty \rho(S_t|S_t) V_t dS_t, \tag{8}$$

$$= \int_0^\infty \delta(S - S_t) V_t dS_t, \tag{9}$$

$$= V_t. \tag{10}$$

[1]When equation (7) is being solved to find the discounted expected cost of FiT, V_t should be replaced by f_t in equation (2).

Appendix B: Solving the GBM partial differential equation for shared upside policy

Obtaining the discounted expected value and discounted expected Cost of FiT for the both Shared Upside and Cap & Floor policies (using equation (7)) is not trivial. In this section details of how these solutions are obtained are provided. The way in which they are derived is similar to that used to find solutions to the Black-Scholes Partial Differential Equation (PDE) for European call and put options Wilmott et al. (1993).

Firstly consider the following transformations:

$$\begin{aligned} S &= Ke^{x}, && (11)\\ \tau &= t - \frac{l}{\frac{\sigma^2}{2}}, && (12)\\ P &= Kw(x,l). && (13) \end{aligned}$$

Using these transformations equation (7) becomes

$$\frac{\partial w}{\partial l} = \frac{\partial^2 w}{\partial x^2} + \frac{\partial w}{\partial x}(E_1 - 1) - E_2 w \quad (14)$$

where $E_1 = \frac{\mu}{\frac{\sigma^2}{2}}$ and $E_2 = \frac{r}{\frac{\sigma^2}{2}}$. The terminal condition (Equation (10)), now an initial condition, becomes

$$w(x,0) = V_t^{'}, \quad (15)$$

where $V_t^{'}$ is the REFIT payoff adjusted by the transformations in equations (11) - (13). Now consider the following change of variable

$$w(x,l) = e^{-\frac{1}{2}(E_1-1)x-(\frac{1}{4}(E_1-1)^2+E_2)l}u(x,l). \quad (16)$$

This change means equation (14) becomes

$$\frac{\partial u}{\partial l} = \frac{\partial^2 u}{\partial x^2}, \quad (17)$$

with $-\infty < x < \infty$, $l > 0$ and initial condition

$$u(x,0) = u_0(x) = V_t^{''}, \quad (18)$$

where $V_t^{''}$ is the REFIT payoff adjusted by the change of variable in equation (16). Equation (17) is the well known diffusion problem which has solution

$$u(x,l) = \frac{1}{2\sqrt{\pi l}} \int_{-\infty}^{\infty} u_0(s) e^{\frac{-(x-s)^2}{4l}} ds, \quad (19)$$

Wilmott et al. (1993).

Shared upside policy

We will firstly solve equation (19) for the Shared Upside policy. The payoff of this policy is $V_t = \max(K, K+\theta(S-K))$ which means that, taking into account the transformations in equations (11) - (13) and the change of variable in equation (16), the initial condition of the diffusion equation (Equation (18)) becomes

$$u_0(x) = e^{\frac{1}{2}(E_1-1)} + \theta \max(e^{\frac{1}{2}(E_1+1)x} - e^{\frac{1}{2}(E_1-1)x}, 0), \quad (20)$$

which means that equation (19) becomes

$$u(x,l) = \frac{1}{2\sqrt{\pi l}} \int_{-\infty}^{\infty} (e^{\frac{1}{2}(E_1-1)} + \theta \max(e^{\frac{1}{2}(E_1+1)s} - e^{\frac{1}{2}(E_1-1)s}, 0)) e^{\frac{-(x-s)^2}{4l}} ds. \quad (21)$$

When the change of variable $x^{'} = (s-x)/\sqrt{2l}$ is considered, equation (21) becomes

$$u(x,l) = \frac{1}{2\sqrt{\pi l}} \int_{-\infty}^{\infty} (e^{\frac{1}{2}(E_1-1)} + \theta \max(e^{\frac{1}{2}(E_1+1)(x+x^{'}\sqrt{2l})} - e^{\frac{1}{2}(E_1-1)(x+x^{'}\sqrt{2l})}, 0)) e^{\frac{1}{2}x^{'2}} dx^{'}, \quad (22)$$

which can be broken up into three integrals as follows

$$u(x,l) = \frac{1}{2\sqrt{\pi l}} \int_{-\infty}^{\infty} e^{\frac{1}{2}(E_1-1)} x^{'2} dx^{'}$$

$$+\theta(\frac{1}{2\sqrt{\pi l}} \int_{-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1+1)(x+x^{'}\sqrt{2l})} e^{\frac{1}{2}x^{'2}} dx^{'}$$

$$-\frac{1}{2\sqrt{\pi l}} \int_{-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1-1)(x+x^{'}\sqrt{2l})} e^{\frac{1}{2}x^{'2}} dx^{'}), \tag{23}$$

which we label I_0, I_1 and I_2 respectively, i.e.,

$$u(x,l) = I_0 + \theta(I_1 - I_2). \tag{24}$$

We now consider the first integral I_0

$$I_0 = \frac{1}{2\sqrt{\pi l}} \int_{-\infty}^{\infty} e^{\frac{1}{2}(E_1-1)-\frac{1}{2}x^{'2}} dx^{'}, \tag{25}$$

$$= \frac{e^{\frac{1}{2}(E_1-1)x+\frac{1}{4}(E_1-1)^2 l}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x^{'}-\frac{((E_1-1)^2\sqrt{2l})}{2})^2} dx^{'}, \tag{26}$$

$$= \frac{e^{\frac{1}{2}(E_1-1)x+\frac{1}{4}(E_1-1)^2 l}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{\Lambda^2}{2}} dx^{'}, \tag{27}$$

$$= e^{\frac{1}{2}(E_1-1)x+\frac{1}{4}(E_1-1)^2 l}. \tag{28}$$

Now consider the second integral I_1

$$I_1 = \frac{1}{2\sqrt{\pi l}} \int_{-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1+1)-\frac{1}{2}x^{'2}} dx^{'}, \tag{29}$$

$$= \frac{e^{\frac{1}{2}(E_1+1)x+\frac{1}{4}(E_1+1)^2 l}}{\sqrt{2\pi}} \int_{-x/\sqrt{2l}}^{\infty} e^{-\frac{1}{2}(x^{'}-\frac{((E_1+1)^2\sqrt{2l})}{2})^2} dx^{'}, \tag{30}$$

$$= \frac{e^{\frac{1}{2}(E_1+1)x+\frac{1}{4}(E_1+1)^2 l}}{\sqrt{2\pi}} \int_{-x/\sqrt{2l}}^{\infty} e^{-\frac{\Lambda^2}{2}} dx^{'}, \tag{31}$$

$$= e^{\frac{1}{2}(E_1+1)x+\frac{1}{4}(E_1+1)^2 l} N(d_1), \tag{32}$$

where

$$d_1 = \frac{x}{\sqrt{2l}} + \frac{1}{2}(E_1+1)\sqrt{2l}, \tag{33}$$

and

$$N(\Lambda) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\Lambda} e^{\frac{1}{2}\lambda^2} d\lambda, \tag{34}$$

represents the cumulative distribution function of the standard normal distribution. The solution for the integral I_2 is obtained in a similar manner to that of I_1 with $E_1 + 1$ replaced by $E_1 - 1$ throughout giving

$$I_2 = e^{\frac{1}{2}(E_1-1)x+\frac{1}{4}(E_1-1)^2 l} N(d_2), \tag{35}$$

where

$$d_2 = \frac{x}{\sqrt{2l}} + \frac{1}{2}(E_1-1)\sqrt{2l}. \tag{36}$$

Using equations (24), (28), (32) and (35) gives the following solution to equation (21):

$$u(x,l) = e^{\frac{1}{2}(E_1-1)x+\frac{1}{4}(E_1-1)^2 l} + \theta(e^{\frac{1}{2}(E_1+1)x+\frac{1}{4}(E_1+1)^2 l} N(d_1) - e^{\frac{1}{2}(E_1-1)x+\frac{1}{4}(E_1-1)^2 l} N(d_2)). \tag{37}$$

When the change of variable in equation (16) and the transformations $x = \ln(\frac{S}{K})$, $l = \frac{1}{2}\sigma^2(t-\tau)$, and $P = Kw(x,l)$ are recovered, equation (37) becomes

$$P(S_\tau, \tau, t) = Ke^{-r(t-\tau)} + \theta(S_\tau e^{(\mu-r)(t-\tau)} N(d_1) - Ke^{-r(t-\tau)} N(d_2)), \tag{38}$$

with

$$d_1 = \frac{\ln(\frac{S_\tau}{K}) + (\mu + \frac{\sigma^2}{2})(t-\tau)}{\sigma\sqrt{t-\tau}}, \quad (39)$$

$$d_2 = \frac{\ln(\frac{S_\tau}{K}) + (\mu - \frac{\sigma^2}{2})(t-\tau)}{\sigma\sqrt{t-\tau}}. \quad (40)$$

Equation (38) represents the discounted expected value of REFIT under the Shared-upside policy. Note: in Section 3 of the main text τ is assumed equal to zero.

Cost of FiT for shared upside policy

We now consider solving equation (19) for the Cost of FiT to the policymakers, under the shared upside policy, i.e., when $f_t = \max(0, K - S_t) - (1-\theta)\max(0, S_t - K)$. When the transformations in equations (11) - (13) and the change of variable in equation (16) are taken into account for this cost, the initial condition of the diffusion equation (Equation (18)) becomes

$$u_0(x) = \max(e^{\frac{1}{2}(E_1-1)x} - e^{\frac{1}{2}(E_1+1)x}, 0) - (1-\theta)\max(e^{\frac{1}{2}(E_1+1)x} - e^{\frac{1}{2}(E_1-1)x}, 0). \quad (41)$$

When this initial condition and the change of variable $x^{'} = (s-x)/\sqrt{2l}$ is considered, the solution to the diffusion equation (19) becomes

$$u(x,l) = \frac{1}{2\sqrt{\pi l}}\int_{-\infty}^{\infty}(\max(e^{\frac{1}{2}(E_1-1)(x+x^{'}\sqrt{2l})} - e^{\frac{1}{2}(E_1+1)(x+x^{'}\sqrt{2l})}, 0) -$$
$$(\max(e^{\frac{1}{2}(E_1+1)(x+x^{'}\sqrt{2l})} - e^{\frac{1}{2}(E_1-1)(x+x^{'}\sqrt{2l})}, 0)e^{\frac{1}{2}x^{'2}}dx^{'}. \quad (42)$$

In similar manner to equation (23), equation (42) can be broken into four integrals as follows

$$u(x,l) = \frac{1}{2\sqrt{\pi l}}\int_{x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1-1)(x+x^{'}\sqrt{2l})}e^{\frac{1}{2}x^{'2}}dx^{'}$$
$$-\frac{1}{2\sqrt{\pi l}}\int_{x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1+1)(x+x^{'}\sqrt{2l})}e^{\frac{1}{2}x^{'2}}dx^{'},$$
$$-(1-\theta)(\frac{1}{2\sqrt{\pi l}}\int_{-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1+1)(x+x^{'}\sqrt{2l})}e^{\frac{1}{2}x^{'2}}dx^{'}$$
$$-\frac{1}{2\sqrt{\pi l}}\int_{-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1-1)(x+x^{'}\sqrt{2l})}e^{\frac{1}{2}x^{'2}}dx^{'}),$$

which we label I_3, I_4, I_1 and I_2 respectively, i.e.,

$$u(x,l) = I_3 - I_4 - (1-\theta)(I_1 - I_2). \quad (43)$$

The integrals I_1 and I_2 are as previously defined while the integrals I_3, I_4 are obtained in a similar manner to I_2 and I_1 respectively but the lower limit of integrals changed from $-x/\sqrt{2l}$ to $x/\sqrt{2l}$. Hence,

$$I_3 = e^{\frac{1}{2}(E_1-1)x+\frac{1}{4}(E_1-1)^2 l}N(-d_2), \quad (44)$$

$$I_4 = e^{\frac{1}{2}(E_1+1)x+\frac{1}{4}(E_1+1)^2 l}N(-d_1). \quad (45)$$

Hence, the solution to equation (42) is

$$u(x,l) = e^{\frac{1}{2}(E_1-1)x+\frac{1}{4}(E_1-1)^2 l}N(-d_2) - e^{\frac{1}{2}(E_1+1)x+\frac{1}{4}(E_1+1)^2 l}N(-d_1) -$$
$$(1-\theta)e^{\frac{1}{2}(E_1+1)x+\frac{1}{4}(E_1+1)^2 l}N(d_1) - e^{\frac{1}{2}(E_1-1)x+\frac{1}{4}(E_1-1)^2 l}N(d_2). \quad (46)$$

When the change of variable in equation (16) and the transformations $x = \ln(\frac{S}{K})$, $l = \frac{1}{2}\sigma^2(t-\tau)$, and $P = Kw(x,l)$ are recovered equation (46) becomes

$$EF(S_\tau, \tau, t) = Ke^{-r(t-\tau)}N(-d_2) - S_\tau e^{(\mu-r)(t-\tau)}N(-d_1) -$$
$$(1-\theta)(S_\tau e^{(\mu-r)(t-\tau)}N(d_1) - Ke^{-r(t-\tau)}N(d_2)). \quad (47)$$

Using the fact that $N(-d_1) = 1 - N(d_1)$ and $N(-d_2) = 1 - N(d_2)$ equation (47) can be rewritten as

$$EF(S_\tau, \tau, t) = Ke^{-r(t-\tau)} - S_\tau e^{(\mu-r)(t-\tau)} + \theta(S_\tau e^{(\mu-r)(t-\tau)}N(d_1) - Ke^{-r(t-\tau)}N(d_2)). \quad (48)$$

Equation (48) represents the discounted expected cost of REFIT under the Shared-upside policy. Note: Note: in Section 3 of the main text τ is assumed equal to zero.

Cap & floor policy

We now consider solving equation (19) for the Cap & floor policy which has the payoff $V_t = \max(K, \min(S_t, \bar{S}))$. When the transformations in equations (11) - (13) and the change of variable in equation (16) are taken into account for this payoff, the initial condition of the diffusion equation (Equation (18)) becomes

$$u_0(x) = e^{\frac{1}{2}(E_1-1)} + \max(e^{\frac{1}{2}(E_1+1)x} - e^{\frac{1}{2}(E_1-1)x}, 0) - \max(e^{\frac{1}{2}(E_1+1)x} - \frac{\bar{S}}{K} e^{\frac{1}{2}(E_1-1)x}, 0). \quad (49)$$

When this initial condition and the change of variable $x' = (s-x)/\sqrt{2l}$ is considered, the solution to the diffusion equation (19) becomes

$$u(x,l) = \frac{1}{2\sqrt{\pi l}} \int_{-\infty}^{\infty} (e^{\frac{1}{2}(E_1-1)} + \max(e^{\frac{1}{2}(E_1+1)(x+x'\sqrt{2l})} - e^{\frac{1}{2}(E_1-1)(x+x'\sqrt{2l})}, 0) -$$
$$\max(e^{\frac{1}{2}(E_1+1)(x+x'\sqrt{2l})} - \frac{\bar{S}}{K} e^{\frac{1}{2}(E_1-1)(x+x'\sqrt{2l})}, 0)) e^{\frac{1}{2}x'^2} dx'. \quad (50)$$

Again, in similar manner to equation (23), equation (50) can be broken into five integrals as follows

$$\begin{aligned} u(x,l) &= \frac{1}{2\sqrt{\pi l}} \int_{-\infty}^{\infty} e^{\frac{1}{2}(E_1-1)x'^2} dx' \\ &+ \frac{1}{2\sqrt{\pi l}} \int_{-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1+1)(x+x'\sqrt{2l})} e^{\frac{1}{2}x'^2} dx' \\ &- \frac{1}{2\sqrt{\pi l}} \int_{-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1-1)(x+x'\sqrt{2l})} e^{\frac{1}{2}x'^2} dx', \\ &- \frac{1}{2\sqrt{\pi l}} \int_{\ln(\frac{\bar{S}}{K})-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1+1)(x+x'\sqrt{2l})} e^{\frac{1}{2}x'^2} dx' \\ &+ \frac{\bar{S}}{K} \frac{1}{2\sqrt{\pi l}} \int_{\ln(\frac{\bar{S}}{K})-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1-1)(x+x'\sqrt{2l})} e^{\frac{1}{2}x'^2} dx', \end{aligned} \quad (51)$$

which we label I_0, I_1, I_2, I_5 and I_6 respectively, i.e.,

$$u(x,l) = I_0 + I_1 - I_2 - I_5 + I_6. \quad (52)$$

The integrals I_0, I_1 and I_2 are as previously defined while the integrals I_5 and I_6 are derived in a similar manner to I_1 and I_2 respectively (See equations (29) - (40)), except with the lower limits on the integrals being equal to $\ln(\frac{\bar{S}}{K}) - x/\sqrt{2l}$. Thus,

$$I_5 = e^{\frac{1}{2}(E_1+1)x+\frac{1}{4}(E_1+1)^2 l} N(d_3), \quad (53)$$
$$I_6 = \frac{\bar{S}}{K} e^{\frac{1}{2}(E_1-1)x+\frac{1}{4}(E_1-1)^2 l} N(d_4), \quad (54)$$

where

$$d_3 = \frac{x - \ln(\frac{\bar{S}}{K})}{\sqrt{2l}} + \frac{1}{2}(E_1+1)\sqrt{2l}, \quad (55)$$
$$d_4 = \frac{x - \ln(\frac{\bar{S}}{K})}{\sqrt{2l}} + \frac{1}{2}(E_1-1)\sqrt{2l}. \quad (56)$$

Using equations (28), (52) and (44) - (54) the following is a solution to equation (50)

$$u(x,l) = e^{\frac{1}{2}(E_1-1)x+\frac{1}{4}(E_1-1)^2 l} + e^{\frac{1}{2}(E_1+1)x+\frac{1}{4}(E_1+1)^2 l} N(d_1) - e^{\frac{1}{2}(E_1-1)x+\frac{1}{4}(E_1-1)^2 l} N(d_2)$$
$$- e^{\frac{1}{2}(E_1+1)x+\frac{1}{4}(E_1+1)^2 l} N(d_3) + \frac{\bar{S}}{K} e^{\frac{1}{2}(E_1-1)x+\frac{1}{4}(E_1-1)^2 l} N(d_4). \quad (57)$$

When the change of variable in equation (16) and the transformations $x = \ln(\frac{S}{K})$, $l = \frac{1}{2}\sigma^2(t-\tau)$, and $P = Kw(x,l)$ are recovered equation (57) becomes

$$P(S_\tau, \tau, t) = Ke^{-r(t-\tau)} + S_\tau e^{(\mu-r)(t-\tau)} N(d_1) - Ke^{-r(t-\tau)} N(d_2)$$

$$-S_\tau e^{(\mu-r)(t-\tau)}N(d_3)+\bar{S}e^{-r(t-\tau)}N(d_4), \tag{58}$$

with

$$d_3 = \frac{\ln(\frac{S_\tau}{\bar{S}})+(\mu+\frac{\sigma^2}{2})(t-\tau)}{\sigma\sqrt{t-\tau}}, \tag{59}$$

$$d_4 = \frac{\ln(\frac{S_\tau}{\bar{S}})+(\mu-\frac{\sigma^2}{2})(t-\tau)}{\sigma\sqrt{t-\tau}}. \tag{60}$$

Equation (58) represents the discounted expected value of REFIT under the Cap & floor policy. Note: in Section 3 of the main text τ is assumed equal to zero.

Cost of Fit cap & floor policy

We now consider solving equation (19) for the Cost of FiT to the policymakers, under the cap & floor policy, i.e., when $f_t = \max(0, K-S_t) - \max(0, S_t - \bar{S})$. When the transformations in equations (11) - (13) and the change of variable in equation (16) are taken into account for this payoff, the initial condition of the diffusion equation (Equation (18)) becomes

$$u_0(x) = \max(e^{\frac{1}{2}(E_1-1)x} - e^{\frac{1}{2}(E_1+1)x}, 0) - \max(e^{\frac{1}{2}(E_1+1)x} - \frac{\bar{S}}{K}e^{\frac{1}{2}(E_1-1)x}, 0). \tag{61}$$

When this initial condition and the change of variable $x^{'} = (s-x)/\sqrt{2l}$ is considered, the solution to the diffusion equation (19) becomes

$$u(x,l) = \frac{1}{2\sqrt{\pi l}}\int_{-\infty}^{\infty}(\max(e^{\frac{1}{2}(E_1-1)(x+x^{'}\sqrt{2l})} - e^{\frac{1}{2}(E_1+1)(x+x^{'}\sqrt{2l})}, 0) -$$
$$(\max(e^{\frac{1}{2}(E_1+1)(x+x^{'}\sqrt{2l})} - \frac{\bar{S}}{K}e^{\frac{1}{2}(E_1-1)(x+x^{'}\sqrt{2l})}, 0)e^{\frac{1}{2}x^{'2}}dx^{'}. \tag{62}$$

Again, in similar manner to equation (23), equation (62) can be broken into four integrals as follows

$$u(x,l) = \frac{1}{2\sqrt{\pi l}}\int_{x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1-1)(x+x^{'}\sqrt{2l})}e^{\frac{1}{2}x^{'2}}dx^{'}$$
$$-\frac{1}{2\sqrt{\pi l}}\int_{x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1+1)(x+x^{'}\sqrt{2l})}e^{\frac{1}{2}x^{'2}}dx^{'},$$
$$-\frac{1}{2\sqrt{\pi l}}\int_{\ln(\frac{\bar{S}}{K})-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1+1)(x+x^{'}\sqrt{2l})}e^{\frac{1}{2}x^{'2}}dx^{'}$$
$$+\frac{\bar{S}}{K}\frac{1}{2\sqrt{\pi l}}\int_{\ln(\frac{\bar{S}}{K})-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1-1)(x+x^{'}\sqrt{2l})}e^{\frac{1}{2}x^{'2}}dx^{'}, \tag{63}$$

which we label I_3, I_4, I_5 and I_6 respectively, i.e.,

$$u(x,l) = I_3 - I_4 - I_5 + I_6. \tag{64}$$

Each of these integrals are as previously defined. Hence the solution to equation (62) is

$$u(x,l) = e^{\frac{1}{2}(E_1-1)x+\frac{1}{4}(E_1-1)^2 l}N(-d_2) - e^{\frac{1}{2}(E_1+1)x+\frac{1}{4}(E_1+1)^2 l}N(-d_1) -$$
$$-e^{\frac{1}{2}(E_1+1)x+\frac{1}{4}(E_1+1)^2 l}N(d_3) + \frac{\bar{S}}{K}e^{\frac{1}{2}(E_1-1)x+\frac{1}{4}(E_1-1)^2 l}N(d_4). \tag{65}$$

When the change of variable in equation (16) and the transformations $x = \ln(\frac{S}{K})$, $l = \frac{1}{2}\sigma^2(t-\tau)$, and $P = Kw(x,l)$ are recovered equation (65) becomes

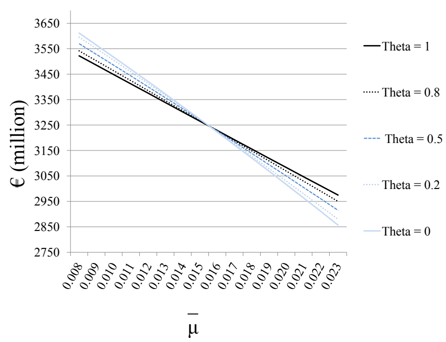
$$EF(S_\tau,\tau,t) = Ke^{-r(t-\tau)}(1-N(d_2)) - S_\tau e^{(\mu-r)(t-\tau)}(1-N(d_1)) - S_\tau e^{(\mu-r)(t-\tau)}N(d_3) + \bar{S}e^{-r(t-\tau)}N(d_4). \tag{66}$$

Equation (66) represents the discounted expected cost of REFIT under the Cap & Floor policy. Note: Note: in Section 3 of the main text τ is assumed equal to zero.

Appendix C: Sensitivity to share of market price exposure

Figure 1: Shared upside: Sensitivity of policy cost to share of market price exposure

(a) Rate of Market Price Growth



(b) Rate of Market Price Volatility

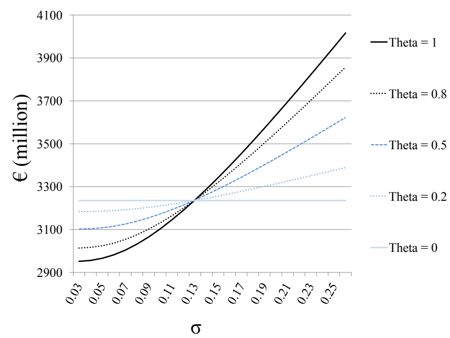
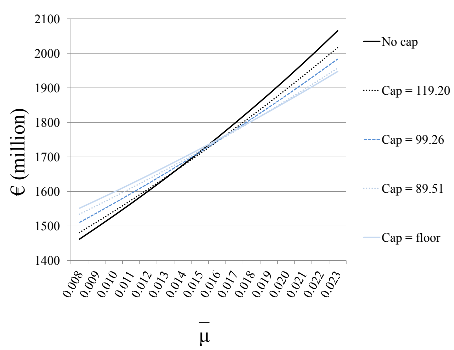


Figure 2: Cap & floor: Sensitivity of investor profit to share of market price exposure

(a) Rate of Market Price Growth



(b) Rate of Market Price Volatility

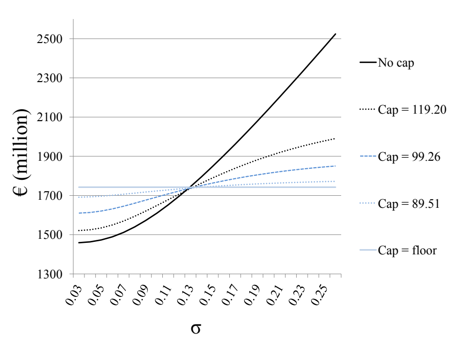
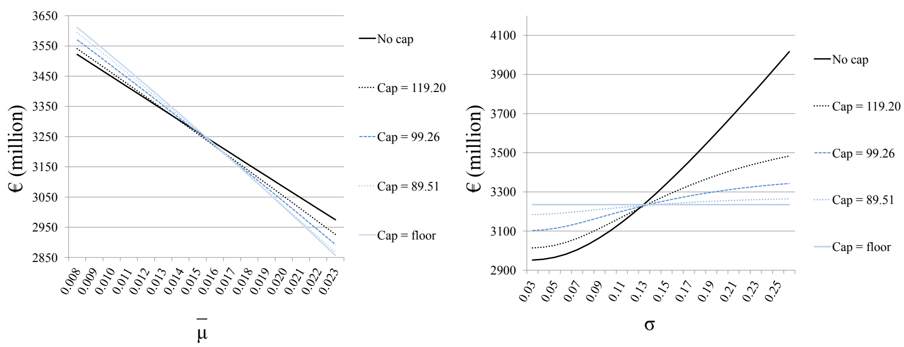


Figure 3: Cap & floor: Sensitivity of policy cost to share of market price exposure

(a) Rate of Market Price Growth (b) Rate of Market Price Volatility



References

Wilmott, P. (2000). *Paul Wilmott on Quantatiive Finance*. West Sussex, England: Wiley.

Wilmott, P., J. Dewynne, and S. Howison (1993). *Option Pricing: Mathematical Models and Computation*. Oxford, England: Oxford Financial Press.