APPENDIX

INVESTMENT WITHOUT POLICY UNCERTAINTY

The value of the option to invest in the presence ($\zeta = 1$) or absence ($\zeta = 0$) of a subsidy is described in (A–1).

$$F^{(j)}_{\zeta,0,0}(E) = \begin{cases} (1-\rho dt)\mathbb{E}_E\left[F^{(j)}_{\zeta,0,0}(E+dE)\right] & , E < \varepsilon^{(j)}_{\zeta,0,0} \\ \frac{Ek^{(j)}_{\zeta,0,0}(1+y\mathbb{1}_{\{\zeta=1\}})}{\rho-\mu} - I\left(k^{(j)}_{\zeta,0,0}\right) & , E \geq \varepsilon^{(j)}_{\zeta,0,0} \end{cases} \tag{A–1}$$

By expanding the first branch on the right–hand side of (A–1) using Itô's lemma, we obtain the differential equation for $F^{(j)}_{\zeta,0,0}(E)$, which, together with its solution for $E < \varepsilon^{(j)}_{\zeta,0,0}$, is indicated in (A–2).

$$\frac{1}{2}\sigma^2 E^2 F^{(j)''}_{\zeta,0,0}(E) + \mu P F^{(j)'}_{\zeta,0,0}(E) - \rho F^{(j)}_{\zeta,0,0}(E) = 0 \quad \Rightarrow \quad F^{(j)}_{\zeta,0,0}(E) = A^{(j)}_{\zeta,0,0}E^{\beta_1} + C^{(j)}_{\zeta,0,0}E^{\beta_2} \tag{A–2}$$

Note that the value of the project becomes very small as $E \to 0$. Since $\beta_2 < 0$, we have $E \to 0 \Rightarrow C^{(j)}_{\zeta,0,0}E^{\beta_2} \to \infty$. Consequently, $C^{(j)}_{\zeta,0,0} = 0$, and, thus, the expression for $F^{(j)}_{\zeta,0,0}(E)$ is indicated in (13). By applying value–matching and smooth–pasting conditions between the two branches of (13), we obtain the expression for the endogenous constant, $A^{(j)}_{\zeta,0,0}$, and the optimal investment threshold, $\varepsilon^{(j)}_{\zeta,0,0}$.

$$A^{(j)}_{\zeta,0,0} = \frac{1}{\varepsilon^{(j)\beta_1}_{\zeta,0,0}}\left[\frac{\varepsilon^{(j)}_{\zeta,0,0}k^{(j)}_{\zeta,0,0}\left(1+y\mathbb{1}_{\{\zeta=1\}}\right)}{\rho-\mu} - I\left(k^{(j)}_{\zeta,0,0}\right)\right] \tag{A–3}$$

$$\varepsilon^{(j)}_{\zeta,0,0}\left(k^{(j)}_{\zeta,0,0}\right) = \frac{I\left(k^{(j)}_{\zeta,0,0}\right)}{k^{(j)}_{\zeta,0,0}\left(1+y\mathbb{1}_{\{\zeta=1\}}\right)}\frac{\beta_1(\rho-\mu)}{\beta_1-1} \tag{A–4}$$

Finally, by inserting (A–4) into (11) we obtain the expression for the optimal capacity.

$$k^{(j)}_{\zeta,0,0} = \left[\frac{a_j}{b_j}\frac{1}{\gamma_j(\beta_1-1)-\beta_1}\right]^{\frac{1}{\gamma_j-1}}, \quad \gamma_j(\beta_1-1)-\beta_1 > 0 \tag{A–5}$$

Moreover, from (A–5) we see that the existence of an optimal solution to the investment problem requires that the cost function is strictly convex, i.e., $\gamma_j(\beta_1-1)-\beta_1 > 0 \Leftrightarrow \gamma_j > \frac{\beta_1}{\beta_1-1} > 1$. ■

Proposition 1 *$\varepsilon^{(j)}_{1,0,0} < \varepsilon^{(j)}_{0,0,0}$ and $k^{(j)}_{1,0,0} = k^{(j)}_{0,0,0}$.*

Proof: In the presence of a subsidy, the value of the option to invest is indicated in (13). The value–matching and smooth–pasting conditions between the two branched of (13) are:

$$A^{(j)}_{1,0,0}\varepsilon^{(j)\beta_1}_{1,0,0} = \frac{\varepsilon^{(j)}_{1,0,0}k^{(j)}_{1,0,0}(1+y)}{\rho-\mu} - I\left(k^{(j)}_{1,0,0}\right) \tag{A–6}$$

$$\beta_1 A^{(j)}_{1,0,0}\varepsilon^{(j)\beta_1}_{1,0,0} = \frac{\varepsilon^{(j)}_{1,0,0}k^{(j)}_{1,0,0}(1+y)}{\rho-\mu} + \underbrace{\frac{\varepsilon^{(j)}_{1,0,0}k^{(j)\prime}_{1,0,0}(1+y)}{\rho-\mu} - a_j k^{(j)\prime}_{\zeta,m,n} - \gamma_j b_j k^{(j)\gamma_j-1}_{\zeta,m,n} k^{(j)\prime}_{\zeta,m,n}}_{=0} \tag{A–7}$$

The expression for the endogenous constant, $A^{(j)}_{1,0,0}$, and the optimal investment threshold, $\varepsilon^{(j)}_{1,0,0}$, is indicated in (A–8).

$$A^{(j)}_{1,0,0} = \frac{\varepsilon^{(j)^{1-\beta_1}}_{1,0,0} k^{(j)}_{1,0,0}(1+y)}{\beta_1(\rho-\mu)} \quad \text{and} \quad \varepsilon^{(j)}_{1,0,0}\left(k^{(j)}_{1,0,0}\right) = \frac{(\rho-\mu)I\left(k^{(j)}_{1,0,0}\right)}{k^{(j)}_{1,0,0}(1+y)}\frac{\beta_1}{\beta_1-1} \tag{A–8}$$

Notice that $\varepsilon^{(j)}_{1,0,0} = \varepsilon^{(j)}_{0,0,0}/(1+y)$ and that by inserting the expression for $\varepsilon^{(j)}_{1,0,0}$ into (11) we obtain:

$$k^{(j)}_{1,0,0}\left(\varepsilon^{(j)}_{1,0,0}\right) = \left[\frac{1}{b_j\gamma_j}\left(\frac{\varepsilon^{(j)}_{1,0,0}(1+y)}{\rho-\mu} - a_j\right)\right]^{\frac{1}{\gamma_j-1}} = k^{(j)}_{0,0,0}\left(\varepsilon^{(j)}_{0,0,0}\right) \tag{A–9}$$

■

INVESTMENT UNDER A RETRACTABLE SUBSIDY

First, we determine the expected value of the active project in the presence of a retractable subsidy. Notice that, within an infinitesimal time interval dt, either the subsidy will be retracted with probability λdt and the instantaneous revenue will decrease by $EK^{(j)}_{1,1,0}y$, or no policy intervention will take place with probability $1-\lambda dt$ and the reduction in the instantaneous revenue will be zero. Consequently, the expected reduction in the instantaneous revenue over a small time interval dt is $\lambda EK^{(j)}_{1,1,0}ydt$ and the expected present value of this reduction is $\frac{\lambda EK^{(j)}_{1,1,0}y}{\rho-\mu}$. By subtracting this from the expected revenues under a permanent subsidy, $\frac{EK^{(j)}_{1,1,0}(1+y)}{\rho-\mu}$, we obtain the expected value of the revenues under sudden subsidy retraction, i.e., $\frac{EK^{(j)}_{1,1,0}[1+(1-\lambda)y]}{\rho-\mu}$. ■

Proposition 2 $\lambda \geq 0 \Rightarrow \varepsilon^{(j)}_{1,1,0} \leq \varepsilon^{(j)}_{0,0,0}$ *and* $k^{(j)}_{1,1,0} \leq k^{(j)}_{0,0,0}$, *while, for low values of* λ, $\varepsilon^{(j)}_{1,1,0} \leq \varepsilon^{(j)}_{1,0,0}$.

Proof: The value of the option to invest in the presence of a retractable subsidy is indicated in (17). Notice that $\lambda = 0 \Rightarrow F^{(j)}_{1,1,0}(E) = F^{(j)}_{1,0,0}(E)$, and, therefore, $\varepsilon^{(j)}_{1,1,0} = \varepsilon^{(j)}_{1,0,0} < \varepsilon^{(j)}_{0,0,0}$ and $k^{(j)}_{1,1,0} = k^{(j)}_{1,0,0} = k^{(j)}_{0,0,0}$. From (15), we know that $\lambda \nearrow \Rightarrow \overline{k}^{(j)}_{1,1,0} \searrow$, which implies that a higher λ lowers the expected value of the project, and, in turn, both the amount of installed capacity and the optimal investment threshold. Hence, for small values of λ, $\varepsilon^{(j)}_{1,1,0} \leq \varepsilon^{(j)}_{1,0,0}$, whereas $\lambda \to 1 \Rightarrow \varepsilon^{(j)}_{1,1,0} \to \varepsilon^{(j)}_{0,0,0}$. ■

Proposition 3 $\frac{F^{(j)}_{1,0,0}(E)-F^{(j)}_{1,1,0}(E)}{F^{(j)}_{1,0,0}(E)} \in \left[0, \frac{A^{(j)}_{1,0,0}-A^{(j)}_{0,0,0}}{A^{(j)}_{1,0,0}}\right)$

Proof: In the presence of a retractable subsidy, the value of the option to invest is:

$$F^{(j)}_{1,1,0}(E) = A^{(j)}_{0,0,0}E^{\beta_1} + B^{(j)}_{1,1,0}E^{\delta_1} \quad , E < \varepsilon^{(j)}_{1,1,0} \tag{A–10}$$

If $\lambda = 0$, then the subsidy will never be retracted. This implies that $F^{(j)}_{1,1,0}(E) = F^{(j)}_{1,0,0}(E)$, and, in turn, that the relative loss in option value is zero. By contrast, as λ increases, the likelihood of subsidy retraction increases, and, as a result, $B^{(j)}_{1,1,0}E^{\delta_1} \to 0$, and, in turn, $F^{(j)}_{1,1,0}(E) \to A^{(j)}_{0,0,0}E^{\beta_1}$, which implies that the relative loss in option value is $\frac{A^{(j)}_{1,0,0}-A^{(j)}_{0,0,0}}{A^{(j)}_{1,0,0}}$. ■

INVESTMENT UNDER SUDDEN PROVISION OF A PERMANENT SUBSIDY

The extra instantaneous revenue from subsidy provision is $EK^{(j)}_{0,0,1}y$ and will be realised with probability λdt, whereas with probability $1-\lambda dt$, no subsidy will be provided. Hence, the expected value of the subsidy is $\lambda EK^{(j)}_{0,0,1}ydt$ and its expected present value is $\frac{\lambda EK^{(j)}_{0,0,1}y}{\rho-\mu}$. Consequently, the expected value of the revenues under sudden provision of a permanent subsidy consist of the expected revenues without the subsidy, $\frac{EK^{(j)}_{0,0,1}}{\rho-\mu}$, and the extra revenues due to the subsidy, $\frac{\lambda EK^{(j)}_{0,0,1}y}{\rho-\mu}$, i.e., $\frac{EK^{(j)}_{0,0,1}(1+\lambda y)}{\rho-\mu}$. ■

Proposition 4 $\lambda \geq 0 \Rightarrow \varepsilon^{(j)}_{0,0,1} \geq \varepsilon^{(j)}_{1,0,0}$ *and* $k^{(j)}_{0,0,1} \geq k^{(j)}_{1,0,0}$, *while, for low values of* λ, $\varepsilon^{(j)}_{0,0,1} \geq \varepsilon^{(j)}_{0,0,0}$.

Proof: The value of the option to invest under sudden provision of a permanent subsidy is indicated in (21). Notice that $\lambda = 0 \Rightarrow F^{(j)}_{0,0,1}(E) = F^{(j)}_{0,0,0}(E)$, and, therefore, $\varepsilon^{(j)}_{0,0,1} = \varepsilon^{(j)}_{0,0,0} > \varepsilon^{(j)}_{1,0,0}$ and $k^{(j)}_{0,0,1} = k^{(j)}_{0,0,0} = k^{(j)}_{1,0,0}$. As λ increases, the likelihood of subsidy provision increases, thereby raising the expected value of the project, and, in turn, the incentive to install greater capacity. Indeed, $\lambda \nearrow \Rightarrow \overline{k}^{(j)}_{0,0,1} \nearrow$, and, therefore, at low values of λ we have $\varepsilon^{(j)}_{0,0,1} \geq \varepsilon^{(j)}_{0,0,0}$. By contrast, at high values of λ it is very likely that the subsidy will be provided, and, therefore, $\lambda \to 1 \Rightarrow \varepsilon^{(j)}_{0,0,1} \to \varepsilon^{(j)}_{1,0,0}$. ■

Proposition 5 $\frac{F^{(j)}_{1,0,0}(E)-F^{(j)}_{0,0,1}(E)}{F^{(j)}_{1,0,0}(E)} \in \left[-\frac{B^{(j)}_{1,0,0}}{A^{(j)}_{1,0,0}}, 0\right)$.

Proof: Under sudden provision of a permanent subsidy, the value of the option to invest is:

$$F^{(j)}_{0,0,1}(E) = A^{(j)}_{1,0,0}E^{\beta_1} + B^{(j)}_{0,0,1}E^{\delta_1} \quad , E < \varepsilon^{(j)}_{1,0,0} \tag{A–11}$$

If $\lambda = 0$, then the subsidy will never be provided. This implies that $F^{(j)}_{0,0,1}(E) = \left(A^{(j)}_{1,0,0} + B^{(j)}_{0,0,1}\right)E^{\beta_1}$, and, in turn, that the relative loss in option value is maximised. By contrast, as λ increases, the likelihood of subsidy provision increases, and, as a result, $B^{(j)}_{0,0,1}E^{\delta_1} \to 0$, and, in turn, $F^{(j)}_{0,0,1}(E) \to A^{(j)}_{1,0,0}E^{\beta_1}$, which implies that the relative loss in option value is zero. ■

INVESTMENT UNDER INFINITE PROVISIONS AND RETRACTIONS

The dynamics of the value of the option to invest under infinite provision and retractions for $\zeta = 0, 1$, are described in (A–12) and (A–13), respectively.

$$F^{(j)}_{0,\infty,\infty}(E) = (1-\rho dt)\left[\lambda dt \mathbb{E}_E\left[F^{(j)}_{1,\infty,\infty}(E+dE)\right] + (1-\lambda dt)\mathbb{E}_E\left[F^{(j)}_{0,\infty,\infty}(E+dE)\right]\right] \quad \text{(A–12)}$$

$$F^{(j)}_{1,\infty,\infty}(E) = (1-\rho dt)\left[\lambda dt \mathbb{E}_E\left[F^{(j)}_{0,\infty,\infty}(E+dE)\right] + (1-\lambda dt)\mathbb{E}_E\left[F^{(j)}_{1,\infty,\infty}(E+dE)\right]\right] \quad \text{(A–13)}$$

By expanding the right–hand side of (A–12) and (A–13) using Itô's lemma, we have

$$\frac{1}{2}\sigma^2 E^2 F^{(j)\prime\prime}_{0,\infty,\infty}(E) + \mu E F^{(j)\prime}_{0,\infty,\infty}(E) - (\lambda+\rho)F^{(j)}_{0,\infty,\infty}(E) + \lambda F^{(j)}_{1,\infty,\infty}(E) = 0 \quad \text{(A–14)}$$

$$\frac{1}{2}\sigma^2 E^2 F^{(j)\prime\prime}_{1,\infty,\infty}(E) + \mu E F^{(j)\prime}_{1,\infty,\infty}(E) - (\lambda+\rho)F^{(j)}_{1,\infty,\infty}(E) + \lambda F^{(j)}_{0,\infty,\infty}(E) = 0 \quad \text{(A–15)}$$

and by adding and subtracting (A–14) and (A–15) we obtain (A–16) and (A–17), respectively, where $F_a(E) = F^{(j)}_{1,\infty,\infty}(E) + F^{(j)}_{0,\infty,\infty}(E)$ and $F_b(E) = F^{(j)}_{1,\infty,\infty}(E) - F^{(j)}_{0,\infty,\infty}(E)$.

$$\frac{1}{2}\sigma^2 E^2 F''_a(P) + \mu E F'_a(P) - \rho F_a(E) = 0 \quad \text{(A–16)}$$

$$\frac{1}{2}\sigma^2 E^2 F''_b(P) + \mu E F'_b(P) - (\rho+2\lambda) F_b(E) = 0 \quad \text{(A–17)}$$

The solution to (A–16) and (A–17) can be obtained by setting $F_a(E) = A^{(j)}_a E^{\beta_1}$ and $F_b(E) = A^{(j)}_b E^{\eta}$. Thus, we obtain:

$$F^{(j)}_{1,\infty,\infty}(E) = \frac{1}{2}\left[A^{(j)}_a E^{\beta_1} + A^{(j)}_b E^{\eta}\right] \quad \text{(A–18)}$$

$$F^{(j)}_{0,\infty,\infty}(E) = \frac{1}{2}\left[A^{(j)}_a E^{\beta_1} - A^{(j)}_b E^{\eta}\right] \quad \text{(A–19)}$$

where η is the positive root of the quadratic $\frac{1}{2}\sigma^2 x(x-1) + \mu x - (\rho+2\lambda) = 0$. ■