Online Appendix

North American Natural Gas Markets Under LNG Demand Growth and Infrastructure Restrictions

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Description: This online appendix file provides the sensitivity analysis, key input parameter values, lists of exogenously defined capacity expansions, discussion of the HOD scenario, and the full model formulation along with the corresponding KKT conditions.

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1 Sensitivity analysis

We acknowledge that the model results are sensitive to its parameterization and calibration. In this section, we provide a sensitivity analysis of some of the key parameters to observe how the most relevant summary statistics change. We choose the HLN scenario for the sensitivity analysis, since this is the scenario that leads to the greatest capacity investments, and hence should reflect the effects of changing parameters more visibly than the other scenarios. We run this analysis based on three key sets of parameters: capacity expansion cost parameters, production cost function parameters, and internal demand growth rates. In each run for the first two of these parameter sets, we either increase or decrease the value of the one set of the aforementioned parameters by 50% and leave the remaining parameters as is. For the demand growth rate sensitivity, since we already have a scenario that couples HLN with high internal demand, i.e., HOD, we test only the decreasing internal natural gas demand case. Specifically, we subtract one percentage point from two-thirds of each region's demand growth rate in REF (two-thirds of REF rates yield the base demand growth before calibration) for this analysis, which yields a decrease in every region's demand over time except for Mexico. Table 1 contains the results.

Table 1. Sensitivity analysis for the HLN scenario in 2050.

Summary statistic	HLN Parameters	Expansion costs +50%	Expansion costs -50%	Production costs +50%	Production costs -50%	Low internal demand
Average price ($/Mcf)	6.24	7.11	5.03	7.01	5.74	3.61
Total production (Tcf)	54.85	52.83	57.54	53.15	55.88	42.02
Total end use consumption (Tcf)	39.76	38.04	42.08	38.29	40.65	26.19
Atlantic LNG shipments (Tcf)	2.22	2.07	2.42	2.09	2.29	2.66
Pacific LNG shipments (Tcf)	11.33	11.21	11.46	11.26	11.37	11.58
Total production capacity expansion (Bcf/d)	45.38	38.75	56.28	45.37	45.54	21.96
Total LNG export capacity expansion (Bcf/d)	31.25	30.00	36.72	31.12	31.35	31.92
Total pipeline capacity expansion (Bcf/d)	15.72	14.93	24.13	14.89	16.36	15.04

When we examine our sensitivity analysis on expansion and production costs, we see that all key summary statistics are more sensitive to the capacity expansion cost parameters than they are to the production cost function parameters. One notable result is that LNG shipments are relatively less sensitive to inputs compared to other statistics. This primarily results from the fact that there is a lucrative arbitrage opportunity due to strong LNG demand. We observe the variables regarding the market dynamics, i.e., production, end use consumption and average North American price to be sensitive to both expansion and production cost parameters; however, the degree of capacity expansion shows little sensitivity to production costs. Generally, the changes in all variables are intuitive for these two sources of variability, but considerable differences in the magnitudes of certain variables occur in different sensitivity runs.

In the decreasing demand analysis, we observe that the lowest average North American prices occur in this scenario, as expected. We also see that this significantly decreases the incentive to invest in production capacity expansions, as smaller additions to current capacity are sufficient to meet both internal and LNG demands. Pipeline and LNG export capacity expansion levels does not appear to be affected significantly by the local demand changes; however, how much the local production scales up shows very considerable variability under decreasing demand analysis. This observation supports the claim that LNG demand is indeed a strong driver of natural gas infrastructure development, especially in pipeline development, and as expected, in LNG export capacity.

2 Values of key input parameters

Table 2. Golombek et al. (1995) production cost function parameters used in the model (costs are in \$/Mcf).

	Parameters		
Region	α	β	γ
Northeast	1	0	1
Southeast	1.3	0	1
Midwest	1.3	0	1
Southwest	0.2	0	1
Central	1.3	0	1
Western U.S.	1.3	0	1
Eastern Canada	2	0	2
Western Canada	0.85	0	1
Mexico	3	0	3

Table 3. Pipeline projects currently approved and/or under construction (EIA 2019)[1] that are exogenously added to the pipeline capacities in the model.

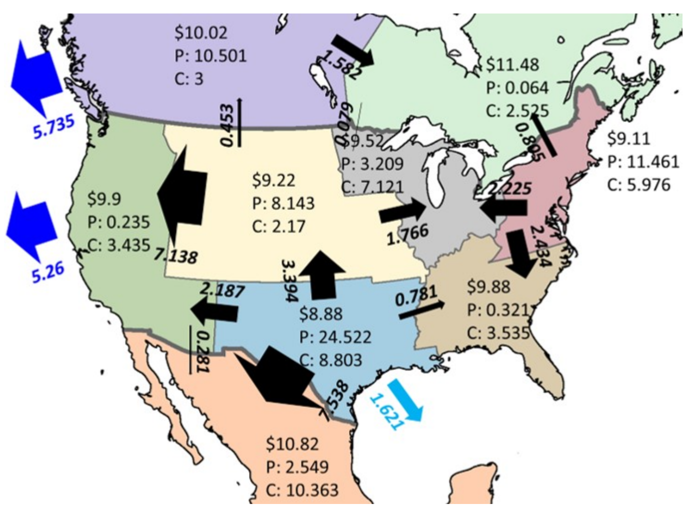
Regions	Pipeline projects	Capacity added (Bcf/d)	Expected in-service year
Northeast to Southeast	Atlantic Sunrise Project Phase 1B	450	2018
	Atlantic Sunrise Project Phase II	850	2018
	Broad Run Expansion Project	200	2018
	Leach XPress Project	1530	2018
	Mountaineer XPress Pipeline Phase 2	2200	2019
	Atlantic Coast Pipeline	1500	2020
Northeast to Midwest	Rover Pipeline Project Phase 2	1550	2018
Northeast to Eastern Canada	Atlantic Bridge project Phase 2	106	2021
Southeast to Southwest	Broad Run Expansion Project	200	2018
	Southwest Louisiana Supply Project	900	2018
	Gulf Coast Connector Expansion Project	475	2019
	Gulf Xpress	875	2019
	Lone Star Expansion	300	2019
Midwest to Northeast	Leach XPress Project	1530	2018
Midwest to Central	NGPL Gulf Coast Southbound Phase I	460	2018
	Valley Expansion Project	33	2018
Southwest to Western U.S.	Permian North Expansion Project	182	2018
Southwest to Mexico	Impulsora Pipeline Crossing Project	1120	2018
	KM Border Pipeline Expansion	150	2018
	TGP Rio Bravo Hidalgo Expansion	383	2018
	Valley Crossing Pipeline	2600	2019
Central to Southwest	NGPL Gulf Coast Southbound Phase I	460	2018
Central to Western Canada	South Saskatchewan Access Project	30	2018
Western U.S. to Mexico	Sierrita Pima Expansion	323	2020
Eastern Canada to Northeast	Portland Xpress Project Phase 1	40	2018

Table 4. LNG export facilities currently approved and/or under construction (EIA 2019, Navigant Consulting 2015, NEB 2019)[2] that are exogenously added to the LNG export capacities in the model.

LNG export facility	Region	Capacity added (Bcf/d)	Expected in-service year
Sabine Pass (Train 5)	Southwest	690	2018
Cove Point (Train 1)	Northeast	760	2018
Elba Island (Trains 1-6)	Southeast	200	2018
Elba Island (Trains 7-10)	Southeast	130	2019
Corpus Christi (Train 1)	Southwest	660	2018
Corpus Christi (Train 2)	Southwest	660	2019
Cameron (Train 1)	Southwest	660	2018
Cameron (Trains 2-3)	Southwest	1320	2019
Freeport (Trains 1-2)	Southwest	1420	2019
Freeport (Train 3)	Southwest	710	2020
Kitimat	Western Canada	3720	2021

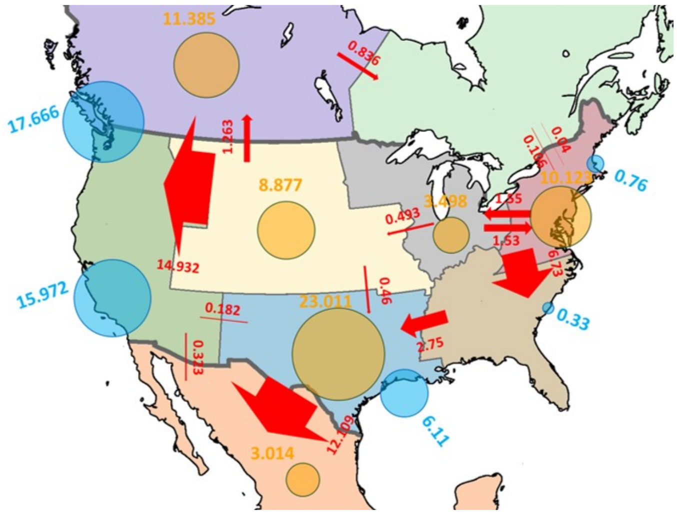
3 High Overall Demand (HOD) scenario results

Figure 1. Production, consumption, and prices in 2050 in the HOD scenario.



[1]The 01/10/2019 release of the U.S. Natural Gas Pipeline Projects document by EIA is used.
[2]The 09/18/2018 release of the U.S. Liquefaction Capacity document by EIA is used.

Figure 2. Cumulative infrastructure additions from 2017 through 2050 in the HOD scenario. These results include exogenously specified capacity additions that are currently approved and/or under construction.



We briefly compare the results from HOD (see Figure 1) against those from HLN to examine what happens if North American demand increases more significantly, similar to the LNG demands. The most noticeable result in this scenario is that the prices increase drastically compared to the HLN scenario, demonstrating the strong effect of local demand on North American prices. The average North American price increases by 57.32% compared to HLN under HOD, with the most drastic increase of 63.53% occuring in the Southwest region. This scenario also leads to the highest increase in production capacity expansions, totaling 59.91 Bcf/d across North America by 2050. The Southwest region alone accounts for almost 40% of this increase, with a total added capacity of 23.01 Bcf/d in 2050. The more abundant natural gas supplies counteract the higher regional demands. The largest consumption increase compared to the HLN scenario occurs in Mexico (57.74%), while total end-use consumption increases by 18.04% in all North America compared to HLN.

Unsurprisingly, the higher North American gas demands in the HOD scenario (see Figure 2) induce the most total pipeline investment. The HOD results include significant pipeline additions from Central to Western U.S. (14.93 Bcf/d), similar to HLN, and from Southwest to Mexico (12.11

Bcf/d including the existing projects in 2017).

HOD leads to slightly less investment in total LNG export capacity than HLN because higher North American demand in HOD intensifies competition for gas supplies between North American consumers and liquefiers seeking gas to export.

4 Full model formulation

The mathematical model consists of three sets of optimization problems and sets of market clearing conditions, each associated with one player type. Note that all variables denoting prices are explained in the Decision Variables section of the respective problems since they are endogenously determined by the entire model as they are dual variables to the associated market clearing constraints. However, from each player's point of view, these dual variables are treated as parameters since players are price takers in a perfect competition environment.

4.1 Supplier's Problem

4.1.1 Indices and Sets

N: Set of regions; $n \in N$
I: Set of suppliers; $i \in I$
S: Set of seasons; $s \in S$
T: Set of time intervals (years); $t \in T$

4.1.2 Parameters

$Q_{is}^{n,max,prod}$: Initial daily production capacity of supplier i in region n in season s in the base year
$CAP_{it}^{n,PROD}$: Maximum daily capacity expansion that can be added by supplier i in region n in time interval t
δ_t: Discount factor for time interval t
$days_s$: Number of days in season s

4.1.3 Decision Variables

$q_{ist}^{n,prod}$: Daily production of supplier i in region n in season s and time interval t

$\Delta_{it}^{n,PROD}$: Daily capacity expansion by supplier i in region n in time interval t

$\pi_{st}^{n,ws}$: Wholesale price of natural gas in region n in season s and time interval t (dual to associated market clearing constraints)

4.1.4 Functions

Function $C_{ist}^{n,prod}(\cdot)$ denotes a cost associated with the production carried out by the supplier. Similarly, an $E_{it}^{n,PROD}(\cdot)$ function denotes the cost of expanding capacity.

$$
\begin{aligned}
C_{ist}^{n,prod}(\cdot) =&(\alpha_{ist}^{n,prod} + \gamma_{ist}^{n,prod})q_{ist}^{n,prod} + \frac{1}{2}\beta_{ist}^{n,prod}(q_{ist}^{n,prod})^2 \\
&+ \gamma_{ist}^{n,prod}(Q_{is}^{n,max,prod} + \sum_{t'<t}\Delta_{it'}^{n,PROD} - q_{ist}^{n,prod})\ln\Big(1 - \frac{q_{ist}^{n,prod}}{Q_{is}^{n,max,prod} + \sum_{t'<t}\Delta_{it'}^{n,PROD}}\Big) \\
E_{it}^{n,PROD}(\cdot) =&(\alpha_{it}^{n,cap} + \gamma_{it}^{n,cap})\Delta_{it}^{n,PROD} + \frac{1}{2}\beta_{it}^{n,cap}(\Delta_{it}^{n,PROD})^2 \\
&+ \gamma_{it}^{n,cap}(CAP_{it}^{n,PROD} - \Delta_{it}^{n,PROD})\ln\Big(1 - \frac{\Delta_{it}^{n,PROD}}{CAP_{it}^{n,PROD}}\Big) \\
\frac{\partial C_{ist}^{n,prod}(\cdot)}{\partial q_{ist}^{n,prod}} =&\alpha_{ist}^{n,prod} + \beta_{ist}^{n,prod}q_{ist}^{n,prod} - \gamma_{ist}^{n,prod}\ln\Big(1 - \frac{q_{ist}^{n,prod}}{Q_{is}^{n,max,prod} + \sum_{t'<t}\Delta_{it'}^{n,PROD}}\Big) \\
\frac{\partial C_{ist}^{n,prod}(\cdot)}{\partial \Delta_{it'}^{n,PROD}} =&\gamma_{ist}^{n,prod}\ln\Big(1 - \frac{q_{ist}^{n,prod}}{Q_{is}^{n,max} + \sum_{\hat{t}<t}\Delta_{i\hat{t}}^{n,PROD}}\Big) + \gamma_{ist}^{n,prod}\frac{q_{ist}^{n,prod}}{Q_{is}^{n,max,prod} + \sum_{\hat{t}<t}\Delta_{i\hat{t}}^{n,PROD}} \\
&\text{for } t' < t \\
\frac{\partial E_{it}^{n,PROD}(\cdot)}{\partial \Delta_{it}^{n,PROD}} =&\alpha_{it}^{n,cap} + \beta_{it}^{n,cap}\Delta_{it}^{n,PROD} - \gamma_{it}^{n,cap}\ln\Big(1 - \frac{\Delta_{it}^{n,PROD}}{CAP_{it}^{n,PROD}}\Big)
\end{aligned}
$$

4.1.5 Optimization Problem

$$\underset{q_{ist}^{n,prod},\Delta_{it}^{n,PROD}}{\text{Maximize}}$$

$$\sum_{s\in S,t\in T}\delta_t days_s\Big(\pi_{st}^{n,ws}q_{ist}^{n,prod} - C_{ist}^{n,prod}(.)\Big) - \sum_{t\in T}\delta_t E_{it}^{n,PROD}(\Delta_{it}^{n,PROD})$$

$s.t.$

$$q_{ist}^{n,prod} \leq Q_{is}^{n,max,prod} + \sum_{t'<t}\Delta_{it'}^{n,PROD} \qquad \forall s \in S, t \in T \qquad (\alpha 1_{ist}^n)$$

$$\Delta_{it}^{n,PROD} \leq CAP_{it}^{n,PROD} \qquad \forall t \in T \qquad (\alpha 2_{it}^n)$$

$$q_{ist}^{n,prod} \geq 0 \qquad \forall s \in S, t \in T$$

$$\Delta_{it}^{n,PROD} \geq 0 \qquad \forall t \in T$$

4.2 Traders's Problem

4.2.1 Indices and Sets

N: Set of regions; $n \in N$

$out(n)$: Set of accessible regions from region n; $m \in out(n)$

K: Set of traders; $k \in K$

S: Set of seasons; $s \in S$

T: Set of time intervals (years); $t \in T$

4.2.2 Decision Variables

$v_{kst}^{n,purch,tra}$: Amount of gas bought from the local supplier by trader k in region n in season s and time interval t

$v_{kst}^{n,purch,int}$: Amount of gas bought from other traders by trader k in region n in season s and time interval t

f_{kst}^{nm}: Total gas flow between trader k in region n and region m in season s and time interval t

$s_{kst}^{nm,tra}$: Gas sales by trader k in region n to traders in region m in season s and time interval t

$s_{kst}^{n,spot}$: Local spot gas sales by trader k in region n in season s and time interval t

$s_{kst}^{n,inj}$: Gas injection to local storages by trader k in region n in season s time interval t (injection can only take place in season 2, hence $s_{k1t}^{n,inj} = 0$)

$s_{kst}^{n,ext}$: Gas extraction from storages by trader k in region n in season s time interval t (extraction can only take place in season 1, hence $s_{k2t}^{n,ext} = 0$)

$s_{kst}^{n,liq}$: Gas sales by trader k in region n to local liquefiers in season s and time interval t

$\pi_{st}^{n,spot}$: Spot price of natural gas in region n in season s and time interval t (dual to associated market clearing constraints)

$\pi_{st}^{n,liq}$: Liquefiers' price of natural gas in region n in season s and time interval t when they buy from the trader (dual to associated market clearing constraints)

$\pi_{st}^{n,tra}$: Traders' price of natural gas in region n in season s and time interval t when they buy from another trader (dual to associated market clearing constraints)

$\eta_{st}^{n,inj}$: Storage injection fee of natural gas in region n in season s time interval t (dual to associated market clearing constraints, only applicable in season 2)

$\eta_{st}^{n,ext}$: Storage extraction fee of natural gas in region n in season s time interval t (dual to associated market clearing constraints, only applicable in season 1)

$\pi_{st}^{n,ws}$: Wholesale price of natural gas in region n in season s and time interval t (dual to associated market clearing constraints)

τ_{st}^{nm}: Pipeline transmission fee between region n and region m in season s and time interval t (dual to associated market clearing constraints)

4.2.3 Optimization Problem

$$\underset{v_{kst}^{n,purch,tra}, v_{kst}^{n,purch,int}, f_{kst}^{nm}, s_{kst}^{nm,tra}, s_{kst}^{n,spot}, s_{kst}^{n,inj}, s_{kst}^{n,ext}, s_{kst}^{n,liq}}{\text{Maximize}}$$

$$\sum_{\substack{m \in out(n) \\ s \in S, t \in T}} \delta_t \left(\pi_{st}^{m,tra} s_{kst}^{nm,tra} - \tau_{st}^{nm} f_{kst}^{nm} \right) + \sum_{s \in S, t \in T} \delta_t \left(\pi_{st}^{n,liq} s_{kst}^{n,liq} + \pi_{st}^{n,spot} s_{kst}^{n,spot} \right.$$
$$\left. - \pi_{st}^{n,ws} v_{kst}^{n,purch,tra} - \pi_{st}^{n,tra} v_{kst}^{n,purch,int} \right) - \sum_{t \in T} \delta_t \left(\eta_{2t}^{n,inj} s_{k2t}^{n,inj} + \eta_{1t}^{n,ext} s_{k1t}^{n,ext} \right)$$

$s.t.$

$$f_{kst}^{nm} = s_{kst}^{nm,tra} \qquad \forall s, t \qquad (\theta 1_{kst}^{nm})$$
$$s_{kst}^{n,liq} + s_{kst}^{n,spot} + s_{kst}^{n,inj} + \sum_{m \in out(n)} s_{kst}^{nm,tra} = v_{kst}^{n,purch,tra} + v_{kst}^{n,purch,int} + s_{kst}^{n,ext} \qquad \forall s \in S, t \in T \quad (\theta 2_{kst}^{n})$$
$$v_{kst}^{n,purch,tra}, v_{kst}^{n,purch,int}, s_{kst}^{n,spot}, s_{kst}^{n,storage}, s_{kst}^{n,liq} \geq 0 \qquad \forall s \in S, t \in T$$
$$f_{kst}^{nm}, s_{kst}^{nm,tra} \geq 0 \qquad \forall m \in out(n) s \in S, t \in T$$

4.3 Storage Operator's Problem

4.3.1 Indices and Sets

N: Set of regions; $n \in N$

D: Set of storage operators; $d \in D$

S: Set of seasons; $s \in S$

T: Set of time intervals (years); $t \in T$

4.3.2 Parameters

$Q_d^{n,max,ext}$: Initial extraction capacity by storage operator d in region n

$Q_d^{n,max,inj}$: Initial injection capacity by storage operator d in region n

$L_d^{n,stor}$: Loss factor by storage operator d in region n in time interval t

$c_{dt}^{n,ext}$: Extraction cost by storage operator d in region n in time interval t

$c_{dt}^{n,inj}$: Injection cost by storage operator d in region n in time interval t

$e_{dt}^{n,EXT}$: Extraction capacity expansion cost by storage operator d in region n in time interval t

$e_{dt}^{n,INJ}$: Injection capacity expansion cost by storage operator d in region n in time interval t

$CAP_{dt}^{n,EXT}$: Maximum extraction capacity expansion that can be added by storage operator d in region n in time interval t

$CAP_{dt}^{n,INJ}$: Maximum injection capacity expansion that can be added by storage operator d in region n in time interval t

4.3.3 Decision Variables

$v_{dst}^{n,ext}$: Amount of gas extracted by storage operator d in region n in season s time interval t (extraction can only take place in season 1, hence $v_{d2t}^{n,extract} = 0$)

$v_{dst}^{n,inj}$: Amount of gas injected by storage operator d in region n in season s time interval t (injection can only take place in season 2, hence $v_{d1t}^{n,inject} = 0$)

$\Delta_{dt}^{n,EXT}$: Extraction capacity expansion by storage operator d in region n in time interval t

$\Delta_{dt}^{n,INJ}$: Injection capacity expansion by storage operator d in region n in time interval t

$\eta_{st}^{n,inj}$: Storage injection fee of natural gas in region n in season s time interval t (dual to associated market clearing constraints, only applicable in season 2)

$\eta_{st}^{n,ext}$: Storage extraction fee of natural gas in region n in season s time interval t (dual to associated market clearing constraints, only applicable in season 1)

4.3.4 Optimization Problem

$$\underset{v_{dst}^{n,ext},v_{dst}^{n,inj},\Delta_{dt}^{n,EXT},\Delta_{dt}^{n,INJ}}{\text{Maximize}}$$

$$\sum_{t\in T} \delta_t \Big(\eta_{1t}^{n,ext} v_{d1t}^{n,ext} + \eta_{2t}^{n,inj} v_{d2t}^{n,inj} - c_{dt}^{n,ext} v_{d1t}^{n,ext} - c_{dt}^{n,inj} v_{d2t}^{n,inj} - e_{dt}^{n,EXT} \Delta_{dt}^{n,EXT} - e_{dt}^{n,INJ} \Delta_{dt}^{n,INJ}\Big)$$

$s.t.$

$$v_{d1t}^{n,ext} \le Q_d^{n,max,ext} + \sum_{t'<t} \Delta_{dt'}^{n,EXT} \qquad \forall t \in T \qquad (\sigma 1_{dt}^n)$$

$$v_{d2t}^{n,inj} \le Q_d^{n,max,inj} + \sum_{t'<t} \Delta_{dt'}^{n,INJ} \qquad \forall t \in T \qquad (\sigma 2_{dt}^n)$$

$$v_{d1t}^{n,ext} = (1 - L_d^{n,stor}) v_{d2t}^{n,inj} \qquad \forall t \in T \qquad (\sigma 3_{dt}^n)$$

$$\Delta_{dt}^{n,EXT} \le CAP_{dt}^{n,EXT} \qquad \forall t \in T \qquad (\sigma 4_{dt}^n)$$

$$\Delta_{dt}^{n,INJ} \le CAP_{dt}^{n,INJ} \qquad \forall t \in T \qquad (\sigma 5_{dt}^n)$$

$$v_{d1t}^{n,extract}, v_{d2t}^{n,inject}, \Delta_{dt}^{n,EXT}, \Delta_{dt}^{n,INJ} \ge 0 \qquad \forall t \in T$$

4.4 Liquefier's Problem

4.4.1 Indices and Sets

N: Set of regions; $n \in N$

R: Set of LNG markets; $r \in R$

$reach(n)$: Set of reachable LNG markets from region n

L: Set of liquefiers; $l \in L$

S: Set of seasons; $s \in S$

T: Set of time intervals (years); $t \in T$

4.4.2 Parameters

$Q_{ls}^{n,max,liq}$: Initial liquefaction capacity by liquefier l in region n in season s
$c_{lst}^{n,liq}$: Cost of liquefaction by liquefier l in region n in season s and time interval t
$L_l^{n,liq}$: Loss factor by liquefier l in region n in time interval t
$e_{lt}^{n,LIQ}$: Cost of liquefaction capacity expansion by liquefier l in region n in time interval t
$CAP_{lt}^{n,LIQ}$: Maximum liquefaction capacity expansion that can be added by liquefier l in region n in time interval t

4.4.3 Decision Variables

$v_{lst}^{n,purch,LNG}$: Amount of gas bought from local trader by liquefier l in region n in season s and time interval t
$v_{lst}^{n,LNG}$: total LNG shipped from liquefier l in region n in season s and time interval t
$s_{lst}^{nr,LNG}$: LNG sales by liquefier l in region n to LNG market r in season s and time interval t
$\Delta_{lt}^{n,LIQ}$: Daily capacity expansion by liquefier l in region n in time interval t
$\pi_{st}^{r,LNG}$: LNG sales price of natural gas at LNG demand point r in season s and time interval t (dual to associated market clearing constraints)
$\pi_{st}^{n,liq}$: Liquefiers' price of natural gas in region n in season s and time interval t when they buy from the trader (dual to associated market clearing constraints)
φ_{st}^{nr}: LNG transshipment fee between region n and LNG demand point r in season s and time interval t (dual to associated market clearing constraints)

4.4.4 Optimization Problem

$$\underset{s_{lst}^{nr,LNG},v_{lst}^{n,LNG},v_{lst}^{n,purch,LNG},\Delta_{lt}^{n,LIQ}}{\text{Maximize}}$$

$$\sum_{\substack{r\in reach(n)\\ s\in S,t\in T}} \delta_t\left(\pi_{st}^{r,LNG} s_{lst}^{nr,LNG} - \varphi_{st}^{nr} s_{lst}^{nr,LNG}\right) - \sum_{s\in S,t\in T} \delta_t\left(c_{lst}^{n,liq} v_{lst}^{n,LNG} + \pi_{st}^{n,liq} v_{lst}^{n,purch,LNG}\right)$$
$$- \sum_{t\in T} \delta_t e_{lt}^{n,LIQ} \Delta_{lt}^{n,LIQ}$$

$s.t.$

$$v^{n,LNG}_{lst} = \sum_{r \in reach(n)} s^{nr,LNG}_{lst} \qquad \forall s \in S, t \in T \qquad (\lambda 1^n_{lst})$$

$$v^{n,LNG}_{lst} \leq Q^{n,max,liq}_{ls} + days_s \sum_{t'<t} \Delta^{n,LIQ}_{lt'} \qquad \forall s \in S, t \in T \qquad (\lambda 2^n_{lst})$$

$$(1 - L^{n,liq}_l) v^{n,purch,LNG}_{lst} = v^{n,LNG}_{lst} \qquad \forall s \in S, t \in T \qquad (\lambda 3^n_{lst})$$

$$\Delta^{n,LIQ}_{lt} \leq CAP^{n,LIQ}_{lt} \qquad \forall t \in T \qquad (\lambda 4^n_{lt})$$

$$s^{nr,LNG}_{lst} \geq 0 \qquad \forall r \in reach(n), s \in S, t \in T$$

$$v^{n,LNG}_{lst}, v^{n,purch,LNG}_{lst} \geq 0 \qquad \forall s \in S, t \in T$$

$$\Delta^{n,LIQ}_{lt} \geq 0 \qquad \forall t \in T$$

4.5 Tanker Network Operator's Problem

4.5.1 Indices and Sets

N: Set of regions; $n \in N$
R: Set of LNG markets; $r \in R$
$reach(n)$: Set of reachable LNG markets from region n
S: Set of seasons; $s \in S$
T: Set of time intervals (years); $t \in T$

4.5.2 Parameters

$Q^{nr,max,tanker}_s$: Initial LNG flow capacity between region n and LNG market r in season s
$c^{nr,tanker}_{st}$: Cost of LNG transshipment between region n and LNG market r in season s and time interval t
$e^{nr,TANKER}_t$: Tanker capacity expansion cost between region n and LNG market r in time interval t
$CAP^{nr,TANKER}_t$: Maximum capacity expansion that can be added by LNG tanker operator between region n and LNG market r in time interval t

4.5.3 Decision Variables

$v^{nr,flow,LNG}_{st}$: Total LNG shipped from region n to LNG market r in season s and time interval t
$\Delta^{nr,TANKER}_t$: Daily capacity expansion by LNG tanker operator between region n and LNG market r in time interval t
φ^{nr}_{st}: LNG transshipment fee between region n and LNG demand point r in season s and time interval t (dual to associated market clearing constraints)

4.5.4 Optimization Problem

$$\underset{v_{st}^{nr,flow,LNG},\Delta_t^{nr,TANKER}}{\text{Maximize}}$$

$$\sum_{\substack{n\in N\\ r\in reach(n)\\ s\in S,t\in T}} \delta_t\Big(\varphi_{st}^{nr} v_{st}^{nr,flow,LNG} - c_{st}^{nr,tanker} v_{st}^{nr,flow,LNG}\Big) - \sum_{\substack{n\in N\\ r\in reach(n)\\ t\in T}} \delta_t e_t^{nr,TANKER} \Delta_t^{nr,TANKER}$$

$s.t.$

$$v_{st}^{nr,flow,LNG} \le Q_s^{nr,max,tanker} + days_s \sum_{t'<t} \Delta_{t'}^{nr,TANKER} \qquad \forall n \in N, r \in reach(n), s \in S, t \in T \qquad (\rho 1_{st}^{nr})$$

$$\Delta_t^{nr,TANKER} \le CAP_t^{nr,TANKER} \qquad \forall n \in N, r \in reach(n), t \in T \qquad (\rho 2_t^{nr})$$

$$v_{st}^{nr,flow,LNG} \ge 0 \qquad \forall n \in N, r \in reach(n), s \in S, t \in T$$

$$\Delta_t^{nr,TANKER} \ge 0 \qquad \forall n \in N, r \in reach(n), t \in T$$

4.6 Pipeline Network Operator's Problem

4.6.1 Indices and Sets

N: Set of regions; $n \in N$

$out(n)$: Set of accessible regions from region n; $m \in out(n)$

S: Set of seasons; $s \in S$

T: Set of time intervals (years); $t \in T$

4.6.2 Parameters

$Q_s^{nm,max,pipe}$: Initial pipeline flow capacity between region n and region m in time interval t

$c_{st}^{nm,pipe}$: Cost of LNG transshipment between region n and region m in season s and time interval t

$e_t^{nm,PIPE}$: Pipeline capacity expansion cost between region n and region m in time interval t

$CAP_t^{nm,PIPE}$: Maximum capacity expansion that can be added by pipeline network operator between region n and region m in time interval t

4.6.3 Decision Variables

$v_{st}^{nm,flow,pipe}$: Total gas shipped from region n to region m in season s and time interval t

$\Delta_t^{nm,PIPE}$: Daily capacity expansion by network operator between region n and region m in time interval t

τ_{st}^{nm}: Pipeline transmission fee between region n and region m in season s and time interval t (dual to associated market clearing constraints)

4.6.4 Optimization Problem

$$\underset{v_{st}^{nm,flow,pipe},\Delta_t^{nm,PIPE}}{\text{Maximize}}$$

$$\sum_{\substack{n\in N\\ m\in out(n)\\ s\in S,t\in T}} \delta_t\Big(\tau_{st}^{nm} v_{st}^{nm,flow,pipe} - c_{st}^{nm,flow,pipe} v_{st}^{nm,flow,pipe}\Big) - \sum_{\substack{n\in N\\ m\in out(n)\\ t\in T}} \delta_t e_t^{nm,PIPE} \Delta_t^{nm,PIPE}$$

$s.t.$

$$v_{st}^{nm,flow,pipe} \le Q_s^{nm,max,pipe} + days_s \sum_{t'<t} \Delta_{t'}^{nm,PIPE} \qquad \forall n\in N, m\in out(n), s\in S, t\in T \qquad (\beta 1_{st}^{nm})$$

$$\Delta_t^{nm,PIPE} \le CAP_t^{nm,PIPE} \qquad \forall n\in N, m\in out(n), t\in T \qquad (\beta 2_t^{nm})$$

$$v_{st}^{nm,flow} \ge 0 \qquad \forall n\in N, m\in out(n), s\in S, t\in T$$

$$\Delta_t^{nm,PIPE} \ge 0 \qquad \forall n\in N, m\in out(n), t\in T$$

4.7 Market Clearing Conditions

4.7.1 Sets

N: Set of regions; $n\in N$

$out(n)$: Set of accessible regions from region n; $m\in out(n)$

$reach(n)$: Set of reachable LNG markets from region n; $r\in reach(n)$

$in(n)$: Set of regions that can access region n; $m\in in(n)$

$supplier(n)$: Set of all suppliers in region n

$trader(n)$: Set of all traders in region n

$storage(n)$: Set of all storage operators in region n

$liquefier(n)$: Set of all liquefiers in region n

4.7.2 Parameters

$L^{nr,ship}$: LNG transshipment loss from region n to LNG market r

$a_{ist}^{n,spot}$: a parameter of aggregated regional demand function in the form of $q=a-bp$, where q is the quantity demanded and p is the price.

$b_{ist}^{n,spot}$: b parameter of aggregated regional demand function in the form of $q=a-bp$, where q is the quantity demanded and p is the price.

$a_{ist}^{r,LNG}$: a parameter of LNG market demand function in the form of $q=a-bp$, where q is the quantity demanded and p is the price.

$b_{ist}^{r,LNG}$: b parameter of LNG market demand function in the form of $q=a-bp$, where q is the quantity demanded and p is the price.

4.7.3 Wholesale Markets

$$\sum_{i \in supplier(n)} days_s q_{ist}^{n,prod} = \sum_{k \in trader(n)} v_{kst}^{n,purch,tra} \qquad \forall n \in N, s \in S, t \in T$$

where $\pi_{st}^{n,ws}$ is the dual variable.

4.7.4 Aggregated (Residential, Commercial, Industrial, Power) Gas Markets

$$\sum_{k \in trader(n)} s_{kst}^{n,spot} = a_{st}^{n,spot} - b_{st}^{n,spot} \pi_{st}^{n,spot} \qquad \forall n \in N, s \in S, t \in T$$

where $\pi_{st}^{n,spot}$ is the dual variable.

4.7.5 Storage Market

$$\sum_{d \in storage(n)} v_{d2t}^{n,inj} = \sum_{k \in trader(n)} s_{k2t}^{n,inj} \qquad \forall n, t$$

$$\sum_{d \in storage(n)} v_{d1t}^{n,ext} = \sum_{k \in trader(n)} s_{k1t}^{n,ext} \qquad \forall n \in N, t \in T$$

where $\eta_{2t}^{n,inj}$ and $\eta_{2t}^{n,inj}$ are the dual variables, respectively.

4.7.6 Liquefiers' Market

$$\sum_{k \in trader(n)} s_{kst}^{n,liq} = \sum_{l \in liquefier(n)} v_{lst}^{n,purch,LNG} \qquad \forall n \in N, s \in S, t \in T$$

where $\pi_{st}^{n,liq}$ is the dual variable.

4.7.7 Traders' Market

$$\sum_{\substack{m \in in(n) \\ k \in trader(m)}} s_{kst}^{mn,trader} = \sum_{k \in trader(n)} v_{kst}^{n,purch,int} \qquad \forall n \in N, s \in S, t \in T$$

where $\pi_{st}^{n,tra}$ is the dual variable.

4.7.8 LNG Markets

$$\sum_{\substack{n \in N \\ l \in liquefier(n)}} (1 - L^{nr,ship}) s_{lst}^{nr,LNG} = a_{st}^{r,LNG} - b_{st}^{r,LNG} \pi_{st}^{r,LNG} \qquad \forall r \in R, s \in S, t \in T$$

where $\pi_{st}^{r,LNG}$ is the dual variable.

4.7.9 LNG Tanker Operations

$$\sum_{l \in liquefier(n)} s_{lst}^{nr,LNG} = v_{st}^{nr,flow,LNG} \qquad \forall n \in N, r \in reach(n), s \in S, t \in T$$

where φ_{st}^{nr} is the dual variable.

4.7.10 Pipeline Operations

$$\sum_{k \in trader(n)} f_{kst}^{nm} = v_{st}^{nm,flow,pipe} \qquad \forall n \in N, m \in out(n), s \in S, t \in T$$

where τ_{st}^{nm} is the dual variable.

4.8 KKT Conditions

4.8.1 Supplier's Set:

$$0 \leq \delta_t days_s \Big(-\pi_{st}^{n,ws} + \frac{\partial C_{ist}^{n,prod}(.)}{\partial q_{ist}^{n,prod}} \Big) + \alpha 1_{ist}^n \perp q_{ist}^{n,prod} \geq 0 \qquad \forall n \in N, i \in supplier(n), s \in S, t \in T$$

$$0 \leq \sum_{t'>t, s \in S} \delta_{t'} days_s \frac{\partial C_{ist'}^{n,prod}(.)}{\partial \Delta_{it}^{n,PROD}} + \delta_t \frac{\partial E_{it}^{n,PROD}(\Delta_{it}^{n,PROD})}{\partial \Delta_{it}^{n,PROD}} - \sum_{t'>t, s \in S} \alpha 1_{ist'}^n + \alpha 2_{it}^n \perp \Delta_{it}^{n,PROD} \geq 0,$$
$$\forall n \in N, i \in supplier(n), t \in T$$

$$0 \leq Q_{is}^{n,max,prod} + \sum_{t'<t} \Delta_{it'}^{n,PROD} - q_{ist}^{n,prod} \perp \alpha 1_{ist}^n \geq 0 \qquad \forall n \in N, i \in supplier(n), s \in S, t \in T$$

$$0 \leq CAP_{it}^{n,PROD} - \Delta_{it}^{n,PROD} \perp \alpha 2_{it}^n \geq 0 \qquad \forall n \in N, i \in supplier(n), t \in T$$

4.8.2 Trader's Set:

$$0 \leq \delta_t \tau_{st}^{nm} + \theta 1_{kst}^{nm} \perp f_{kst}^{nm} \geq 0 \qquad \forall n \in N, m \in out(n), k \in trader(n), s \in S, t \in T$$
$$0 \leq -\delta_t \pi_{st}^{m,tra} - \theta 1_{kst}^{nm} + \theta 2_{kst}^{n} \perp s_{kst}^{nm,tra} \geq 0 \qquad \forall n \in N, m \in out(n), k \in trader(n), s \in S, t \in T$$
$$0 \leq -\delta_t \pi_{st}^{n,spot} + \theta 2_{kst}^{n} \perp s_{kst}^{n,spot} \geq 0 \qquad \forall n \in N, k \in trader(n), s \in S, t \in T$$
$$0 \leq -\delta_t \pi_{st}^{n,liq} + \theta 2_{kst}^{n} \perp s_{kst}^{n,liq} \geq 0 \qquad \forall n \in N, k \in trader(n), s \in S, t \in T$$
$$0 \leq \delta_t \pi_{st}^{n,ws} - \theta 2_{kst}^{n} \perp v_{kst}^{n,purch,tra} \geq 0 \qquad \forall n \in N, k \in trader(n), s \in S, t \in T$$
$$0 \leq \delta_t \pi_{st}^{n,tra} - \theta 2_{kst}^{n} \perp v_{kst}^{n,purch,int} \geq 0 \qquad \forall n \in N, k \in trader(n), s \in S, t \in T$$
$$0 \leq \delta_t \eta_{2t}^{n,inj} + \theta 2_{k2t}^{n} \perp s_{k2t}^{n,inj} \geq 0 \qquad \forall n \in N, k \in trader(n), s = 2, t \in T$$
$$0 \leq \delta_t \eta_{1t}^{n,ext} - \theta 2_{k1t}^{n} \perp s_{k1t}^{n,ext} \geq 0 \qquad \forall n \in N, k \in trader(n), s = 1, t \in T$$
$$0 = f_{kst}^{nm} - s_{kst}^{nm,trader}, \theta 1_{kst}^{nm} \text{ free} \qquad \forall n \in N, m \in out(n), k \in trader(n), s \in S, t \in T$$
$$0 = \sum_{m \in out(n)} s_{kst}^{nm,tra} + s_{kst}^{n,liq} + s_{kst}^{n,spot} + s_{kst}^{n,inj} - v_{kst}^{n,purch,tra} - v_{kst}^{n,purch,int} - s_{kst}^{n,ext}, \theta 2_{kst}^{n} \text{ free}$$
$$\forall n \in N, k \in trader(n), s \in S, t \in T$$

4.8.3 Storage Operator's Set

$$0 \leq \delta_t(-\eta_{1t}^{n,ext} + c_{dt}^{n,ext}) + \sigma 1_{dt}^{n} + \sigma 3_{dt}^{n} \perp v_{d1t}^{n,ext} \geq 0 \qquad \forall n \in N, d \in storage(n), t \in T$$
$$0 \leq \delta_t(-\eta_{2t}^{n,inj} + c_{dt}^{n,inj}) + \sigma 2_{dt}^{n} - (1 - L_d^{n,stor})\sigma 3_{dt}^{n} \perp v_{d2t}^{n,inj} \geq 0 \qquad \forall n \in N, d \in storage(n), t \in T$$
$$0 \leq \delta_t e_{dt}^{n,EXT} + \sigma 4_{dt}^{n} - \sum_{t' > t} \sigma 1_{dt'}^{n} \perp \Delta_{dt}^{n,EXT} \geq 0 \qquad \forall n \in N, d \in storage(n), t \in T$$
$$0 \leq \delta_t e_{dt}^{n,INJ} + \sigma 5_{dt}^{n} - \sum_{t' > t} \sigma 2_{dt'}^{n} \perp \Delta_{dt}^{n,INJ} \geq 0 \qquad \forall n \in N, d \in storage(n), t \in T$$
$$0 \leq Q_d^{n,max,ext} + \sum_{t' < t} \Delta_{dt'}^{n,EXT} - v_{d1t}^{n,ext} \perp \sigma 1_{dt}^{n} \geq 0 \qquad \forall n \in N, d \in storage(n), t \in T$$
$$0 \leq Q_d^{n,max,inj} + \sum_{t' < t} \Delta_{dt'}^{n,INJ} - v_{d2t}^{n,inj} \perp \sigma 2_{dt}^{n} \geq 0 \qquad \forall n \in N, d \in storage(n), t \in T$$
$$0 = v_{d1t}^{n,ext} - (1 - L_d^{n,stor}) v_{d2t}^{n,inj}, \sigma 3_{dt}^{n} \text{ free} \qquad \forall n \in N, d \in storage(n), t \in T$$
$$0 \leq CAP_{dt}^{n,EXT} - \Delta_{dt}^{n,EXT} \perp \sigma 5_{dt}^{n} \geq 0 \qquad \forall n \in N, d \in storage(n), t \in T$$
$$0 \leq CAP_{dt}^{n,INJ} - \Delta_{dt}^{n,INJ} \perp \sigma 6_{dt}^{n} \geq 0 \qquad \forall n \in N, d \in storage(n), t \in T$$

4.8.4 Liquefier's Set

$$0 \leq \delta_t(-\pi_{st}^{r,LNG} + \varphi_{st}^{nr}) - \lambda 1_{lst}^n \perp s_{lst}^{nr,LNG} \geq 0 \qquad \forall n \in N, r \in reach(n), l \in liquefier(n), s \in S, t \in T$$
$$0 \leq \delta_t c_{lst}^{n,liq} + \lambda 1_{lst}^n + \lambda 2_{lst}^n - \lambda 3_{lst}^n \perp v_{lst}^{n,LNG} \geq 0 \qquad \forall n \in N, l \in liquefier(n), s \in S, t \in T$$
$$0 \leq \delta_t \pi_{st}^{n,liq} + (1 - L_l^{n,LNG})\lambda 3_{lst}^n \perp v_{lst}^{n,purch,LNG} \geq 0 \qquad \forall n \in N, l \in liquefier(n), s \in S, t \in T$$
$$0 \leq \delta_t e_{lt}^{n,LIQ} - \sum_{t'>t, s \in S} days_s \lambda 2_{lst'}^n + \lambda 4_{lt}^n \perp \Delta_{lt}^{n,LNG} \geq 0 \qquad \forall n \in N, l \in liquefier(n), t \in T$$
$$0 = v_{lst}^{n,LNG} - \sum_{r \in reach(n)} s_{lst}^{nr,LNG}, \lambda 1_{lst}^n \text{ free} \qquad \forall n \in N, l \in liquefier(n), s \in S, t \in T$$
$$0 \leq Q_{ls}^{n,max,LNG} + \sum_{t'<t} days_s \Delta_{lt'}^{n,LIQ} - v_{lst}^{n,LNG} \perp \lambda 2_{lst}^n \geq 0 \qquad \forall n \in N, l \in liquefier(n), s \in S, t \in T$$
$$0 = (1 - L_l^{n,LNG}) v_{lst}^{n,purch,LNG} - v_{lst}^{n,LNG}, \lambda 3_{lst}^n \text{ free} \qquad \forall n \in N, l \in liquefier(n), s \in S, t \in T$$
$$0 \leq CAP_{lt}^{n,LIQ} - \Delta_{lt}^{n,LIQ} \perp \lambda 4_{lt}^n \geq 0 \qquad \forall n \in N, l \in liquefier(n), t \in T$$

4.8.5 Tanker Network Operator's Set

$$0 \leq \delta_t(-\varphi_{st}^{nr} + c_{st}^{nr,flow,LNG}) + \rho 1_{st}^{nr} \perp v_{st}^{nm,flow,LNG} \geq 0 \qquad \forall n \in N, r \in reach(n), s \in S, t \in T$$
$$0 \leq \delta_t e_t^{nr,TANKER} - \sum_{t'>t, s \in S} days_s \rho 1_{st'}^{nr} + \rho 2_t^{nr} \perp \Delta_t^{nr,TANKER} \geq 0, \forall n \in N, r \in reach(n), t \in T$$
$$0 \leq Q_s^{nr,max,tanker} + \sum_{t'<t} days_s \Delta_{t'}^{nr,TANKER} - v_{st}^{nr,flow,LNG} \perp \rho 1_{st}^{nr} \geq 0 \qquad \forall n \in N, r \in reach(n), s \in S, t \in T$$
$$0 \leq CAP_t^{nr,TANKER} - \Delta_t^{nr,TANKER} \perp \rho 2_t^{nr} \geq 0 \qquad \forall n \in N, r \in reach(n), t \in T$$

4.8.6 Pipeline Network Operator's Set

$$0 \leq \delta_t(-\tau_{st}^{nm} + c_{st}^{nm,flow,pipe}) + \beta 1_{st}^{nm} \perp v_{st}^{nm,flow,pipe} \geq 0 \qquad \forall n \in N, m \in out(n), s \in S, t \in T$$
$$0 \leq \delta_t e_t^{nm,PIPE} - \sum_{t'>t, s \in S} days_s \beta 1_{st'}^{nm} + \beta 2_t^{nm} \perp \Delta_t^{nm,PIPE} \geq 0 \qquad \forall n \in N, m \in out(n), t \in T$$
$$0 \leq Q_s^{nm,max,pipe} + \sum_{t'<t} days_s \Delta_{t'}^{nm,PIPE} - v_{st}^{nm,flow,pipe} \perp \beta 1_{st}^{nm} \geq 0 \qquad \forall n \in N, m \in out(n), s \in S, t \in T$$
$$0 \leq CAP_t^{nm,PIPE} - \Delta_t^{nm,PIPE} \perp \beta 2_t^{nm} \geq 0, \qquad \forall n \in N, m \in out(n), t \in T$$

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