Competition in Electricity Markets with Renewable Energy Sources: Online Appendix

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Part I. Proofs omitted from main text

1. MERIT ORDER EFFECT VS. DIVERSIFICATION

Proof of Theorem 1 We present the proof for the duopoly case, extension to $n \geq 2$ is straightforward. With the concave (downward) inverse demand P and the convex cost C, profit of each (diverse) thermal producer is given by

$$\Pi_i = (q_i + \delta R/2)P(Q + R) - C(q_i), \tag{1}$$

where each conventional generator owns $\delta R/2$ units of renewable supply, $\delta \in [0, 1]$.

We first note that

$$\frac{\partial p}{\partial R} = \left(\frac{\partial Q}{\partial R} + 1\right) P'(Q + R), \tag{2}$$

where $p \equiv P(Q + R)$. Since P' is downward (i.e. $P' < 0$), thus to show $\frac{\partial p}{\partial R} \leq 0$, we next prove that $\frac{\partial Q}{\partial R} + 1 \geq 0$.

FOC implies

$$0 = \frac{\partial \Pi_i}{\partial q_i} = P(Q + R) + (q_i + \delta R/2)P'(Q + R) - C'(q_i)$$

By symmetry $q_1 = q_2$, thus the above equation is equivalent to

$$0 = \frac{\partial \Pi_i}{\partial q_i} = P(Q + R) + (\frac{1}{2})(Q + \delta R)P'(Q + R) - C'(Q/2) \tag{3}$$

Taking derivative from (3) with respect to R implies

$$\begin{aligned} 0 = &\left(1 + \frac{\partial Q}{\partial R}\right) P'(Q + R) + (\frac{1}{2}) \left(1 + \frac{\partial Q}{\partial R}\right) (Q + \delta R)P''(Q + R) \\ &+ (\frac{1}{2}) \left(\delta + \frac{\partial Q}{\partial R}\right) P'(Q + R) - (\frac{1}{2}) \left(\frac{\partial Q}{\partial R}\right) C'' \left(\frac{Q}{2}\right) \end{aligned}$$

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Rearranging terms yields

$$0 = \left[3P'(Q+R) + (Q+\delta R)P''(Q+R) - C''\left(\frac{Q}{2}\right)\right]\frac{\partial Q}{\partial R} + \left[(2+\delta)P'(Q+R) + (Q+\delta R)P''(Q+R)\right]$$

Consequently,

$$\frac{\partial Q}{\partial R} = -\frac{(2+\delta)P'(Q+R) + (Q+\delta R)P''(Q+R)}{3P'(Q+R) + (Q+\delta R)P''(Q+R) - C''\left(\frac{Q}{2}\right)} \tag{4}$$

Recall that the cost function is convex, thus $C'' \geq 0$. As a result, with linear cost, $C'' = 0$, since $P' < 0$ and $P'' < 0$, thus (4) implies

$$-1 \leq \frac{\partial Q}{\partial R} < 0 \quad \Rightarrow \quad 1 + \frac{\partial Q}{\partial R} \geq 0, \quad \text{and with } \delta = 1, \quad -1 = \frac{\partial Q}{\partial R}. \tag{5}$$

Thus, using (2), we obtain $\frac{\partial p}{\partial R} \leq 0$. Moreover, when all renewable supply generates profits for only conventional power generators (i.e. $\delta = 1$), $\frac{\partial p}{\partial R} = 0$, neutralizing the MoE. However, with strictly convex cost (i.e. $C'' > 0$), for $\delta \in [0,\ 1]$:

$$\begin{aligned} 0 > \frac{\partial Q}{\partial R} &= -\frac{(2+\delta)P'(Q+R) + (Q+\delta R)P''(Q+R)}{3P'(Q+R) + (Q+\delta R)P''(Q+R) - C''\left(\frac{Q}{2}\right)} \\ &> -\frac{(2+\delta)P'(Q+R) + (Q+\delta R)P''(Q+R)}{3P'(Q+R) + (Q+\delta R)P''(Q+R)} \\ &\geq -1. \end{aligned}$$

As a result, $\frac{\partial Q}{\partial R} + 1 > 0$, consequently, using (2), $\frac{\partial p}{\partial R} < 0$ for all $\delta \in [0,\ 1]$. Therefore, full neutralization may not be obtained with strictly convex cost functions.

To wrap up the proof we next show $\frac{\partial p}{\partial \delta} > 0$, diversification amplifies the prices. Since

$$\frac{\partial p}{\partial \delta} = \left(\frac{\partial Q}{\partial \delta}\right)\underbrace{P'(Q+R)}_{<0}$$

to prove the claim is then sufficient to show $\frac{\partial Q}{\partial \delta} < 0$. Taking a derivative from (3) with respect to δ implies

$$0 = \left(\frac{\partial Q}{\partial \delta}\right)P'(Q+R) + (\frac{1}{2})\left(\frac{\partial Q}{\partial \delta}\right)(Q+\delta R)P''(Q+R) + (\frac{1}{2})\left(R + \frac{\partial Q}{\partial \delta}\right)P'(Q+R) - (\frac{1}{2})\left(\frac{\partial Q}{\partial \delta}\right)C''\left(\frac{Q}{2}\right)$$

Rearranging terms gives

$$0 = \left[3P'(Q+R) + (Q+\delta R)P''(Q+R) - C''\left(\frac{Q}{2}\right)\right]\frac{\partial Q}{\partial \delta} + RP'(Q+R)$$

Therefore,

$$\frac{\partial Q}{\partial \delta} = -\frac{RP'(Q+R)}{3P'(Q+R) + (Q+\delta R)P''(Q+R) - C''\left(\frac{Q}{2}\right)} < 0, \tag{6}$$

completing the proof of the first part.

To prove the second part we note that since $P' \neq 0$, thus $\frac{\partial p}{\partial R} = \left(\frac{\partial Q}{\partial R} + 1\right) P'(Q+R) = 0$ if and only if $\frac{\partial Q}{\partial R} = -1$. Therefore, when $\delta \to 1$, using (4), we obtain $\frac{\partial Q}{\partial R} = -1$ if and only if $C'' = 0$. Thus, under any condition ensuring a unique interior equilibrium, neutralization result prevails when (i) thermal producers are diversified, (ii) cost of production (via thermal sources) is either linear or constant, i.e. $C'' = 0$.

It is worthy to note that, inspired by the standard Cournot model, a unique equilibrium is ensured in our model if: (i) $C'' - P'(Q+R) > 0$, (ii) $\frac{P'(Q+R)+(q_i+\delta\frac{R}{2})P''(Q+R)}{C''-P'(Q+R)} < \frac{1}{n}$, where n denotes the number of thermal generators.

2. DERIVATIONS OF THE REDUCED-FROM MODELS

Proof of Theorem 2 Since $\Pi_i = (\alpha - \beta(\sum_i q_i + R))(q_i + \delta R/n) - \gamma q_i$, thus FOC implies $\alpha - \beta(q_i + \sum_{j \neq i} q_j + R) - \beta(q_i + \delta R/n) - \gamma = 0$, for all $i = 1, 2, \cdots, n$. Taking a sum over all i implies $n(\alpha-\gamma) - n\beta(Q+R) - \beta(Q+\delta R) = 0$. Hence, at the equilibrium, $Q = \frac{n(\alpha-\gamma)-\beta(\delta R+nR)}{\beta(n+1)}$. By symmetry, $q_i = Q/n = \frac{\alpha-\gamma-\beta(\delta R/n+R)}{\beta(n+1)}$, for all $i = 1, 2, \cdots, n$. Further, plugging Q into $p = \alpha - \beta(Q+R)$ implies $p = \frac{\alpha+\beta(-R+\delta R)+n\gamma}{n+1}$, as desired.

Proof of Theorem 3 As discussed when thermal producers are competitive $\mathcal{W}(CE) = \frac{(\alpha-\gamma)^2}{2\beta} + \gamma R$, that is independent of δ. Next, we consider the case in which thermal producers are strategic. Thus, at the corresponding Nash equilibrium the welfare is given by

$$\begin{aligned}\mathcal{W}(NE) &= (Q_{NE} + \delta R)p_{NE} - \gamma Q_{NE} + p_{NE}(1-\delta)R + \frac{(\alpha - p_{NE})^2}{2\beta} \\ &= (p_{NE} - \gamma)Q_{NE} + Rp_{NE} + \frac{(\alpha - p_{NE})^2}{2\beta}\end{aligned}$$

where (as shown in Theorem 2) the overall production is given by $Q_{NE} = \frac{n}{(n+1)\beta}(\alpha - \gamma - \beta(R + \delta R/n))$ and the resulting spot price satisfies $p_{NE} = \frac{1}{n+1}(\alpha + \beta(-R+\delta R) + n\gamma)$. Therefore

$$\begin{aligned}\frac{\partial \mathcal{W}(NE)}{\partial \delta} &= \left(\frac{\partial p_{NE}}{\partial \delta}\right) Q_{NE} + (p_{NE} - \gamma)\frac{\partial Q_{NE}}{\partial \delta} + \frac{\partial p_{NE}}{\partial \delta} R - \frac{1}{\beta}(\alpha - p_{NE})\frac{\partial p_{NE}}{\partial \delta} \\ &= \underbrace{(p_{NE} - \gamma)}_{>0} \underbrace{\frac{\partial Q_{NE}}{\partial \delta}}_{<0} \\ &< 0, \end{aligned} \tag{7}$$

note that $p_{NE} > \gamma$, because $\alpha + \beta(-R + \delta R) - \gamma > 0$. Equation (7) immediately implies $\frac{\partial}{\partial \delta}\left(\frac{\mathcal{W}(CE)}{\mathcal{W}(NE)}\right) > 0$, completing the proof.

Proof of Theorem 4 To have a better understanding of the proof steps we first consider the duopoly case. The oligopoly case is more involved but it follows similar steps.

Consider the duopoly case, i.e. $n = 2$. By adding forward contract to the previous case the profit of each generator becomes $\Pi_i(q_1, q_2) = (\alpha - \beta(q_1 + q_2 + R))(q_i - q_i^f + \delta R/2) + q_i^f p_i^f - \gamma q_i$. In this case, the economy has two dates, $t = 1, 2$: generators sign forward contract at $t = 1$ and the market clearing price $p = \alpha - \beta(\sum_{i=1}^n q_i + R)$ is characterized at the final date $t = 2$. The solution strategy is to work backward. Thus, given $(q_1^f, p_1^f, q_2^f, p_2^f)$, FOC implies

$$0 = \frac{\partial \Pi_i}{\partial q_i} = \alpha - \gamma - \beta(q_1 + q_2 + R) - \beta(q_i - q_i^f + \delta R/2). \tag{8}$$

Summing over $i \in \{1,2\}$ and rearranging terms imply $Q = \frac{1}{3\beta}\left(2\alpha - 2\beta R - \beta(-Q^f + \delta R) - 2\gamma\right)$. Plugging Q into (8) and some alegra yield

$$q_i = \frac{1}{3\beta}\left(\alpha - \gamma - \beta(Q^f - 3q_i^f + R + \delta R/2)\right).$$

Now, given the optimal supply at the final date, we next characterize the optimal forward contract for each generator. Note that assuming no possibility for arbitrage implies at $t = 1$ the forward quantity q_i^f is signed at the market price, i.e. $p_i^f = p$. Thus, optimal q_i^f maximizes $p(q_i + \delta R/2) - \gamma q_i$, where $p = \alpha - \beta(q_1 + q_2 + R)$. Since $\frac{\partial p}{\partial q_i^f} = -\beta/3$ and $\frac{\partial q_i}{\partial q_i^f} = 2/3$, thus FOC gives $\frac{\partial p}{\partial q_i^f}(q_i + \delta R/2) + p\frac{\partial q_i}{\partial q_i^f} - \gamma\frac{\partial q_i}{\partial q_i^f} = -\frac{\beta}{3}(q_i + \delta R/2) + (p - \gamma)\frac{2}{3} = 0$. Simplifying this equation after plugging (8) into it, implies

$$q_1^f = q_2^f = \frac{1}{5\beta}\left(\alpha - \gamma + \beta(-R + \delta R)\right).$$

Plugging q_i^f into (8) yields $q_i = \frac{2}{5\beta}\left(\alpha - \gamma + \beta(-R - \delta R/4)\right)$. Finally, market price becomes $p = \alpha - \beta(q_1 + q_2 + R) = \frac{1}{5}\left(\alpha + 4\gamma + \beta(-R + \delta R)\right)$.

We next consider the oligopoly case, i.e. $n \geq 2$. Given the profit of producer i, i.e. $\Pi_i(q_i, q_{-i}) = (\alpha - \beta(q_i + \sum_{j\neq i} q_j + R))(q_i - q_i^f + \delta R/n) + q_i^f p_i^f - \gamma q_i$, employing the FOC implies $\alpha - \gamma - \beta\left(\sum_{j\neq i} q_j + R - q_i^f + \delta R/n\right) = 2\beta q_i$. Thus, rearranging terms yields

$$\alpha - \gamma - \beta\left(2q_i + \sum_{j\neq i} q_j + R - q_i^f + \delta R/n\right) = 0. \tag{9}$$

Let $Q \equiv \sum_{j=1}^n Q_j$. Taking a sum over all i from (9) implies

$$Q = \frac{n(\alpha - \gamma - \beta R) - \beta(-Q^f + \delta R)}{(n+1)\beta}, \tag{10}$$

where $Q^f \equiv \sum_{i=1}^n q_i^f$.

Next, from (9) we obtain

$$\alpha - \beta(Q + R) - \beta(-q_i^f + \delta R/n) - \gamma = \beta q_i.$$

The LHS of the above equation can be simplified as follows:

$$\begin{aligned}
\text{LHS} &= (\alpha - \gamma) - \beta R - \beta(q_i^f + \delta R/n) - \beta Q \\
&= \frac{1}{n+1}\left((n+1)[(\alpha - \gamma) - \beta R - \beta(-q_i^f + \delta R/n)] - n(\alpha - \gamma) + n\beta R + \beta(-Q^f + \delta R)\right) \\
&= \frac{1}{n+1}\left((\alpha - \gamma) - \beta\left[Q^f - (n+1)q_i^f + R + \delta R/n\right]\right)
\end{aligned}$$

Therefore, we have

$$q_i = \frac{1}{(n+1)\beta}\left(\alpha - \gamma - \beta\left[Q^f - (n+1)q_i^f + R + \delta R/n\right]\right). \tag{11}$$

Next, we move to the contracting stage.

Contracting stage Equipped with the results from the production stage, we next find optimal forward quantities, i.e. $q_1^f, q_2^f, \cdots, q_n^f$. Importantly, due to the no arbitrage assumption $p_i^f = p$. Thus, producer i's optimal choice for q_i^f should maximize

$$\left(\alpha - \beta(Q(q_i^f, q_{-i}^f) + R)\right)(q_i(q_i^f, q_{-i}^f) + \delta R/n) - \gamma q_i(q_i^f, q_{-i}^f).$$

Thus, the FOC gives

$$(\alpha - \beta(Q(q_i^f, q_{-i}^f) + R))\frac{\partial q_i(q_i^f, q_{-i}^f)}{\partial q_i^f} - \beta\frac{\partial Q(q_i^f, q_{-i}^f)}{\partial q_i^f}\left(q_i(q_i^f, q_{-i}^f) + \delta R/n\right) - \gamma\frac{\partial q_i(q_i^f, q_{-i}^f)}{\partial q_i^f} = 0 \tag{12}$$

Since $\frac{\partial q_i(q_i^f, q_{-i}^f)}{\partial q_i^f} = \frac{n}{n+1}$ (see (11)) and $\frac{\partial Q(q_i^f, q_{-i}^f)}{\partial q_i^f} = \frac{1}{n+1}$ (see (10)), thus (12) yeilds

$$(\alpha - \beta(Q(q_i^f, q_{-i}^f) + R))\frac{n}{n+1} - \gamma\frac{n}{n+1} - (q_i(q_i^f, q_{-i}^f) + \delta R/n)\frac{\beta}{n+1} = 0 \tag{13}$$

multiplying in $(n+1)$ and rearranging terms give $-\beta(q_i(q_i^f, q_{-i}^f) + nQ(q_i^f, q_{-i}^f)) + n(\alpha - \gamma) - \beta(\delta R/n + nR) = 0$. By symmetry $q_1(q_1^f, q_{-1}^f) = q_2(q_2^f, q_{-2}^f) = \cdots = q_n(q_n^f, q_{-n}^f)$, thus

$$-\beta(n^2+1)q_i(q_i^f, q_{-i}^f) + n(\alpha - \gamma) - \beta(\delta R/n + nR) = 0. \tag{14}$$

Moreover, (11) gives that $-\beta(n^2+1)q_i(q_i^f, q_{-i}^f) = -\frac{n^2+1}{n+1}\left(\alpha - \gamma - \beta\left[-q_i^f + R + \delta R/n\right]\right)$, note that by symmetry $q_1^f = q_2^f = \cdots = q_n^f$.

Plugging this into (14) gives

$$-(n^2+1)\left[\alpha - \gamma - \beta(-q_i^f + R + \delta R/n)\right] + (n^2+n)(\alpha - \gamma) - (n+1)\beta(\delta R/n + nR) = 0. \tag{15}$$

Rearranging terms implies

$$\begin{aligned}(n^2+1)\beta q_i^f &= (n-1)(\alpha - \gamma) + (n^2+1)\beta(R + \delta R/n) - (n+1)\beta(\delta R/n + nR)\\ &= (n-1)[\alpha - \gamma + \beta(-R + \delta R)].\end{aligned}$$

Thus

$$q_i^f = \frac{n-1}{(n^2+1)\beta}\left(\alpha - \gamma + \beta(-R + \delta R)\right). \tag{16}$$

Thus, we finally find q_i by plugging q_i^f into (11). That is

$$\begin{aligned} q_i &= \frac{1}{(n+1)\beta}\left(\alpha - \gamma - \beta(-q_i^f + R + \delta R/n)\right) \\ &= \frac{1}{(n+1)\beta}\left(\alpha - \gamma - \beta\left[-\frac{n-1}{(n^2+1)\beta}\left(\alpha - \gamma + \beta(-R + \delta R)\right) + R + \delta R/n\right]\right) \\ &= \frac{n}{(n^2+1)\beta}\left(\alpha + \beta\left(-R - \frac{\delta R}{n^2}\right) - \gamma\right). \end{aligned}$$

With the characterization of q_i, $i = 1, 2\cdots, n$, the proof is complete.

3. CORRELATED SHOCKS AND INCOMPLETE INFORMATION WITH ENDOGENOUS FORWARD CONTRACT

3.1 Equilibrium characterization

Proof of Theorem 5 It is useful to construct the equilibrium sequentially and work backward. We start by deriving the optimal behavior at the production stage, for any given set of forward contract level $(q_i^f, p_i^f)_{i=1,\cdots,n}$ that summarizes the behavior at the production stage. Next, we analyze the optimal behavior at the contracting stage.

Production stage For a given profile of contracting profile $((q_1^f, p_1^f), \cdots, (q_n^f, p_n^f))$, the production stage is a game with imperfect competition in the class of normal linear-quadratic games with correlated types, once a generator information R_i is established as the type. A strategy q_i is a mapping from the information set into the real space: $q_i : \mathbb{R} \to \mathbb{R}_+$ for a given individual contracting choice (q_i^f, p_i^f). We focus on linear equilibria at the production stage throughout, that is for each generator $i = 1, \cdots, n$, there exists a_i and b_i such that $q_i(R_i) = b_i - a_i\theta_i$. Optimality for generator i at the production stage requires that strategy q_i maximizes the conditional expected utility, taking all other generators strategies q_{-i} as given. Furthermore, applying the projection theorem,[1] with the linear information structure conditional expectations $E(\theta_j|\theta_i)$ are linear in θ_i, for all i, j.

Producer i chooses q_i to maximize

$$E_{\theta_{-i}}(\Pi_i|R_i) = E\{p(q_i - q_i^f + \delta_i R_i) + p_i^f q_i^f - \gamma q_i|R_i\}$$

where $q_j(\theta_j) = b_j - a_j\theta_j$, for all $j \neq i$, and $p = \alpha - \beta(q_i + R_i + \sum_{j\neq i} R_j + \sum_{j\neq i} q_j)$. Recall that $R_i = R/n + \theta_i$, for all $i = 1, 2, \cdots, n$, thus, the first order conditions (FOC) gives

$$\alpha - \gamma - \beta\left(\sum_{j\neq i} E[q_j(\theta_j)|R_i] + \sum_{j\neq i} E[\theta_j|R_i] + \theta_i + R\right) - \beta(-q_i^f + \delta R/n + \delta\theta_i) = 2\beta q_i \tag{17}$$

Using the projection theorem:

$$\begin{aligned} E[q_j(\theta_j)|R_i] = E[q_j(\theta_j)|\theta_i] &= b_j - a_j\, Cov(\theta_i, \theta_j)\, \sigma^{-2}\, \theta_i = b_j - a_j \kappa_{i,j}\theta_i \\ E[\theta_j|R_i] = E[\theta_j|\theta_i] &= Cov(\theta_i, \theta_j)\, \sigma^{-2}\, \theta_i = \kappa_{i,j}\theta_i. \end{aligned} \tag{18}$$

[1] Let θ and ν be random variables such that $(\theta, \nu) \sim \mathcal{N}(\mu, \Sigma)$ such that:

$$\mu \equiv \begin{pmatrix} \mu_\theta \\ \mu_\nu \end{pmatrix} \qquad \Sigma \equiv \begin{pmatrix} \Sigma_{\theta,\theta} & \Sigma_{\theta,\nu} \\ \Sigma_{\nu,\theta} & \Sigma_{\nu,\nu} \end{pmatrix}$$

are expectations and variance-covariance matrix, then the conditional density of θ given ν is normal with conditional mean $\mu_\theta + \Sigma_{\theta,\nu}\Sigma_{\nu,\nu}^{-1}(\nu - \mu_\nu)$ and variance-covariance matrix $\Sigma_{\theta,\theta} - \Sigma_{\theta,\nu}\Sigma_{\nu,\nu}^{-1}\Sigma_{\nu,\theta}$, provided that $\Sigma_{\nu,\nu}$ is non-singular.

Plugging (18) into (17) and rearranging terms gives

$$\left(\alpha - \gamma - \beta\left(\sum_{j\neq i} b_j + R - q_i^f + \delta R/n\right)\right) - \theta_i\beta\left((1+\delta) + \sum_{j\neq i}\kappa_{i,j} - \sum_{j\neq i} a_j\kappa_{i,j}\right)$$
$$= (2\beta b_i) - \theta_i(2\beta a_i) \tag{19}$$

Next, to find a_i, we only need to equate the coefficient of θ_i in the LHS and RHS of (19), that implies (note that $\beta > 0$)

$$\sum_{j\neq i}\kappa_{i,j}a_j + 2a_i = (1+\delta) + \sum_{j\neq i}\kappa_{i,j} \equiv v_i \tag{20}$$
$$\Rightarrow \quad A\boldsymbol{a} = \boldsymbol{v},$$

where $A \equiv \frac{1}{\sigma^2}\Sigma + I$, and I denotes the identity matrix. Since A is positive define, it is invertible and thus

$$\boldsymbol{a} = A^{-1}\boldsymbol{v}, \tag{21}$$

that is

$$\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} 2 & \kappa_{1,2} & \cdots & \kappa_{1,n} \\ \kappa_{2,1} & 2 & \cdots & \kappa_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \kappa_{n,1} & \kappa_{n,2} & \cdots & 2 \end{pmatrix}^{-1} \begin{pmatrix} 1+\delta+\sum_{j\neq 1}\kappa_{1,j} \\ 1+\delta+\sum_{j\neq 2}\kappa_{2,j} \\ \vdots \\ 1+\delta+\sum_{j\neq n}\kappa_{n,j} \end{pmatrix}.$$

Thus,

$$\boldsymbol{a} = \left(\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1}\left(\delta\mathbf{1} + \frac{1}{\sigma^2}\Sigma\mathbf{1}\right) = \left(\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1}\left((\delta-1)\mathbf{1} + \mathbf{1} + \frac{1}{\sigma^2}\Sigma\mathbf{1}\right)$$
$$= \mathbf{1} + (\delta-1)\left(\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1}\mathbf{1}.$$

Similarly, we derive b_i. Equating the terms that are independent of θ_i in the LHS and RHS of (19) implies

$$\alpha - \gamma - \beta\left(\sum_{j\neq i} b_j + R - q_i^f + \delta R/n\right) = 2\beta b_i$$
$$\Rightarrow \quad \alpha - \gamma - \beta\left(2b_i + \sum_{j\neq i} b_j + R - q_i^f + \delta R/n\right) = 0. \tag{22}$$

Let $b \equiv \sum_{j=1}^n b_j$. Taking a sum over all i from (22) gives

$$b = \frac{n\alpha - n\gamma - n\beta R - \beta(-Q^f + \delta R)}{(n+1)\beta} \tag{23}$$

where $Q^f \equiv \sum_{i=1}^n q_i^f$. In addition, from (22) we have

$$\alpha - \beta b - \beta R - \beta(-q_i^f + \delta R/n) - \gamma = \beta b_i.$$

The LHS of the above equation can be simplified as follows:

$$\begin{aligned}
\text{LHS} &= (\alpha-\gamma) - \beta R - \beta(q_i^f + \delta R/n) - \beta b \\
&= \frac{1}{n+1}\left((n+1)[(\alpha-\gamma) - \beta R - \beta(-q_i^f + \delta R/n)] - n(\alpha-\gamma) + n\beta R + \beta(-Q^f + \delta R)\right) \\
&= \frac{1}{n+1}\left((\alpha-\gamma) - \beta R - \beta\left[\left(-(n+1)q_i^f + \frac{n+1}{n}\delta R\right) + Q^f - \delta R\right]\right) \\
&= \frac{1}{n+1}\left((\alpha-\gamma) - \beta\left[Q^f - (n+1)q_i^f + R + \delta R/n\right]\right)
\end{aligned}$$

Therefore, we finally have

$$b_i = \frac{1}{(n+1)\beta}\left(\alpha - \gamma - \beta\left[Q^f - (n+1)q_i^f + R + \delta R/n\right]\right). \tag{24}$$

The above equations summarize the unique linear equilibrium in the production stage.
Stage 1: Contracting stage Equipped with the results from the production stage, we next evaluate the amount of optimal forward contract q_i^f for each generator $i = 1, 2, \cdots, n$, at the average market price $p_i^f = E_\theta[p]$ (due to the no arbitrage assumption). This is achieved by computing expected payoff of each generator. Thus, generator i's optimal choice for q_i^f should maximize

$$E_\theta\left[\left(\alpha - \beta(q_i(q_i^f, q_{-i}^f) + R_i + \sum_{j\neq i} R_j + \sum_{j\neq i} q_j(q_j^f, q_{-j}^f))\right)(q_i(q_i^f, q_{-i}^f) + \delta R_i) - \gamma q_i(q_i^f, q_{-i}^f)\right].$$

The optimal choice in the production stage is linear in the observed information and is in the form of $q_i(\theta_i) = b_i - a_i\theta_i$. Characterization of a_i and b_i (see (24) and (37)) gives

$$\frac{\partial q_i(q_i^f, q_{-i}^f)}{\partial q_i^f} = \frac{\partial b_i(q_i^f, q_{-i}^f)}{\partial q_i^f}.$$

By the above equality and the fact that $E[\theta_i] = 0$ and $E[R_i] = R/n$, it is sufficient to find q_i^f maximizing

$$(\alpha - \beta(b(q_i^f, q_{-i}^f) + R))(b_i(q_i^f, q_{-i}^f) + \delta R/n) - \gamma b_i(q_i^f, q_{-i}^f).$$

The FOC gives

$$(\alpha - \beta(b(q_i^f, q_{-i}^f) + R))\frac{\partial b_i(q_i^f, q_{-i}^f)}{\partial q_i^f} - \beta\frac{\partial b(q_i^f, q_{-i}^f)}{\partial q_i^f}\left(b_i(q_i^f, q_{-i}^f) + \delta R/n\right) - \gamma\frac{\partial b_i(q_i^f, q_{-i}^f)}{\partial q_i^f} = 0 \tag{25}$$

Since $\frac{\partial b_i(q_i^f, q_{-i}^f)}{\partial q_i^f} = \frac{n}{n+1}$ (see (24)) and $\frac{\partial b(q_i^f, q_{-i}^f)}{\partial q_i^f} = \frac{1}{n+1}$ (see (23)), thus (25) gives

$$(\alpha - \beta(b(q_i^f, q_{-i}^f) + R))\frac{n}{n+1} - \gamma\frac{n}{n+1} - (b_i(q_i^f, q_{-i}^f) + \delta R/n)\frac{\beta}{n+1} = 0. \tag{26}$$

Multiplying (26) in $(n+1)$ and rearranging terms yield

$$-\beta(b_i + nb(q_i^f, q_{-i}^f)) + n(\alpha - \gamma) - \beta(\delta R/n + nR) = 0.$$

By symmetry $b_1(q_1^f, q_{-1}^f) = b_2(q_2^f, q_{-2}^f) = \cdots = b_n(q_n^f, q_{-n}^f)$, thus

$$-\beta(n^2+1)b_i(q_i^f, q_{-i}^f) + n(\alpha-\gamma) - \beta(\delta R/n + nR) = 0. \tag{27}$$

Further, (24) implies that (note that by symmetry $q_1^f = q_2^f = \cdots = q_n^f$)

$$-\beta(n^2+1)b_i(q_i^f, q_{-i}^f) = -\frac{n^2+1}{n+1}\left(\alpha-\gamma-\beta\left[-q_i^f + R + \delta R/n\right]\right)$$

Plugging this into (27) gives

$$\begin{aligned} &-(n^2+1)\left[\alpha-\gamma-\beta(-q_i^f + R + \delta R/n)\right] + \\ &\quad (n^2+n)(\alpha-\gamma) - (n+1)\beta(\delta R/n + nR) = 0. \end{aligned} \tag{28}$$

Rearranging terms implies

$$\begin{aligned} (n^2+1)\beta q_i^f &= (n-1)(\alpha-\gamma) + (n^2+1)\beta(R+\delta R/n) - (n+1)\beta(\delta R/n + nR) \\ &= (n-1)(\alpha-\gamma) + \beta((n^2+1)R - (n^2+n)R + [(n^2+1)-(n+1)]\delta R/n) \\ &= (n-1)(\alpha-\gamma) + \beta(-(n-1)R + (n-1)\delta R) \\ &= (n-1)[\alpha-\gamma+\beta(-R+\delta R)]. \end{aligned}$$

Thus

$$q_i^f = \frac{n-1}{(n^2+1)\beta}\big(\alpha-\gamma+\beta(-R+\delta R)\big). \tag{29}$$

Finally, we next find b_i. Plugging q_i^f into (24) gives

$$\begin{aligned} b_i &= \frac{1}{(n+1)\beta}\big(\alpha-\gamma-\beta(-q_i^f + R + \delta R/n)\big) \\ &= \frac{1}{(n+1)\beta}\left(\alpha-\gamma-\beta\left[-\frac{n-1}{(n^2+1)\beta}\big(\alpha-\gamma+\beta(-R+\delta R)\big) + R + \delta R/n\right]\right) \\ &= \frac{1}{(n+1)\beta}\left(\alpha-\gamma-\left[\frac{-(n-1)[\alpha-\gamma+\beta(-R+\delta R)] + (n^2+1)\beta(R+\delta R/n)}{n^2+1}\right]\right) \\ &= \frac{1}{(n+1)\beta}\left(\frac{(n^2+n)(\alpha-\gamma) + \beta\left(-(n^2+n)R - \frac{n^2+n}{n^2}\delta R\right)}{n^2+1}\right) \\ &= \frac{n}{(n^2+1)\beta}\left(\alpha+\beta\left(-R-\frac{\delta R}{n^2}\right)-\gamma\right). \end{aligned}$$

With the characterization of b_i, $i = 1, 2\cdots, n$, the proof is complete.

4. DERIVATIONS OF PRICE VOLATILITY AND WELFARE/PROFIT

4.1 Price volatility

Proof of Proposition 1 and Proposition 2 As shown in Theorem 5 the optimal production strategy is in form of $q_i(R_i) = b_i - a_i\theta_i$, where b_i and a_i are scalars. Thus

$$\begin{aligned}
Var(p) &= Var\left(\alpha - \beta\left(\sum_{i=1}^{n}(b_i - a_i\theta_i) + \sum_{i=1}^{n}(R/n + \theta_i)\right)\right) \\
&= \beta^2 Var\left(\sum_{i=1}^{n}(a_i - 1)\theta_i\right) \\
&= \beta^2\left(\boldsymbol{a}^T\Sigma\boldsymbol{a} - 2\boldsymbol{a}^T\Sigma\mathbf{1} + \mathbf{1}^T\Sigma\mathbf{1}\right) \\
&= \beta^2(\boldsymbol{a} - \mathbf{1})^T\Sigma(\boldsymbol{a} - \mathbf{1}) \\
&= \beta^2(\delta - 1)^2\ \mathbf{1}^T\left(\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1}\Sigma\left(\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1}\mathbf{1} \\
&= \beta^2(\delta - 1)^2\ \mathbf{b}^T\Sigma\mathbf{b},
\end{aligned}$$

where $\mathbf{b} \equiv \left(\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1}\mathbf{1}$.

We next derive price volatility when Definition 1 holds. First note that for regular configurations, we have $v_i = 1 + \delta + \sum_{j\neq i}\kappa_{i,j} = 1 + \delta + K$, for all $i = 1, 2, \cdots, n$. Therefore, due to (20), the optimal a_i satisfies

$$2a_i + \sum_{j\neq i} a_j\kappa_{i,j} = v_i = 1 + \delta + K.$$

Thus, by symmetry, we have $a_1 = \cdots = a_n = \frac{1+\delta+K}{2+K}$. Consequently,

$$Var(p) = \beta^2\left(1 - \frac{1 + \delta + K}{2 + K}\right)^2\mathbf{1}^T\Sigma\mathbf{1} = n\sigma^2\beta^2\left(\frac{1 - \delta}{2 + K}\right)^2(1 + K),$$

completing the proof.

Proof of Proposition 3 Since the decay factor $\zeta \in (0\ 1)$, thus

$$K_{\text{cycle}} = 2\left(\zeta + \zeta^2 + \cdots + \zeta^{\frac{n-1}{2}}\right) = 2\zeta\left(\frac{1 - \zeta^{\frac{n-1}{2}}}{1 - \zeta}\right) \leq (n-1)\zeta = K_{\text{complete}}$$

Moreover, by the characterization of price volatility in Proposition 2, it is clear that $\frac{\partial Var(p)}{\partial K} \leq 0$. Thus, price volatility is in decreasing the decay factor because

$$\frac{\partial Var(p)}{\partial\zeta} = \frac{\partial Var(p)}{\partial K}\underbrace{\frac{\partial K}{\partial\zeta}}_{\geq 0} \leq 0.$$

This implies that price volatility in the complete model is less than the cycle model, completing the proof. Note that since $\kappa_{i,j} \in \{0, \kappa\}$, for any $i \neq j$, thus given the definition of regular structures, for any regular configuration with n renewable plants, $K_{\text{cycle}}^{(n)} < K_{\text{regular}}^{(n)} < K_{\text{complete}}^{(n)}$.

Part II. Extra results and Extensions

5. WELFARE (GENERAL ANALYSIS)

Suppose the demand arises from an aggregate consumer whose gross surplus $U(q) \geq 0$ is concave in q, i.e. $U'' < 0$ (we assume $U(0) = 0$). This gives rise to the inverse demand $P(q) = U'(q)$.[2] The rest of the economy is as in Section 2.1 (in the main text): there are n thermal producers in the market, each thermal producer i faces a (convex and increasing) cost function $C(q_i)$ of supplying q_i unit of energy via thermal sources, the economy has a total R units of renewable energy (at zero marginal cost), and each thermal producer owns a fraction δ/n units of R where $\delta \in [0, 1]$.

The welfare in this economy is the sum of three components: the renewable producers surplus[3] (i.e. $(1-\delta)pR$, where $p \equiv P(\sum_{i=1}^n q_i + R)$), the (total) thermal producers surplus (i.e. $\sum_{i=1}^n \Pi_i = \sum_{i=1}^n [p(q_i + \delta R/n) - C(q_i)]$), and the consumer (net) surplus (i.e. $U(\sum_{i=1}^n q_i + R) - (\sum_{i=1}^n q_i + R)p$). As a result

$$\begin{aligned}
\mathcal{W} &\equiv \left(\sum_{i=1}^n q_i + \delta R\right)p - \sum_{i=1}^n C(q_i) + (1-\delta)Rp + U\left(\sum_{i=1}^n q_i + R\right) - \left(\sum_{i=1}^n q_i + R\right)p \\
&= U\left(\sum_{i=1}^n q_i + R\right) - \sum_{i=1}^n C(q_i) \qquad (30)
\end{aligned}$$

Appendix-Theorem 1 *Let $\mathcal{W}(CE)$ denote the welfare at the corresponding competitive equilibrium and $\mathcal{W}(NE)$ denote the welfare at the corresponding Nash equilibrium. Then, the ratio $\frac{\mathcal{W}(CE)}{\mathcal{W}(NE)}$ is increasing in δ. That is, increasing the diversification ratio leads to an increase in the welfare loss.*

Proof of Appendix-Theorem 1 The proof follows in three steps as follows.
Step 1 (characterizing $\mathcal{W}(CE)$): Let $q_1^{CE}, \cdots, q_n^{CE}$ be the quantities produced by thermal producers at the competitive equilibrium. Since $\mathcal{W}(CE)\{q_1^{CE}, \cdots, q_n^{CE}\} = \max_{q_1 \geq 0, \cdots, q_n \geq 0} \mathcal{W}$, thus the first order optimality condition of (30) implies that $q_1^{CE}, \cdots, q_n^{CE}$ should satisfy the following equations:

$$U'\left(\sum_{i=1}^n q_i^{CE} + R\right) - C'(q_i^{CE}) = 0, \quad \forall i = 1, 2, \cdots, n.$$

By symmetry $q_1^{CE} = \cdots = q_n^{CE} = Q^{CE}/n$ (where $Q^{CE} = \sum_{i=1}^n q_i^{CE}$). Therefore, Q^{CE} is characterized from the following equality

$$U'\left(Q^{CE} + R\right) - C'(Q^{CE}/n) = 0. \qquad (31)$$

Step 2 (characterizing $\mathcal{W}(NE)$): Let $q_1^{NE}, \cdots, q_n^{NE}$ be the quantities produced via thermal sources when thermal producers are strategic. Thus,

$$q_i^{NE} \in \arg\max_{q_i \geq 0} \quad (q_i + \delta R/n)P\left(q_i + R + \sum_{j \neq i} q_j^{NE}\right) - C(q_i),$$

given $(q_1^{NE}, \cdots, q_{i-1}^{NE}, q_{i+1}^{NE}, \cdots, q_n^{NE})$.

[2]For example, when $U(q) = \alpha q - \frac{\beta}{2}q^2$, the inverse demand becomes $P(q) = \alpha - \beta q$, the linear inverse demand adopted in the previous sections.

[3]Renewable producers do not have market power. As a result, they sell their production at the level of spot price characterized by the (diversified) thermal producers.

The corresponding first order optimality condition gives

$$P(Q^{NE}+R)+(q_i^{NE}+\delta R/n)P'(Q^{NE}+R)-C'(q_i^{NE})=0,\quad i=1,2,\cdots,n, \tag{32}$$

where $Q^{NE} \equiv \sum_{i=1}^n q_i^{NE}$. Finally, symmetry implies $q_1^{NE} = \cdots = q_n^{NE} = Q^{NE}/n$. Since (by definition) $P(Q^{NE}+R) = U'(Q^{NE}+R)$, thus (32) gives

$$\begin{aligned} U'(Q^{NE}+R)-C'(Q^{NE}/n) &= -(q_i^{NE}+\delta R/n)P'(Q^{NE}+R) \\ &= -(q_i^{NE}+\delta R/n)U''(Q^{NE}+R) \\ &> 0, \end{aligned}$$

where the last inequality is true because $U''<0$.
Step 3 (Effect of δ on $\mathcal{W}(CE)$ and $\mathcal{W}(NE)$): In this step we show $\mathcal{W}(CE)$ does not depend on δ, however, $\mathcal{W}(NE)$ is decreasing in δ. Equation (30) implies

$$\begin{aligned} \frac{\partial \mathcal{W}(T)}{\partial \delta} &= \sum_{i=1}^n \frac{\partial q_i^T}{\partial \delta}\left[U'\left(\sum_{i=1}^n q_i^T + R\right) - C'(q_i^T)\right] \\ &= \frac{\partial Q^T}{\partial \delta}\left(U'(Q^T+R)-C'(Q^T/n)\right) \qquad \text{for } T\in\{CE,NE\}. \end{aligned}$$

Therefore, as shown in Step 2, $U'\left(Q^{CE}+R\right)-C'(Q^{CE}/n)=0$, thus $\frac{\partial \mathcal{W}(CE)}{\partial \delta}=0$, i.e. $\mathcal{W}(CE)$ does not depend on δ. However, as shown in Step 3, $U'\left(Q^{NE}+R\right)-C'(Q^{NE}/n)>0$, thus $\text{sign}\{\frac{\partial \mathcal{W}(NE)}{\partial \delta}\} = \text{sign}\{\frac{\partial Q^{NE}}{\partial \delta}\}$. Moreover, we show in Theorem 1 that $\frac{\partial Q^{NE}}{\partial \delta}<0$, therefore $\frac{\partial \mathcal{W}(NE)}{\partial \delta}<0$, i.e. $\mathcal{W}(NE)$ is decreasing in δ. As a result, $\frac{\partial}{\partial \delta}\left(\frac{\mathcal{W}(CE)}{\mathcal{W}(NE)}\right)>0$, completing the proof.

5.1 Price Volatility: Linear vs. Quadratic costs

We focus on regular configurations, which represents a symmetric correlation structure for the renewable plants. This is defined formally through the covariance matrix of θ_i's as follows.

Definition 1 (Regular configurations) *Renewable plants have a regular configuration if the covariance matrix Σ is row-(sub)stochastic. That is, $\sum_{j\neq i}^n \kappa_{i,j}=K$, where K is fixed and the same for all $i=1,2,\cdots,n$.*

Hence, regular configurations represent a correlation structure in which the total covariance of each θ_i with other θ_j's is the same. It follows from Theorem 5 that for regular configurations, the equilibrium is symmetric, i.e., $a_1=\cdots=a_n$. Moreover the price volatility can be characterized explicitly in terms of K as follows.

Appendix-Theorem 2 *Let the production cost via thermal sources be given by $C(q_i)=\gamma q_i+\frac{\lambda}{2}q_i^2$. Then, the price volatility of any regular configuration is given by*

$$Var(p)=n\sigma^2\beta^2\left(\frac{\beta(1-\delta)+\lambda}{\beta(2+K)+\lambda}\right)^2(1+K). \tag{33}$$

Moreover:

(i) When producers have strictly convex costs (i.e. $\lambda>0$) and are fully diversified (they have full ownership of renewable supply), price volatility does not disappear, i.e., if $\lambda>0$ and $\delta=1$, then $Var(p)\neq 0$. This result holds for any configuration (i.e. there is no need to have a regular configuration).

(ii) When cost is linear (i.e. $\lambda = 0$), price volatility is monotonically decreasing in K, i.e., $\frac{\partial Var(p)}{\partial K} < 0$.

(iii) Let β be fixed. When cost is strictly convex (i.e. $\lambda > 0$), depending on the degree of convexity in the cost function price volatility can be either increasing or decreasing in K. To be precise

$$\text{sign}\{\frac{\partial Var(p)}{\partial K}\} = \begin{cases} + & \text{if } \frac{\lambda}{\beta} > K; \\ - & \text{if } \frac{\lambda}{\beta} < K. \end{cases}$$

This result has two important consequences. First, when cost function is sufficiently convex, i.e. $\lambda > 0$, in contrast to the linear cost (see Proposition 1), price volatility does *not* disappear when $\delta = 1$. This is simply because when thermal producers are fully diversified, i.e. $\delta = 1$, and their cost function is convex, i.e. $\lambda > 0$, then the total supply (via thermal and renewable sources) of each producer i still depends on θ_i (this will be more clear by the following Example). Second, the monotonicity of price volatility in K depends on the extent of convexity in the cost function. That is, assuming β is fixed, depending on the extent of convexity in the cost function, price volatility can be either increasing or decreasing in K. In fact, in contrast to the linear cost function, with increasing the extent of convexity in the cost function, price volatility can become increasing in K. To see this, suppose the cost function from thermal sources, i.e. $C(q_i)$, is sufficiently convex in q_i, so that production from thermal sources is so expensive. Therefore, each diversified thermal producer cuts its production via thermal sources (i.e. $q_i(\theta_i)$ becomes small). As a result, the aggregate production in the economy mostly comes form the aggregate renewable supply. That is, Aggregate production $= \sum_{i=1}^n q_i(\theta) + \sum_{i=1}^n R_i \approx \sum_{i=1}^n R_i = R + \sum_{i=1}^n \theta_i$, where R is constant. Hence, increasing correlation, i.e. K, increases $Var(\sum_{i=1}^n \theta_i) = \mathbf{1}^T\Sigma\mathbf{1} = n\sigma^2(1+K)$, increasing volatility in the aggregate production. Thus, price volatility can increase with increasing K, given λ is sufficiently large.

Proof of Appendix-Theorem 2 The proof follows similar steps as in the proofs of Theorem 5 and Proposition 1. In the first part the analysis is for a general configuration. Next we focus on the regular configurations.

General configuration Given that $C(q_i) = \gamma q_i + \frac{\lambda}{2}q_i^2$, producer i's objective is to choose q_i maximizing

$$E_{\theta_{-i}}(\Pi_i|R_i) = E\{p(q_i - q_i^f + \delta_i R_i) + p_i^f q_i^f - \gamma q_i - \frac{\lambda}{2}q_i^2|R_i\}$$

where $q_j(\theta_j) = b_j - a_j\theta_j$, for all $j \neq i$, and $p = \alpha - \beta(q_i + R_i + \sum_{j\neq i} R_j + \sum_{j\neq i} q_j)$. Since $R_i = R/n + \theta_i$, for all $i = 1, 2, \cdots, n$, thus, the first order optimality condition (FOC) implies

$$\alpha - \gamma - \beta\left(\sum_{j\neq i} E[q_j(\theta_j)|R_i] + \sum_{j\neq i} E[\theta_j|R_i] + \theta_i + R\right) - \beta(-q_i^f + \delta R/n + \delta\theta_i) = (2\beta + \lambda)q_i \tag{34}$$

Using the projection theorem: $E[q_j(\theta_j)|R_i] = b_j - a_j\kappa_{i,j}\theta_i$ and $E[\theta_j|R_i] = \kappa_{i,j}\theta_i$.

As a result rearranging terms in (34) gives

$$\begin{aligned}&\left(\alpha - \gamma - \beta\left(\sum_{j\neq i} b_j + R - q_i^f + \delta R/n\right)\right) - \theta_i\beta\left((1+\delta) + \sum_{j\neq i}\kappa_{i,j} - \sum_{j\neq i} a_j\kappa_{i,j}\right)\\ &\quad = ((2\beta + \lambda)b_i) - \theta_i((2\beta + \lambda)a_i)\end{aligned} \tag{35}$$

To analyze price volatility we only need to find a_i for $i = 1, 2, \cdots n$. Thus, we only need to equate the coefficient of θ_i in the LHS and RHS of (35), that implies (note that $\beta > 0$)

$$\sum_{j\neq i} \kappa_{i,j} a_j + \left(2 + \frac{\lambda}{\beta}\right) a_i = (1+\delta) + \sum_{j \neq i} \kappa_{i,j} \equiv v_i \tag{36}$$
$$\Rightarrow \quad \tilde{A}\boldsymbol{a} = \boldsymbol{v},$$

where $\tilde{A} \equiv \frac{1}{\sigma^2}\Sigma + \left(1 + \frac{\lambda}{\beta}\right)\mathbf{I}$, and $\mathbf{I}$ denotes the identity matrix. Since $\tilde{A}$ is positive define, it is invertible and thus

$$\begin{aligned}
\boldsymbol{a} &= \tilde{A}^{-1}\boldsymbol{v} \\
&= \left(\frac{1}{\sigma^2}\Sigma + (1 + \frac{\lambda}{\beta})\mathbf{I}\right)^{-1} (\delta\mathbf{1} + \frac{1}{\sigma^2}\Sigma\mathbf{1}) \\
&= \left(\frac{1}{\sigma^2}\Sigma + (1 + \frac{\lambda}{\beta})\mathbf{I}\right)^{-1} \left((\delta - (1 + \frac{\lambda}{\beta}))\,\mathbf{I} + \frac{1}{\sigma^2}\Sigma + (1 + \frac{\lambda}{\beta})\,\mathbf{I}\right)\mathbf{1} \\
&= \mathbf{1} + (\delta - \left(1 + \frac{\lambda}{\beta}\right)) \left(\left(1 + \frac{\lambda}{\beta}\right)\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1} \mathbf{1}.
\end{aligned} \tag{37}$$

As shown in the proof of Proposition 1, $Var(p) = (\mathbf{a} - \mathbf{1})^T \Sigma (\mathbf{a} - \mathbf{1})$, thus

$$Var(p) = (\delta - \left(1 + \frac{\lambda}{\beta}\right))^2 \quad \mathbf{1}^T \left(\left(1 + \frac{\lambda}{\beta}\right)\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1} \Sigma \left(\left(1 + \frac{\lambda}{\beta}\right)\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1} \mathbf{1}$$

Thus, when $\lambda > 0$ and $\delta = 1$ (in contrast to the linear cost), $Var(p) \neq 0$.

Regular configuration For regular configurations $a_1 = \cdots = a_n \equiv \tilde{a}$. Thus (36) implies $\tilde{a} = \frac{\beta(1+\delta+K)}{\beta(K+2)+\lambda}$.

Moreover, as shown in the proof of Proposition 1, $Var(p) = \beta^2 Var\left(\sum_{i=1}^n (a_i - 1)\theta_i\right)$, thus for regular configurations we have

$$Var(p) = \beta^2(\tilde{a} - 1)^2 \mathbf{1}^T \Sigma \mathbf{1} = n\sigma^2\beta^2 \left(\frac{\beta(1-\delta) + \lambda}{\beta(2+K) + \lambda}\right)^2 (1+K).$$

The explicit characterization of $Var(p)$ implies that

$$\frac{\partial Var(p)}{\partial K} = \left(n\sigma^2\beta^2 \, \frac{(\beta(1-\delta)+\lambda)^2}{(\beta(2+K)+\lambda)^3}\right)[\lambda - \beta K]$$

completing the proof.

Example (Duopoly with incomplete information and quadratic cost) Let us assume each producer $i \in \{1,2\}$ owns a generator that produces q_i units of thermal energy at cost $C(q_i) = \frac{\lambda}{2}q_i^2$ (where $\lambda > 0$). In this economy thermal producers are also capable to generate energy from renewable plants. To this end, we assume there are two intermittent plants. Let ℓ_1 and ℓ_2 denote the locations of these plants. Each producer i privately observes the available renewable energy R_i at local plant l_i. We assume $R_i = R/2 + \theta_i$, where R is a constant, and θ_i is normally distributed with mean zero and variance σ^2, i.e., $\theta_i \sim \mathcal{N}(0, \sigma^2)$. The vector $\theta = (\theta_1, \theta_2)$ is assumed to be jointly normal and $\text{cov}(\theta_1, \theta_2) = \kappa\sigma^2$, where $\kappa \in [0,1]$. The scalar κ captures the correlation between available renewable energy at plants l_i and l_j.[4]

For ease of exposition we assume $p \equiv \alpha - (q_1 + q_2 + R_1 + R_2)$, consequently,[5] producer i's

[4] It is worth noting that adding forward contract does not have any effect on the price volatility.
[5] In this simple example we assume $\beta = 1$.

(ex-post) payoff becomes:

$$\Pi_i = p(q_i + \delta R_i) - \lambda \frac{q_i^2}{2} = (\alpha - q_1 - q_2 - R_1 - R_2)(q_i + \delta R_i) - \lambda \frac{q_i^2}{2}.$$

Solving this case implies

$$q_i(\theta_i) = \frac{\alpha - R - \delta R/2}{\lambda + 3} - \left(\frac{1 + \delta + \kappa}{\lambda + 2 + \kappa}\right)\theta_i$$

$$Var[p] = 2\sigma^2 \left(\frac{\lambda + 1 - \delta}{\lambda + 2 + \kappa}\right)^2 (1 + \kappa) \tag{38}$$

Therefore, with increasing λ, intuitively, production from thermal sources decreases, i.e. $\frac{\partial E[q_i(\theta_i)]}{\partial \lambda} < 0$.

Furthermore, using Theorem 2, we have

$$\frac{\partial Var(p)}{\partial \kappa} = \left(\frac{2\sigma^2(\lambda + 1 - \delta)^2}{(\lambda + 2 + \kappa)^3}\right)(\lambda - \kappa).$$

Therefore, depending on the extent of convexity in the cost function, price volatility can be decreasing or increasing with respect κ. To be precise:

$$\text{sign}(\frac{\partial Var(p)}{\partial \kappa}) = \text{sign}(\lambda - \kappa)$$

where as for the linear cost price volatility monotonically decreases in κ (see Proposition 2).

5.2 Degrees of convexity and concavity

This section analyzes the effects of convex cost and concave inverse demand functions on the market price, when thermal producers have a diverse energy portfolio. We show, for a given concave inverse demand function, with increasing extent of convexity in the cost function market price goes up. However, for a given convex cost function, with increasing extent of concavity in the inverse demand market price goes down. To this end, we consider two cases as follows. Without loss of generality we assume $n = 2$.

Concavity analysis of the inverse demand Let the cost function be a given convex function (i.e. $C'' \geq 0$), and inverse demand be $p = P(Q) = \alpha - \beta Q^2$, where $\beta > 0$. Thus, the more β is, the more concave the inverse demand $P(Q)$ is. The objective is to show $\frac{\partial p}{\partial \beta} < 0$.

The profit of each (diverse) thermal producer is given by

$$\Pi_i = (q_i + \delta R/2)P(Q + R) - C(q_i) = (q_i + \delta R/2)(\alpha - \beta(Q + R)^2) - C(q_i)$$

FOC then gives $\frac{\partial \Pi_i}{\partial q_i} = (\alpha - \beta(Q + R)^2) + (q_i + \delta R/2)(-2\beta(Q + R)) - C'(q_i) = 0$. Due to symmetry (at equilibrium) $q_1 = q_2$. Thus,

$$0 = (\alpha - \beta(Q + R)^2) + (Q + \delta R)(-\beta(Q + R)) - C'(Q/2).$$

Taking a derivative with respect β implies

$$\left[(Q + R)^2 + (Q + \delta R)(Q + R)\right] + \frac{\partial Q}{\partial \beta}\left[3\beta(Q + R) + \beta(Q + \delta R) + 1/2C''(Q/2)\right] = 0.$$

Thus (note that $C'' \geq 0$),

$$\frac{\partial Q}{\partial \beta} = -\frac{(Q+R)^2 + (Q+\delta R)(Q+R)}{3\beta(Q+R) + \beta(Q+\delta R) + 1/2C''(Q/2)} < 0.$$

What is the effect of inverse demand concavity (controlled by β) on the market price p? Since $p = \alpha - \beta(Q+R)^2$, thus

$$\begin{aligned}
\frac{\partial p}{\partial \beta} &= -(Q+R)^2 - 2\beta(Q+R)\frac{\partial Q}{\partial \beta} \\
&= -(Q+R)^2\left[1 - 2\beta\frac{Q+R+Q+\delta R}{3\beta(Q+R)+\beta(Q+\delta R)+1/2C''(Q/2)}\right] \\
&\leq 0,
\end{aligned}$$

where the last inequality is correct because $[\ldots] \geq 0$ (note that $\delta \in [0,\ 1]$ and $C'' \geq 0$). Therefore, with increasing extent of concavity in the inverse demand market price decreases.

Convexity analysis of the cost function Let the inverse demand be concave (and downward) (i.e. $P'' \leq 0$ and $P' < 0$) and the cost function be $C(q_i) = \gamma q_i + \frac{\lambda}{2}q_i^2$, where $\gamma \geq 0$ and $\lambda \geq 0$. Thus, the more is λ, the more convex is the cost function $C(q_i)$. The objective is to show $\frac{\partial p}{\partial \lambda} > 0$.

The analysis is inline with the previous case. The profit of each (diverse) thermal producer is updated by $\Pi_i = (q_i + \delta R/2)P(Q+R) - C(q_i) = (q_i + \delta R/2)P(Q+R) - \left(\gamma q_i + \frac{\lambda}{2}q_i^2\right)$. The FOC then gives $\frac{\partial \Pi_i}{\partial q_i} = P(Q+R) + (q_i + \delta R/2)P'(Q+R) - (\gamma + \lambda q_i)$. Due to the symmetry (at equilibrium) $q_1 = q_2$. Thus,

$$0 = P(Q+R) + (\frac{1}{2})(Q+\delta R)P'(Q+R) - \left(\gamma + \lambda\frac{Q}{2}\right).$$

Taking a derivative with respect λ and rearranging terms imply

$$\frac{\partial Q}{\partial \lambda} = \frac{Q}{3P'(Q+R) + (Q+\delta R)P''(Q+R) - \lambda} < 0,$$

where the last inequality is because the inverse demand in downward and concave (i.e. $P' < 0$ and $P'' \leq 0$). Thus, with increasing convexity in the cost function aggregate production decreases. As a result, since P is decreasing in Q (i.e. $P' < 0$), thus the market price increases in λ, completing the proof.

5.3 Welfare/Profit analysis

Here, we present two intuitive results about the impact of R and δ on each thermal producer's profit. We show diversified energy portfolios are always beneficial for thermal producers. However, the profit consequences of increasing renewables for diversified thermal producers (i.e. $\delta > 0$) crucially depends on δ. That is, depending on the extent of δ, increasing renewable supply can be beneficial or detrimental for thermal producers. In fact, our model suggests there exists a *unique* threshold δ^* for which when thermal producers have a low share from renewable outcome (i.e. $\delta < \delta^*$) their benefit decreases with increasing renewable supply, but when their share is sufficiently high (i.e. $\delta > \delta^*$), increasing renewable supply is actually beneficial for them.

Appendix-Proposition 1 *The payoff of each non-diversified thermal producer monotonically decreases with increasing renewable supply on the grid, i.e.* $\frac{\partial \Pi_i}{\partial R} < 0$ *if* $\delta = 0$. *However, the payoff of each thermal producer always arises via diversification, i.e.* $\frac{\partial \Pi_i}{\partial \delta} > 0$.

Proof of Appendix-Proposition 1 Due to Theorem 2, $\frac{\partial p}{\partial \delta} = \frac{\beta R}{n+1} > 0$, and $\frac{\partial q_i}{\partial \delta} = \frac{-\beta R/n}{(n+1)\beta}$ thus $\frac{\partial q_i}{\partial \delta} + \frac{R}{n} = \frac{R}{n}(1 - \frac{1}{n+1}) > 0$. Moreover,

$$\frac{\partial \Pi_i}{\partial \delta} = \underbrace{\frac{\partial p}{\partial \delta}}_{\geq 0}\left(q_i + \frac{\delta R}{n}\right) + p\underbrace{\left(\frac{\partial q_i}{\partial \delta} + \frac{R}{n}\right)}_{\geq 0}\underbrace{-\gamma\frac{\partial q_i}{\partial \delta}}_{\geq 0} \geq 0.$$

Next, assume $\delta = 0$. Thus, $\frac{\partial p}{\partial R} = \frac{-\beta}{n+1} < 0$, and $\frac{\partial q_i}{\partial R} = \frac{-1}{(n+1)} < 0$. Moreover,

$$\begin{aligned}\frac{\partial \Pi_i}{\partial R} &= \frac{\partial p}{\partial R} q_i + p\frac{\partial q_i}{\partial R} - \gamma\frac{\partial q_i}{\partial R} \\ &= \frac{-\beta}{n+1} q_i + (p - \gamma)\frac{-1}{n+1} < 0,\end{aligned}$$

where the last inequality holds because $p - \gamma > 0$.

Appendix-Proposition 2 *There exists a unique $\delta^* \in (0,1)$ such that if $\delta < \delta^*$ then with increasing renewable supply the profit of each diversified thermal producer decreases, i.e. $\frac{\partial \Pi_i}{\partial R} < 0$. However, if $\delta > \delta^*$ then each diversified thermal producer will be better off with increasing renewable supply, i.e. $\frac{\partial \Pi_i}{\partial R} > 0$.*

Proof of Appendix-Proposition 2 Due to symmetry, the profit of each thermal producer at the equilibrium can be written as $\Pi_i = (q_i + \delta R/n)p - \gamma q_i = \frac{1}{n}\left[(Q + \delta R)p - \gamma Q\right]$, recall that $Q = \sum_{i=1}^{n} q_i = nq_i$ (at the equilibrium). Further, due to Theorem 2, $p = \frac{1}{(n+1)}(\alpha + \beta(-R + \delta R) + n\gamma)$ and $Q = \frac{n}{(n+1)\beta}(\alpha - \gamma - \beta(R + \delta R/n))$. Plugging p and Q into the profit of the thermal producer i gives

$$\begin{aligned}\Pi_i &= \frac{1}{n}\left[(Q + \delta R)p - \gamma Q\right] \\ &= \frac{1}{\beta n(n+1)}\Bigg[\frac{1}{n+1}\Big(\alpha + \beta(-R + \delta R) + n\gamma\Big)\Big(n(\alpha - \gamma) - \beta(nR + \delta R) + \beta(n+1)\delta R\Big) \\ &\qquad\qquad - \gamma n\Big(\alpha - \gamma - \beta(R + \delta R/n)\Big)\Bigg]\end{aligned}$$

Let us define $\Lambda \equiv [\ldots]$ (note that the roots of $\frac{\partial \Pi_i}{\partial R}$ and $\frac{\partial \Lambda}{\partial R}$ are equivalent). Thus,

$$\begin{aligned}\frac{\partial \Lambda}{\partial R} &= \frac{\beta(\delta - 1)}{n+1}\Big(n(\alpha - \gamma) - \beta(nR + \delta R) + \beta(n+1)\delta R\Big) \\ &\quad + \frac{-\beta(\delta + n) + (n+1)\delta\beta}{n+1}\Big(\alpha + \beta(-R + \delta R) + n\gamma\Big) \\ &\quad + \gamma n\beta\left(1 + \frac{\delta}{n}\right) \\ &\equiv f(\delta).\end{aligned}$$

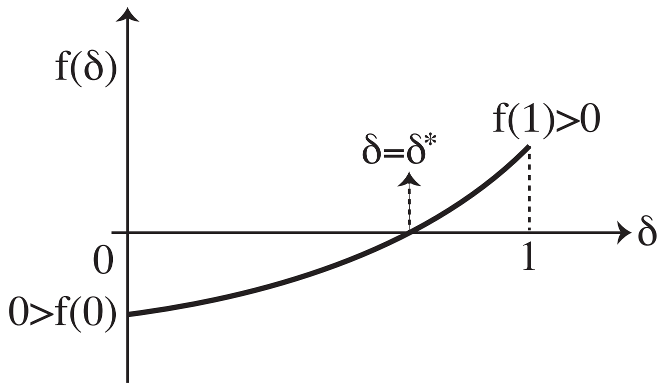
It is easy to see that $f(\delta)$ is quadratic in δ, thus it looks like $f(\delta) = x\delta^2 + y\delta + z$ (where x, y, and z are all independent of δ). First note that $x = \frac{2\beta^2 Rn}{n+1} > 0$, thus $f(\delta)$ is convex. Next we show $f(\delta)$ has a unique zero lying between zero and one. To achieve this we show $f(0) < 0$ and $f(1) > 0$ (see the following figure) since $f(\delta)$ is continuous and quadratic thus has unique zero in $(0,1)$.

$$\begin{aligned} f(0) &= -\beta\left(\frac{n(\alpha-\gamma)-\beta nR}{n+1}\right) - \frac{n\beta}{n+1}(\alpha-\beta R+n\gamma)+\gamma n\beta \\ &= \frac{-2n\beta}{n+1}(\alpha-\gamma-\beta R) \\ &< 0 \end{aligned}$$

where the last inequality holds since $\alpha-\gamma-\beta R > 0$.

$$\begin{aligned} f(1) &= \gamma n\beta\left(1+\frac{1}{n}\right) \\ &> 0. \end{aligned}$$

Figure 1: There exists a unique $\delta^* \in (0,1)$ for which the behavior of Π_i with respect to R switches.



Since $f(1) > 0$ and $f(0) < 0$, thus there exists a unique $\delta^* \in (0,1)$ for which $f(\delta^*) = 0$. As a result, $\frac{\partial \Pi_i}{\partial R}|_{\delta=\delta^*} = 0$, and consequently, $\frac{\partial \Pi_i}{\partial R}|_{\delta<\delta^*} < 0$ and $\frac{\partial \Pi_i}{\partial R}|_{\delta>\delta^*} > 0$, completing the proof.

6. PRICE VOLATILITY: GENERAL SPATIAL CONFIGURATIONS

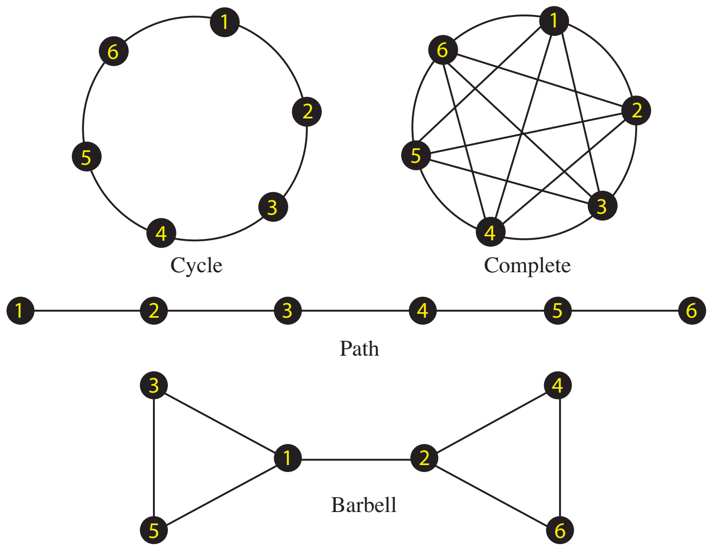
In this section through examples we aim to consider the effect of general correlation structures on the price volatility. Importantly, we show comparison of price volatility among different network structures crucially depends on the way that the underlying structures are normalized. To this end, we consider three normalizations as follows.

6.1 Normalization 1: Fixed distance between any neighboring plants

We consider path, cycle, barbell and complete network structures, depicted in the following figure.

We assume the distance between any two immediate neighbor plants is normalized to 1, and in any network $d(\ell_i,\ell_j)$ is the shortest distance between plants i and j within the network (thus $d(\ell_i,\ell_i) = 0$ for all i). We assume $\sigma^2 = 1$. To capture that correlation in renewable supply at any two plants, i.e. $\kappa_{i,j}$, decays with their distance we assume $\kappa_{i,j} = \zeta^{d(\ell_i,\ell_j)}$ where $0 < \zeta < 1$ is the decay factor and $d(\ell_i,\ell_j)$ is the shortest distance between the plants. For example, the variance-covariance matrix of the above Barbell and Path networks are as follows:

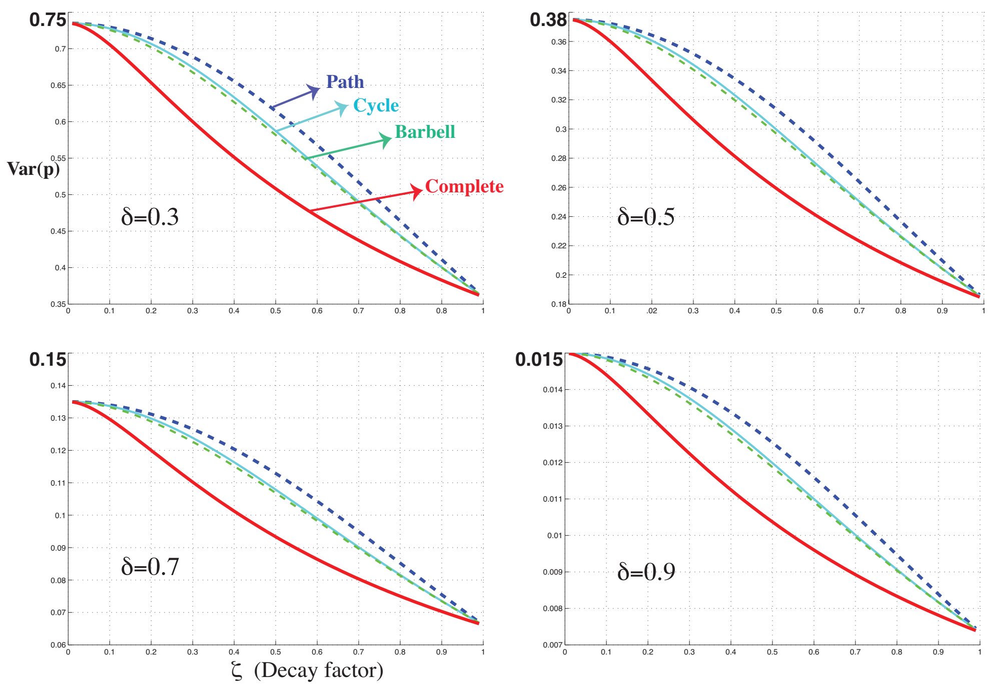
Figure 2: Cycle, Complete, Path and Barbell network structures.



$$\Sigma_{\text{Barbell}} = \begin{pmatrix} 1 & \zeta & \zeta & \zeta^2 & \zeta & \zeta^2 \\ \zeta & 1 & \zeta^2 & \zeta & \zeta^2 & \zeta \\ \zeta & \zeta^2 & 1 & \zeta^3 & \zeta & \zeta^3 \\ \zeta^2 & \zeta & \zeta^3 & 1 & \zeta^3 & \zeta \\ \zeta & \zeta^2 & \zeta & \zeta^3 & 1 & \zeta^3 \\ \zeta^2 & \zeta & \zeta^3 & \zeta & \zeta^3 & 1 \end{pmatrix}, \quad \Sigma_{\text{Path}} = \begin{pmatrix} 1 & \zeta & \zeta^2 & \zeta^3 & \zeta^4 & \zeta^5 \\ \zeta & 1 & \zeta & \zeta^2 & \zeta^3 & \zeta^4 \\ \zeta^2 & \zeta & 1 & \zeta & \zeta^2 & \zeta^3 \\ \zeta^3 & \zeta^2 & \zeta & 1 & \zeta & \zeta^2 \\ \zeta^4 & \zeta^3 & \zeta^2 & \zeta & 1 & \zeta \\ \zeta^5 & \zeta^4 & \zeta^3 & \zeta^2 & \zeta & 1 \end{pmatrix}.$$

In this section we assume $\beta = 1$. Applying Proposition 1 we can characterize the price volatility for the above configurations. Figure 3 visualizes the price volatility of these networks when the decay factor varies from 0 to 1, with different share for thermal producers from renewable supply, i.e. δ. Consistent with Proposition 1 price volatility decreases with increasing δ. In addition price volatility decreases with increasing the decay factor. This is because decay factor inversely related to the distance between the plants. Thus, high decay factor means low distance among the plants, decreasing the price volatility which is due to the lower misscoordinations among (close) competitors. In addition, with changing the decay factor and δ, price volatility in these networks uniformly follows a pattern that $Var_{\text{Path}}(p) \geq Var_{\text{Cycle}}(p) \geq Var_{\text{Barbell}}(p) \geq Var_{\text{Complete}}(p)$.

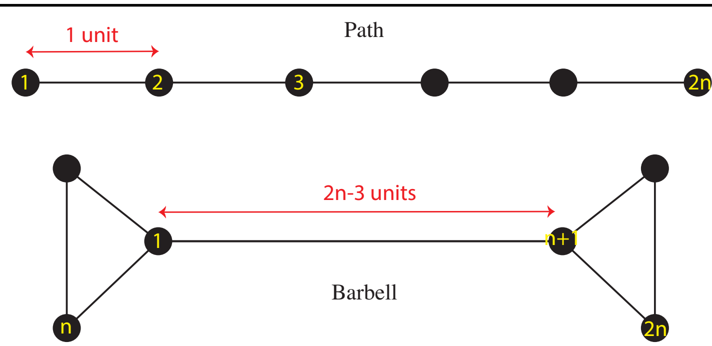
Figure 3: Price volatility in Cycle, Complete, Path and Barbell networks with respect to the decay factor $\zeta \in (0, 1)$ when thermal producers own some share from renewable outcome, i.e. $\delta = .3, .5, .7, .9$. *Price volatility in the Path (Complete) structure is uniformly higher (lower) than the others*.



6.2 Normalization 2: The same distance for the farthest plants

For this normalization we compare Path and Barbell structures, given that the distance for their farthest plants is the same. In the Path network the correlation between any two "neighboring" plants is ζ, where $\zeta \in (0, 1)$ is the decay factor. However, the Barbell network consists of two cliques located far from each other. In each clique the correlation between any two neighbor plants is ζ. But, since by this normalization, in the Path and Barbell networks the distance between the two farthest plants should be the same, thus the correlation between the two farthest plants in these networks is $\zeta^{\text{maximum distance}} = \zeta^{2n-1}$.

Figure 4: Path and Barbell networks. The same distance for the farthest plants.



Now, suppose the decay factor changes from zero to one. Then, as shown in Figures 11 and 12, in contrast to the previous case, price volatility with this normalization does not change uniformly in these network structures. In fact, when decay factor is small (i.e. low correlation in renewable supply for neighboring plants) price volatility in Path is higher than Barbel structures. But, when decay factor is sufficiently high (i.e. high correlation in renewable supply for neighboring plants) then price volatility in the Path structure is lower than the Barbell.

Figure 5: Barbell vs Path networks (Effect of distance). When nodes are close (i.e. high ζ), the price volatility in the path network is lower. However when nodes are far (i.e. low ζ), the price volatility in the barbell network is lower.

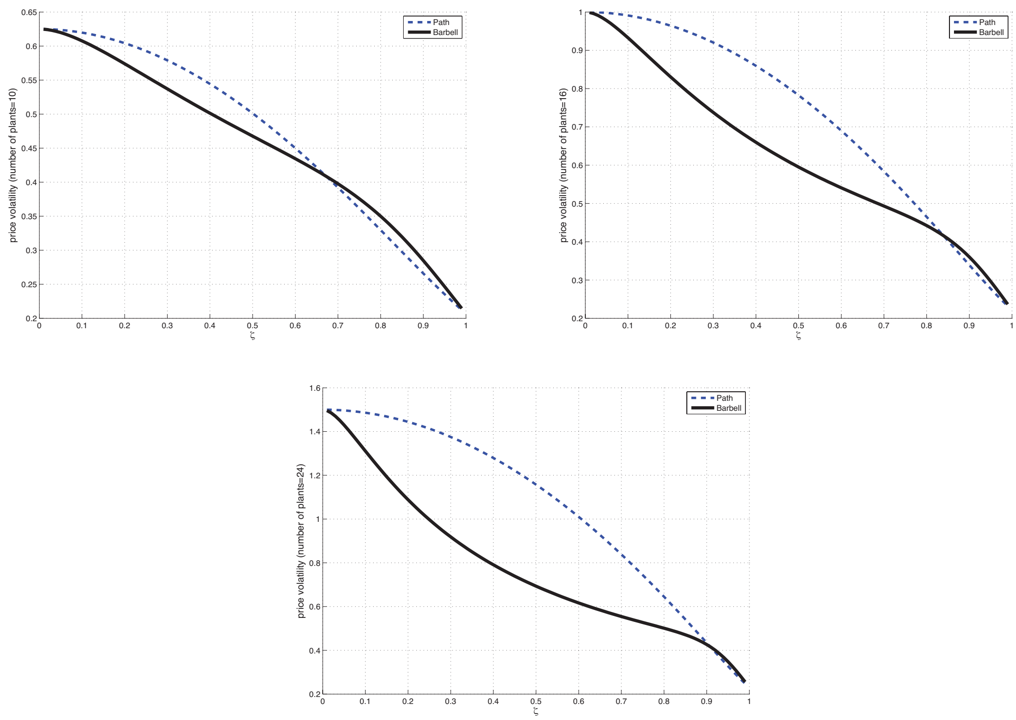
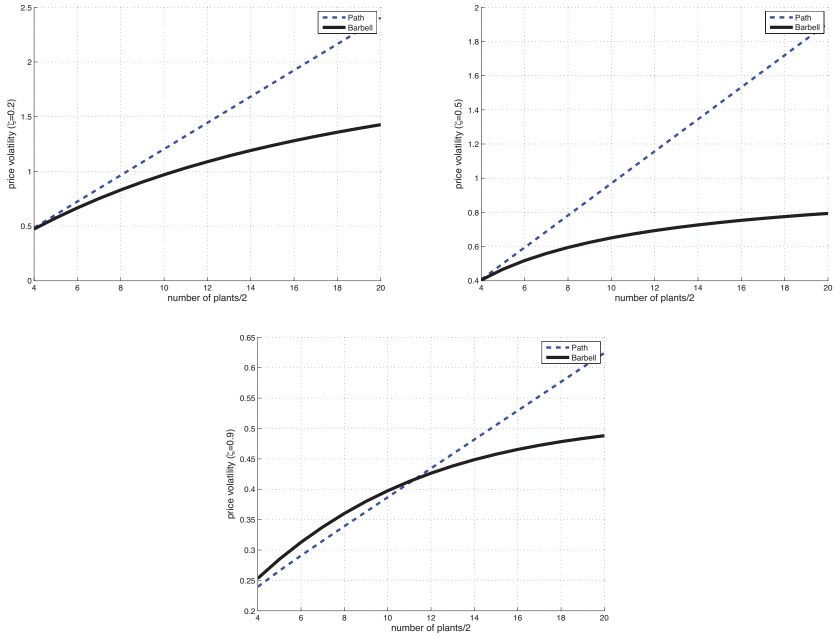


Figure 6: Barbell vs Path networks (Effect of size). The same intuition as in the above.



6.3 Normalization 3: The same total correlation

In this normalization we assume the total correlation in the underlying structures is the *same*, i.e. $\mathbf{1}^T\Sigma_{structure}\mathbf{1}$ = fixed.

The construction is as follows. Let us start with a path network with $2n$ nodes. In the path network the corresponding variance-covariance matrix (i.e. Σ_{Path}) is such that (for a given decay factor $\zeta \in (0,1)$) the $Cov(\theta_i,\theta_j) = \zeta^{d_{ij}}$, where d_{ij} denotes the length of the shortest path from i to j on the path.

$$\Sigma_{\text{Path}} = \begin{pmatrix} 1 & \zeta & \zeta^2 & \cdots & \zeta^{2n-1} \\ \zeta & 1 & \zeta & \cdots & \zeta^{2n-2} \\ \zeta^2 & \zeta & 1 & \cdots & \zeta^{2n-3} \\ \vdots & & & \ddots & \vdots \\ \zeta^{2n-1} & \zeta^{2n-2} & \zeta^{2n-3} & \cdots & 1 \end{pmatrix}$$

Next moving to the barbell and the complete networks, the constructions of their variance-covariance matrices need to satisfy $\mathbf{1}^T\Sigma_{Complete}\mathbf{1} = \mathbf{1}^T\Sigma_{Path}\mathbf{1} = \mathbf{1}^T\Sigma_{Barbell}\mathbf{1}$ (resulting in normalization in the total correlation). As a result, for the Barbell structure that consists of two cliques (each with n nodes), the $Cov(\theta_i,\theta_j) = q_b$ if $i \neq j$ and both belongs to a same clique, however, when i and j are in different cliques then $Cov(\theta_i,\theta_j) = \alpha q_b$ where $\alpha \in (0,1)$ and $q_b \equiv \frac{\mathbf{1}^T\Sigma_{Path}\mathbf{1}-2n}{2n((\alpha+1)n-1)}$. It can be easily shown that with this q_b, $\mathbf{1}^T\Sigma_{Path}\mathbf{1}$ and $\mathbf{1}^T\Sigma_{Barbell}\mathbf{1}$ will be equal. Following the same argument, for the complete network we need to choose $Cov(\theta_i,\theta_j) = q_c$ where $q_c \equiv \frac{\mathbf{1}^T\Sigma_{Path}\mathbf{1}-2n}{2n(2n-1)}$ (for all $i \neq j$), so as to have $\mathbf{1}^T\Sigma_{Complete}\mathbf{1} = \mathbf{1}^T\Sigma_{Path}\mathbf{1}$.

$$\Sigma_{\text{Complete}} = \begin{pmatrix} 1 & q_c & \cdots & & q_c \\ q_c & 1 & q_c & \cdots & q_c \\ \vdots & & \ddots & & \vdots \\ q_c & q_c & \cdots & 1 & q_c \\ q_c & \cdots & & q_c & 1 \end{pmatrix} = q_c(\mathbf{U}_{2n} - \mathbf{I}_{2n}) + \mathbf{I}_{2n}, \quad q_c = \frac{\mathbf{1}^T \Sigma_{Path} \mathbf{1} - 2n}{2n(2n-1)}$$

$$\Sigma_{\text{Barbell}} = \begin{pmatrix} \mathbf{A}_b & \mathbf{C}_b \\ \mathbf{C}_b & \mathbf{A}_b \end{pmatrix}, \quad q_b = \frac{\mathbf{1}^T \Sigma_{Path} \mathbf{1} - 2n}{2n((\alpha+1)n-1)}, \quad y_b = \alpha x_b \quad (\alpha \text{ is exogenous and lies in} (0,1))$$

where $\mathbf{A}_b = q_b(\mathbf{U}_n - \mathbf{I}_n) + \mathbf{I}_n$ and $\mathbf{C}_b = y_b \mathbf{U}_n$.

With the above constructions we can now compare price volatility in the above structures. Importantly, as is evident from the following figure, with this normalization the price volatilities in these structures (the red, green and the blue lines/dots) are *all the same*, implying that network structure becomes actually *ineffective*. The solid black line is the price volatility for the barbell network with the previous normalization in which the distance between its two farthest nodes is equal to the distance between the two end nodes of the path network, taken as a way for the normalization.

Figure 7: Barbell vs Path vs Complete networks (Network structure becomes ineffective when they all have the same *total correlation*). This figure visualizes the effect of total correlation on the price volatility for different network structures.