

Online Appendix A: Spot Prices - One factor Model

We summarize Lucia and Schwartz (2002). Spot electricity prices P_t are characterized as

$$P_t = s(t) + X_t \quad (\text{A.1})$$

where $s(t)$ is a deterministic function¹, and X_t , is a mean-reverting stochastic process with constant volatility σ and, under the natural probability measure P follow:

$$dX_t = -kX_t dt + \sigma dZ_t^P \quad (\text{A.2})$$

It can be shown that under the risk-neutral probability measure Q , process X_t follows:

$$dX_t = k(\alpha^* - X_t)dt + \sigma dZ_t^Q \quad (\text{A.3})$$

where dZ_t^Q are increments of standard independent Brownian motions Z_t^* the mean reversion parameters are k and $X(0)=x_0$, and the drift term is

$$\alpha^* \equiv \frac{-\lambda\sigma}{k} \quad (\text{A.4})$$

We assume the Market Prices of Risk (MPR) of the electricity, which are λ respectively, to be constant over time. Under the risk-neutral measure the spot price P_t follows

$$P_t = s(t) + X_0 e^{-kt} + \alpha^*(1 - e^{-kt}) + \sigma \int_0^t e^{k(s-t)} dZ^Q \quad (\text{A.5})$$

The distribution of P_t is Normal with mean given by :

$$E_0^Q(P_t) = s(t) + X_0 e^{-kt} + \alpha^Q(1 - e^{-kt}) \quad (\text{A.6})$$

The value of any derivative security must be the expected value, under the risk-neutral measure, of its payoffs discounted to the valuation date at the risk-free rate. Assuming a constant risk-free rate r , the value at time zero of a forward contract on the spot price maturing at time T must be

$$V_0^T(P_T) = e^{-rT} E_0^*[P_T - F_0(P_0, T)] \quad (\text{A.7})$$

where $F_0(P_0, T)$ is the forward price set at time zero and T is the time to maturity. Since the value of a forward contract must be zero when it is first entered into, we obtain a closed form expression for computing forward prices with maturity T as follows

¹ Variables $f(t)$ and $fF(t)$ include constant terms, deterministic seasonal components as well as other deterministic factors such as calendar effects.

$$F_0(P_0, T) = E_0^*(P_T) = s(T) + (P_0 - f(0))e^{-kT} + \alpha^*(1 - e^{-kT}) \quad (\text{A.8})$$

The variance of the forward prices are given by

$$\text{Var}_0^T(P_T) = \frac{\sigma^2}{2k}(1 - e^{-2kT}) \quad (\text{A.9})$$

These results are for forward contracts providing electricity in a single point in time (T). Given that the swap contract provides delivery of electricity during a period (e.g. during 31 days in January), we use (A.8) to generate prices during the full delivery period (e.g. we generate thirty-one forward prices in the cases of monthly contracts maturing in January and so on), and take the average. This average is the estimated swap price provided by this model.

Table A1: Estimation of One Factor Model

This table reports the results of regressions (16) and (17). The dependent variable is the average daily EEX spot price (EEX - Phelix Base Hr.01-24 E/Mwh). Our database spans from February 2, 2009 to December 31 2012. Explanatory variables include day of the week dummies as well as the NEG variable that is a dummy variable taking into account negative electricity prices. It is equal to 1 if the price is negative (4 Oct 2009, 26 Dec 2009, 25 Dec 2012 y 26 Dec 2012) and it is zero otherwise. We estimate the coefficients by means of a regression robust to heteroscedasticity, and serial autocorrelation. The results presented correspond to the estimated coefficient, standard errors and t-statistics. The symbol * and ** denotes that the variable is significant at 5% and 1%, respectively.

	Coeff.	s.e.	t-stat
NEG	-79.86**	12.91	-6.18
@WEEKDAY=1	46.25**	0.77	60.09
@WEEKDAY=2	48.06**	0.64	75.30
@WEEKDAY=3	48.14**	0.67	71.49
@WEEKDAY=4	47.48**	0.66	71.76
@WEEKDAY=5	46.22**	0.71	65.28
@WEEKDAY=6	39.84**	0.60	65.99
@WEEKDAY=7	33.60**	0.68	49.21
1 - \hat{k}	0.77**	0.03	29.70
\hat{k}	0.23**	0.025	8.79
$\hat{\sigma}$	5.86		
R²	58.7		

Figure A1: Parameter Stability

The figure shows the recursive estimation of the parameters (mean reversion k and volatility σ) for the Spot Series 7 days EEX - Phelix Base Hr.01-24 E/Mwh. We use a 365-day rolling window. There are 1066 estimates. The last window is: 2/01/12-31/12/12. Average values are $k = 0.23$ and $\sigma = 5.86$.

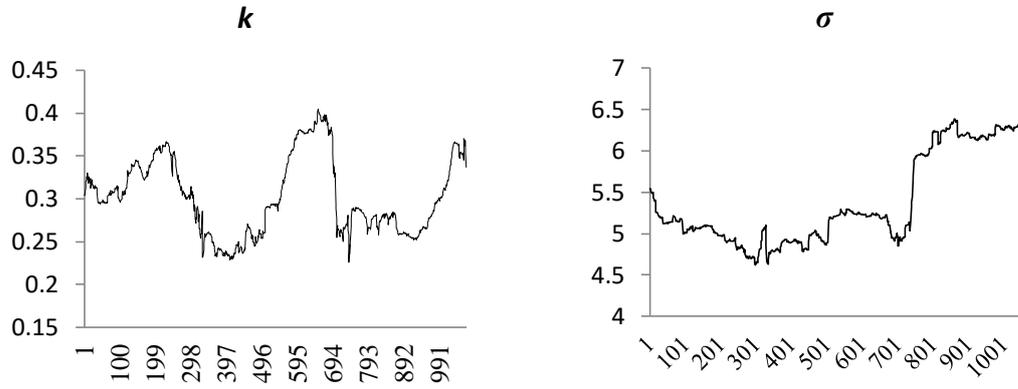
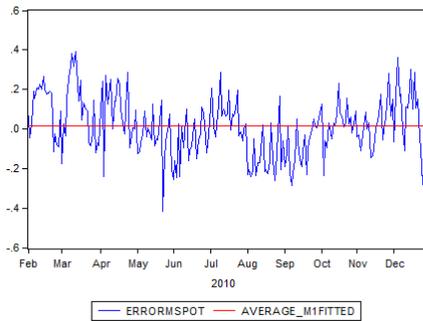


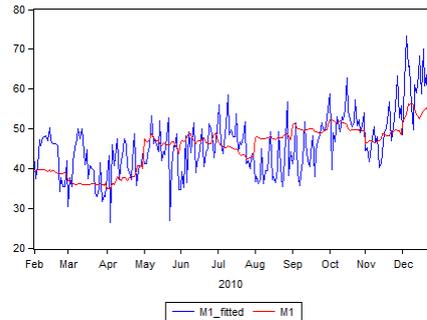
Figure A2: Swap Prices, Fitted Values and Errors in the Spot Model

Using the parameters estimated in Table B1 we compute forward prices we obtain a closed form expression for computing forward prices with maturity T using Equation (A.8) as follows $F_0(P_0, T) = \bar{s}(T) + (P_0 - f(0))e^{-kT} + \hat{\alpha}^*(1 - e^{-kT})$. We then generate swap theoretical prices (average of forward prices during the delivery period) using the previous equation for all contracts (denoted M1_fitted, Q1_fitted and Y1_fitted). As an illustration we present the results for M+1, Q+1 and Y+1 denoted M1, Q1 and Y1 (which are the most liquid contracts within each market segment) and compare them against market prices during 2010. Results for the other contracts and periods are available on request.

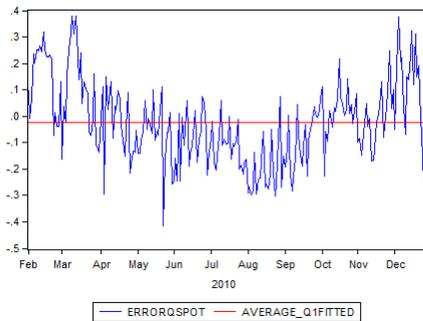
Error M1



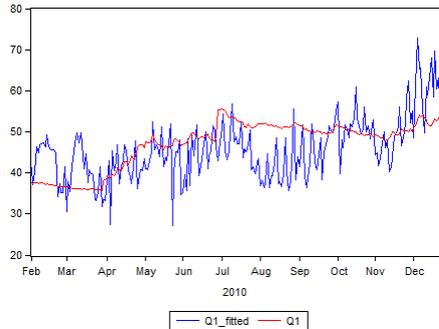
M1 Fitted and M1



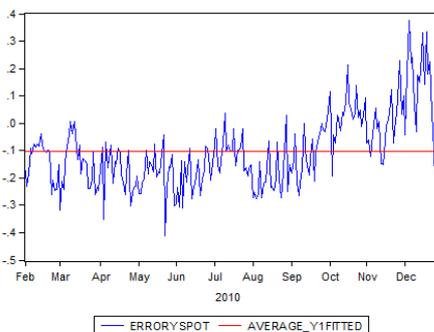
Error Q1



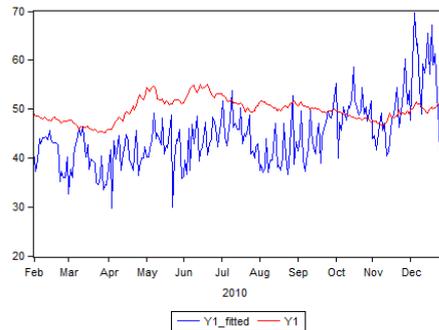
Q1 Fitted and Q1



Error Y1



Y1 Fitted and Y1



Online Appendix B: HJM Model

We tested several alternative specifications for the HJM model; all of them are available on request. We present the best performing specification, based on using three liquid contracts within each segment. Under this specification, the model gives best results in terms of fitting the volatility term structure.² This model is a HJM-based multi-factor stochastic process for electricity swap prices under the real-world probability measure,

$$\frac{dF_i(t, \mathbf{T})}{F_i(t, \mathbf{T})} = \sum_{k=1}^N \alpha_{ki}(t, \mathbf{T}) + \sigma_{ki}(t, \mathbf{T}) dW_t^{ki}, \quad (\text{B.1})$$

Given that we work with liquid contracts within each market segment, we propose specific parameterizations for the volatility functions in (B.1) as follows

$$\frac{dF_i(t, \mathbf{T})}{F_i(t, \mathbf{T})} = \alpha_i + \sigma_{1i}(t, \mathbf{T}_i) dW_t^{1i} \quad (\text{B.2})$$

$$\sigma_{1i}(t, \mathbf{T}_i) = e^{-k_i(T_i-t)} \sigma_{1i} \quad (\text{B.3})$$

where dW_t^{1i} , are independent Brownian motions for all delivery periods, and $\sigma_{1i}(t, \mathbf{T}_i)$ are volatility functions. We choose parameterization (B.2)-(B.3) because one factor explains over 80% of total variation and we look for simple and robust parametrizations. So, we apply a parsimonious representation, that is, one factor. Regarding specific functions to be used, Equation (B.2-B.3) was chosen because of its analytical tractability and at the same time its ability in reflecting the well-known fact that short dated forward returns are more volatile than long dated forwards. To calibrate this model we proceed as follows. We compute returns for contracts available in each case. We use yearly contracts (Y1 to Y3), quarterly contracts (Q1 to Q3) and monthly contracts (M1 to M3). Volatility functions are recovered by eigenvector decomposition of the covariance matrix. This decomposition yields a set of independent factors driving the evolution of the variables underlying covariance matrix Σ . We decompose Σ into n ($n=3$) eigenvectors \mathbf{v}_i (size 3×1) and associated eigenvalues λ_i such that $\Sigma = \mathbf{R}\mathbf{V}\mathbf{R}'$ where columns of \mathbf{R} are eigenvectors and the principal diagonal in \mathbf{V} contains eigenvalues (other elements in \mathbf{V} are zero). We only consider one eigenvalue. The first volatility function is computed by fitting Equation (B.1) to data $\mathbf{v}_1 \sqrt{\lambda_1}$.

Statistics of returns series are in Table B1. Average returns are not statistically different from zero, and so we set $\alpha_i = 0, \forall i$. Estimated standard deviation is annualized by the number of trading days (250) and varies from 13% (Y3) to 32% (M1). Volatility is usually higher for the closest to maturity contracts (Samuelson effect), confirming the well-known fact that short dated forward returns tend to be more volatile than long dated forwards. Figure B1 shows (sample from 2004 to 2012) the term structure of the volatility of market segments. The distribution of returns presents skewness and kurtosis, deviating

² Additional results for other models are available on request.

significantly from the normal distribution.

Eigenvalues resulting from the eigenvector decomposition tell us the importance of each eigenvector and hence the number of factors that we should include in our model. The first eigenvector is the most important, explaining 87%, 89% and 94% of the total variation in the evolution of the swap curve for the monthly, quarterly and yearly contracts respectively, supporting the reasonableness of assumptions (B.2 –B.3).

Figure B2 shows the first principal component function recovered for each contract type. This first principal component acts to shift forward prices and tilts curves. The most important factor (COMP1) is positive for all maturities, but decreasing with maturity. This implies that a positive shock to the system causes all prices to shift up but by decreasing amounts, depending on the maturity. The longer the maturity, the smaller the increase in prices is. Table B2 presents the parameter estimates from the volatility function obtained in the Principal Component Analysis using equations for the entire sample 2004-2012.

Table B2 presents in Panel B the LS estimates of parameters from volatility functions obtained in PCA using equation $\sigma_{1i}(t, \mathbf{T}_i) = e^{-k_i(T_i-t)}\sigma_{1i}$ for the full sample period 2004-2012. Panel A reports the in-sample root mean squared pricing errors (RMSEs). We compute daily errors based on fitted swap prices based on estimated parameters in Panel B. We compute the volatility function implied by the HJM model and by the SFP model and compare results against market prices. In the case of the HJM model, root-mean squared errors (RMSEs) are 6.05%, 19.32%, and 11.96% for monthly, quarterly and yearly contracts respectively. By contrast, RMSEs for those contracts are 0.10%, 0.12% and 0.30% respectively in the case of SFP model. The degree of fit of SFP is substantially higher than HJM's. In the case of the first volatility function, parameter σ_{1i} represents the overall volatility of the forward curve whilst parameter k_i tells us how fast the forward volatility curve decreases with increasing maturity. Parameter σ_{1i} captures the annualized volatility averaged over all contracts of a given class. From Table B1 it is easy to see that average volatility for annual, quarterly and monthly returns is 17%, 19% and 32% respectively. These are very close to estimated parameters σ_{1i} in Table B1, which are 19%, 19% and 38% respectively. The reason of the proximity lies in the low values of the decay factor. Estimated values of this parameter k (0.15, 0.03 and 0.20) suggest a fairly slow decrease in volatility as time to maturity increases. Monthly prices present higher overall volatility and faster decrease in volatility with maturity, followed by quarterly and yearly prices. However, monthly prices present slower volatility attenuation than yearly prices. The degree of fit of the equation is high in the case of yearly prices (99%), followed by monthly (99%) and quarterly prices (83%). To check parameter stability we repeat the calibration exercise using different subsamples 2006-2010 and 2010-2012. A comparison of parameters is in Figure B3, which suggests stability of estimates of parameter. Overall, results suggest that parameters are reasonably stable over time.

Although the model seems to fit the volatility term structure to some extent, cannot recover the

skewness and kurtosis observed in the empirical distributions. By contrast, the SFP model not only fits better the market's volatility term structure, but also is able to take into account skewness and kurtosis.

Table B1: Descriptive Statistics of Returns

The table shows descriptive statistics of returns (1-day changes in the natural logs of swap prices) and the sample covariance matrix of these returns. We study three contracts for each market segment (yearly, quarterly and monthly), contracts M1 to M3, Q1 to Q3 and Y1 to Y3 from 6/1/2004 to 12/31/2012. The "Std. Dev." column reports the standard deviation of the series in annual terms. The nine series are corrected of the rolling effect by means of intervention analysis. p-val is the p-value for the test of zero mean.

	M1	M2	M3	Q1	Q2	Q3	Y1	Y2	Y3
Mean	-0.001	-0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Median	-0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Maximum	0.163	0.149	0.126	0.109	0.099	0.099	0.088	0.070	0.073
Minimum	-0.146	-0.163	-0.239	-0.062	-0.161	-0.074	-0.071	-0.063	-0.064
Std. Dev.	0.329	0.269	0.239	0.193	0.191	0.182	0.174	0.150	0.131
Skewness	0.175	-0.160	-1.837	0.326	-0.905	0.136	0.008	0.163	0.516
Kurtosis	9.999	14.152	37.444	9.801	24.640	10.586	9.424	10.023	14.248
p-val	0.080	0.133	0.195	0.511	0.985	0.984	0.962	0.595	0.998

Table B2: Parameters Estimation and RMSE

The table presents in Panel A the LS estimates of the parameters from the volatility functions obtained in PCA using equation $\sigma_{1i}(t, T_i) = e^{-k_i(T_i-t)}\sigma_{1i}$ for the full sample period 2004-2012. t- statistics are presented in parenthesis. Panel A reports in-sample root mean squared pricing errors for HJM and SFP models. In the case of HJM, the errors are computed daily based on the fitted prices of swap based on estimated parameters in Panel B.

	Yearly Contracts	Quarterly Contracts	Monthly Contracts
Panel B: RMSEs			
HJM	6.05%	19.32%	11.96%
SFP	0.10%	0.12%	0.30%
Panel A: Volatility Function			
σ_1	0.012 (37.34)	0.012 (34.75)	0.024 (67.90)
K	-0.154 (11.32)	-0.030 (2.23)	-0.201 (26.09)
R²	0.992	0.834	0.998

Figure B1: Volatility Functions

The Figure shows the first principal component functions recovered from the above procedure for each contract type (M1, M2, M3; Q1, Q2, Q3 and Y1, Y2, Y3). Sample period 2004-2012

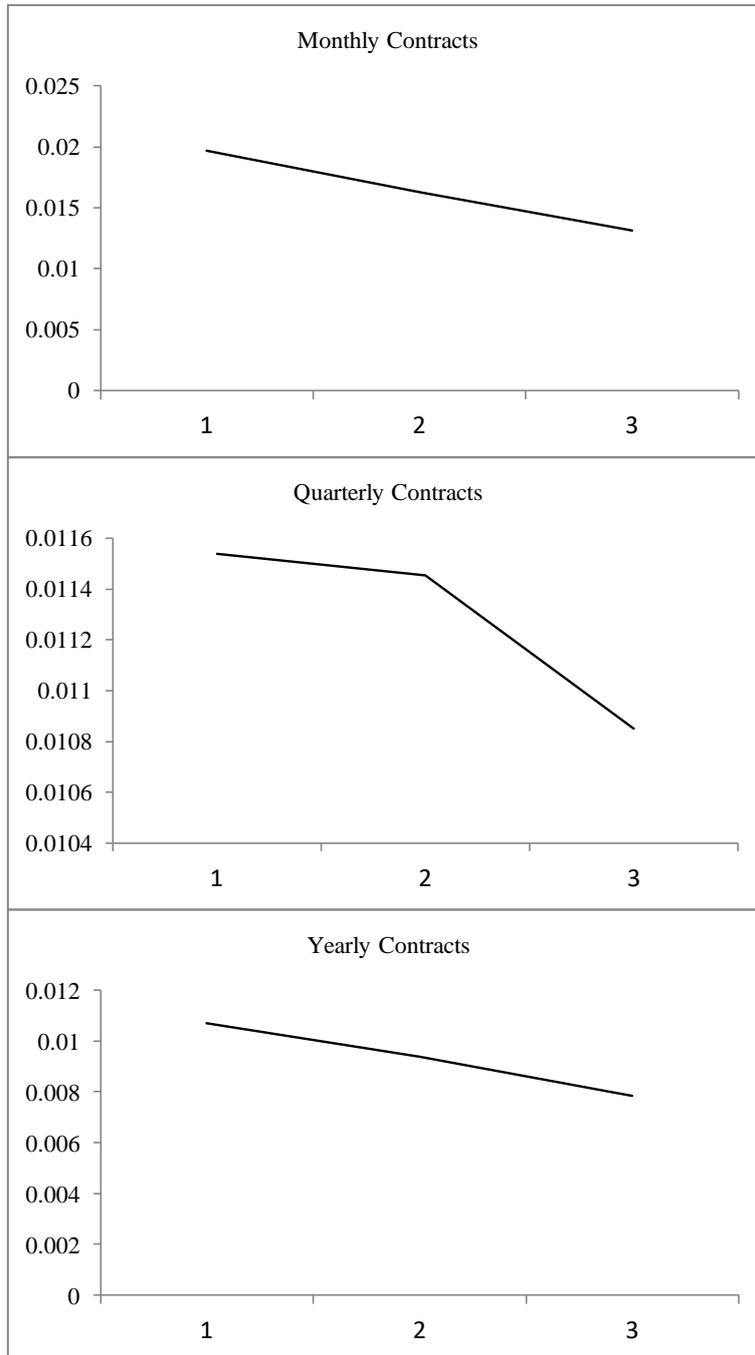
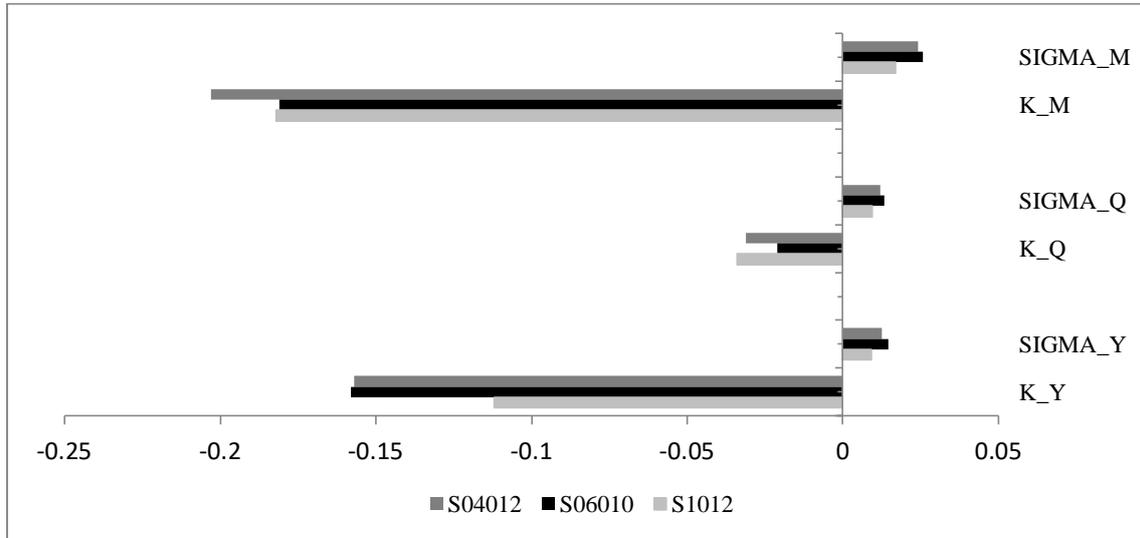


Figure B2: Stability of the parameters by subsamples

To check parameter stability we have repeated the calibration exercise using different subsamples 2006-2010 and 2010-2012. The plot presents a comparison of the parameters.



	S04012	S06010	S1012
SIGMA_M	0.0242	0.0257	0.0169
K_M	-0.203	-0.181	-0.182
SIGMA_Q	0.012	0.0134	0.0094
K_Q	-0.031	-0.021	-0.034
SIGMA_Y	0.0126	0.0146	0.0093
K_Y	-0.157	-0.158	-0.112

Online Appendix C: Cartea and Figueroa (2005) Model

CF denote the electricity spot price at time $0 \leq t \leq T$ by $P(t)$, and assume that it takes the form $P(t) = e^{s(t)}X(t)$ where $s(t)$ is a deterministic function modelling trend and seasonal effects, and $X(t)$ is a stochastic process modelling the random fluctuations around this trend. We choose the following trend model (Benth et al., 2012)

$$s(t) = \alpha_1 + \alpha_2 \frac{t}{250} + \alpha_3 \cos\left(\alpha_4 + 2\pi \frac{t}{250}\right) + \alpha_5 \cos\left(\alpha_6 + 4\pi \frac{t}{250}\right) \quad (C.1)$$

DF specify the $X(t)$ process for the jump-diffusion model as follows

$$d \ln X(t) = -\alpha \ln X(t) dt + \sigma(t) dW(t) + \ln J dQ(t) \quad (C.2)$$

where $\alpha > 0$ is the speed of mean-reversion, W is a Brownian motion, $\sigma(t) > 0$ is a time-dependent volatility, J is a proportional random jump size and $dQ(t)$ is a Poisson process of daily intensity (arrival rate) l with

$$dQ(t) = \begin{cases} 1 & \text{with probability } l dt \\ 0 & \text{with probability } (1-l) dt \end{cases} \quad (C.3)$$

We follow standard practice and assume that the jump size distribution is $\ln J \sim N(\mu_J; \sigma_J)$ and $E[J] = 1$. We estimate parameters using the method in Cartea and Figueroa (2005) with $\sigma(t)$ estimated by rolling historical volatility. Estimated parameters are shown in Table C.1

Table C1: CF Estimated parameters

This table contains estimates of CF model. Sample period is from 6/1/2004 to 12/31/2012. Sample size is 2179 observations. * denotes significance at 5% level and ** at 1% level

Parameter	Estimate
α	0.1347**
Average $\sigma(t)$	0.1618**
μ_j	-0.1971
σ_j	0.6952**
l	0.0188**

Notice that the estimate of daily intensity l implies an average number of seven jumps per year, which is consistent with the empirical evidence. Next, we compute forward prices using Equation 20 in Cartea and Figueroa (2005)

$$F(t, T) = G(T) \left(\frac{S(t)}{G(t)} \right)^{\alpha} e^{-\alpha(T-t)} \exp \left[\int_t^T \left[\frac{1}{2} \sigma^2(s) e^{-2\alpha(T-s)} - \lambda \sigma(s) e^{-\alpha(T-s)} \right] ds + \int_t^T \xi(\alpha, \sigma_f^2) l ds - l(T-t) \right] \quad (C.4)$$

$$\xi(\alpha, \sigma_f^2) = \exp \left[-\frac{\sigma_f^2}{2} e^{-\alpha(T-s)} + \frac{\sigma_f^2}{2} e^{-2\alpha(T-s)} \right]$$

Notice that in C.4 an additional parameter λ , Market Price of Risk (MPR) is included. By using theoretical forward prices and comparing with average market prices, we extract the implied lambda (MPR). In Table C2 we show values of lambda by contract.

Table C2: CF Market Price of Risk

This table contains estimates of the Market Price of Risk for CF model using Equation C.4. Sample period is from 6/1/2004 to 12/31/2012. Sample size is 2179 observations.

	Average	λ (MPR)
SPOT	50.82	
M1	48.74	0.21
M2	49.92	0.17
M3	50.61	0.16
M4	50.99	0.14
M5	51.45	0.13
M6	51.51	0.11
Q1	50.44	0.14
Q2	51.47	0.11
Q3	51.46	0.05
Q4	51.74	0.01
Q5	52.47	-0.05
Q6	52.76	-0.11
Y1	52.01	0.02
Y2	52.61	-0.17
Y3	53.52	-0.41
Y4	55.23	-0.69
Y5	56.69	-1.02
Y6	57.54	-1.39
Average	52	-0.15

Notice that sign and magnitude of MPR varies across forward maturities depending on hedging pressure from producers and consumers (Benth, Cartea and Kiesel 2008). Situations where $MPR < 0$ hold with markets where the consumers' desire to cover their positions 'outweighs that of the producers. Consumers are averse to higher electricity prices and willing to pay a risk premium to avoid such higher prices. We see this situation with contracts Y2-Y6. Conversely, situations where $MPR > 0$ result when the producers' desire of hedge their positions outweighs that of the consumers. We see this situation with contracts M1 to Q3. Positive (Negative) MPR is usual in positive (negative) beta equity markets and implies that forward prices are (upward) downward-biased estimators of future spot prices.

Online Appendix D: Di Poto and Fanone (2012) Model

To give a comparison against a recent market model competitor, we develop an adapted version of the model by Di Poto and Fanone (2012), DF henceforth. We adjust DF in several aspects. First, we do not apply any smoothing algorithm to data. This choice make results comparable to the model proposed in this paper. Second, we use the loading factor as a direct volatility proxy instead of fixing a polynomial parametrization as in DF. Although this deviation does not make a significant difference, eliminates the possible fitting error, so improving DF model performance. Finally, we make use of the first four independent components instead of the first three components of DF. This adjustment should have a positive effect on DF's performance.

D.1. The model

Assume $T < \infty$ and let (Ω, F, P) be a complete filtered probability space, with an increasing and right-continuous filtration $\{F_t\}_{t \in [0, T]}$ where, as usually, F_0 contains all sets of probability zero in F . The model assumes a simple lognormal market representation given by the following stochastic differential equation:

$$d \ln F_c(t) = \delta_c(t)dt + \sum_{k=1}^n \Sigma_{c,k}(t)dL_k(t) \quad (D.1)$$

Where, $F_c(t) = F_c(t, \tau_s^c, \tau_e^c)$ is the price at time t for an electricity future with delivery period $[\tau_s^c, \tau_e^c]$. We assume δ_c and $\Sigma_{c,k}$ to be sufficiently regular functions such that the swap dynamic $\ln F_c$ is square integrable, and dL_k , $k = 1, \dots, n$ are independent Lévy processes.

In addition, the following functional form is assumed to model the mean reverting process, $\delta_c(t)$:

$$\delta_c(t) = \frac{ds_c(t)}{dt} + \alpha_c(s_c(t) - \ln F_c(t)) \quad (D.2)$$

Last expression allows us to model a typical market behavior, that is, mean reversion in the direction of seasonality. Finally, a Normal Inverse Gaussian (NIG) distribution is assumed for the independent increments, $dL_k(t)$.

D.2 Model estimation

We model seasonality by using the following parametric periodical function:

$$s_c(t) = \beta_0 + \beta_1 t + \beta_2 \cos\left(\frac{2\pi(t - \beta_3)}{250}\right) \quad (D.3)$$

We estimate parameters in (D.3) with a least square approach for each futures contract. After the seasonal estimation we remove it by subtracting, $s_c(t)$, to the log-price. Once seasonality is removed, in line with DF, we deal with the autoregressive component of order one, AR(1). As in DF, we do not find evidence of a long memory effect. Estimated mean-reversion parameters, $\widehat{\alpha}_c$ are close to zero and non-significant and we report results in Table D1.

Table D1: Estimated mean reversion parameters

Estimated parameters of the AR(1) process for future contracts with different maturities. Sample period is from 6/1/2004 to 12/31/2012. Sample size is 2179 observations. * denotes significance at 5% level and ** at 1% level.

Parameter	Yearly	Quarterly	Monthly
α_1	0,0059	0,0062	0,0093
α_2	0,0039	0,0071	0,0042
α_3	0,0022	0,0096	0,0031
α_4	0,0026	0,0064	0,0038
α_5	0,0038	0,0088	0,0049
α_6	0,0096	0,0081	0,0120

Last, we apply the Independent Component Analysis (ICA) algorithm to residuals $\mathbf{x} = d \ln F_c(t) - \delta_c(t)dt$, to decompose them in a mixing matrix $\mathbf{A} = \Sigma_{c,k}$ and a source \mathbf{s} . Independent Components (ICs) are obtained by applying the *FastICA* algorithm to the residuals. Table D2 shows model performance by comparing model-generated volatility $\Sigma_{c,k}$ term structure against actual market volatility.

Table D2: Volatility Term Structure

This table compares actual market volatility of swap returns and estimated volatility using the term structure of swap prices variances. Sample period is from 6/1/2004 to 12/31/2012. Sample size is 2179 observations.

	Market	DF Model	Relative Error	Absolute Error
M1	0.3264	0.3125	4.26%	4.26%
M2	0.2613	0.2526	3.34%	3.34%
M3	0.2191	0.213	2.78%	2.78%
M4	0.2002	0.1963	1.97%	1.97%
M5	0.1999	0.1873	6.30%	6.30%
M6	0.1997	0.1855	7.09%	7.09%
Q1	0.2245	0.1751	22.01%	22.01%
Q2	0.1925	0.1634	15.09%	15.09%
Q3	0.1831	0.1632	10.89%	10.89%
Q4	0.1815	0.1542	15.04%	15.04%
Q5	0.1843	0.1326	28.06%	28.06%
Q6	0.1759	0.1217	30.79%	30.79%
Y1	0.1738	0.1861	-7.09%	7.09%
Y2	0.1506	0.1445	4.03%	4.03%
Y3	0.1311	0.1249	4.70%	4.70%
Y4	0.1237	0.1138	8.01%	8.01%
Y5	0.1264	0.1111	12.07%	12.07%
Y6	0.1336	0.1134	15.09%	15.09%
Average			10.25%	11.03%

Online Appendix E: Value at Risk

To compare the performance of alternative models, we compute Value-at-Risk (VaR) at different probability levels over a one-day horizon. Given a portfolio P , a time T and a probability level Q , a loss L^* is selected, at which exists a probability Q effective losses L , are at most L^* in period T . The loss L^* is portfolio's VaR. Formally,

$$Prob[L^* \geq L] = Q \quad (\text{E.1})$$

and therefore VaR_Q is a quintile of asset's returns probability density function, which defines the maximum expected loss with confidence level Q . In the following, and to be consistent with the empirical evidence in our sample, we assume the expected one-day swap return is zero. A comparison of the VaR for standardized returns and for different probability levels, Q is shown in Figure E1, for the Normal distribution and for the NIG distribution with different kurtosis parameter values. With low significance levels (90% and 95%), the values of VaR_Q^{Normal} tend to be higher (in absolute terms) than those of VaR_Q^{NIG} , so the latter measure will probably underestimate risk. However, with high significance levels (99% and beyond) there is a very substantial difference between the two measures, because the VaR_Q^{Normal} strongly underestimates risk in comparison with VaR_Q^{NIG} . The difference between the two measures, for a given Q , is higher; the closer to the unity is the kurtosis parameter ζ .

For the computation of the 1-day VaR for each swap contract, we proceed as follows. We assume the innovations in the spot model and in the HJM model are normal. However, and given the limited success of the spot price model in our sample, we use errors from the HJM model for the VaR calculations. Therefore, we compute $VaR(i, T)_Q^{Normal}$ as follows

$$VaR(i, T)_Q^{Normal} = k(\sigma_{i,T}\sqrt{\Delta t}) \quad (\text{E.2})$$

where the factor k (critical values) depends on Q as presented in Figure E1 and $\sigma_{i,T}$ is the volatility of the innovations of the forward prices generated by means of the HJM model.

To compute the VaR with the SFP model, we consider each swap can be thought as a portfolio containing two stochastic factors and therefore its VaR should be computed using the standard VaR formula for a portfolio (Jorion, 2001). Using Equation (E.1), we define the VaR for the swap in market segment i and maturity T as follows

$$VaR_Q^{NIG} [F_i(t, T)] = \sqrt{VaR_Q[\bar{F}_i(t)]^2 + VaR_Q[\gamma_i(t, T)]^2 + 2Cov(\bar{F}_i(t), \gamma_i(t, T))} \quad (\text{E.3})$$

We compute the cumulative distributions functions for the NIG processes driving $\bar{F}_i(t)$ and $\gamma_i(t, T)$ by means of numerical simulation. We compute the VaR for each component as follows

$$VaR_Q^{NIG}[\bar{F}_i(t)] = k_{\xi,\chi}(\theta_{\bar{F}_i} \sqrt{\Delta t}) \quad (E.4)$$

$$VaR_Q^{NIG}[\gamma_i(t, T)] = k_{\xi,\chi}(\theta_{\gamma_i(T)} \sqrt{\Delta t}) \quad (E.5)$$

where factor $k_{\xi,\chi}$ depends on Q and on skewness and kurtosis. The volatilities $\theta_{\bar{F}_i}$, $\theta_{\gamma_i(T)}$ are the residual standard errors got from Equation (12) and reported in Table 5. We obtain covariance matrix $\Omega = \{\omega_{i,j}\}$ (see Table 6) computed as follows

$$Cov(\bar{F}_i(t), \gamma_i(t, T)) = \theta_{\bar{F}_i} \times \theta_{\gamma_i(T)} \times \omega_{\bar{F}_i, \gamma_i(T)}$$

Next, we compare the Failure Ratios (FR) of alternative models. We define FR as

$$FR = \frac{\text{Realized proportion of VaR failures}}{\text{Expected proportion of VaR failures}} = \frac{N/T}{c}$$

where $1-c$ is the confidence level, T is number of time periods (e.g. $T=100$ days) and a Failure appears when the realized loss (negative return) is larger than the VaR forecast. If the model producing the VaR forecasts is right in assessing the risk, we expect $FR \approx 1$. If the model underestimates, (overestimate) risk, then $FR > 1$ ($FR < 1$). To test the statistical difference from one of the estimated FR, we use a variation of Kupiec (1995) test, suggested by Campbell (2007). Under the assumption the VaR under consideration is accurate, the z -statistic has an approximate standard normal distribution and has a known exact finite sample distribution. The z statistic is the Wald variant of the likelihood ratio statistic proposed by Kupiec (1995). One potential advantage of the Wald test over the likelihood ratio test is that the former is well-defined if there are no VaR violations. Kupiec's test is not defined in this case. The possibility of no violations in a short period, is not trivial. The z -statistic is

$$z = \frac{\sqrt{T}(\frac{N}{T}-c)}{\sqrt{c(1-c)}} \quad (E.6)$$

A positive (negative) z statistic shows the model tends underestimate (overestimate) risk. We present the results in Table 8. VaR_Q^{Normal} , calculated under the assumption of normality understates risk, and this understatement is very strong for high confidence levels (99.5% and 99.99%) suggesting that tail risk is severely underestimated. On the other hand VaR_Q^{NIG} tends to overestimate risk at relatively low significance levels but can account for extreme tail risk. It is worth noting that the overestimation of risk provided by the NIG distribution is proportionally much lower than the underestimation of risk produced by the normal distribution.

One important practical implication of our results is as follows. We know VaR models allow users to control risk and decide how to divide limited resources. Financial intermediaries impose a capital charge to traders based on risk-adjusted capital. This creates a natural incentive for traders to take a

position only when they have strong views on markets. If they have no views, they should abstain from trading. Our results suggest that risk adjusted capital for traders using EEX swap electricity contracts should be adjusted upwards in comparison with the standard practice based on the normality assumption. Traders should also adjust positions as risk changes (in the face of an increasingly volatile environment a sensible response is to scale down positions). Furthermore and given that VaR is also a performance evaluation tool, the evaluators of traders' performance should adjust their measures accordingly.

Figure E1: Critical values k

The figure shows critical values k for Value-at-Risk computations in Equations (29), (31) and (32) for a number of Levels of Significance (Q): Standardized Normal $N(0,1)$ and NIG distributions

