Technical On-Line Appendix: The value of saving oil in Saudi Arabia

This on-line appendix outlines the main features of the Blazquez et al. (2017) Saudi Arabia model and details how it has been extended and developed for the analysis undertaken in the paper.

We represent Saudi Arabia’s economy by a dynamic general equilibrium model of a small open economy with a representative household, three productive sectors (electricity, energy services and final goods) and a government.

**Government**

The government is the owner of the primary energy resources (oil and gas) in the economy. Oil production ($O_t$) is either exported or used domestically to generate electricity ($O_{Et}$) or to produce energy services ($O_{St}$). Production of gas ($\tilde{G}_t$) is entirely used to produce electricity.

The government collects revenues from selling gas and renewables ($\tilde{R}_t$) to produce electricity, from exporting oil at the international oil price ($P_o$), and from selling oil domestically to produce electricity ($O_{Et}$) and energy services ($O_{St}$) at an administered price ($\tilde{P}_o$).

We consider government policies that allow an increase in oil exports. Some of those policies involve a cost. We assume that this cost is paid by the government, entering the expenditure side of the budget constraint. In particular, those policies include (i) increasing oil production, (ii) importing natural gas, (iii) increasing the efficiency of gas power plants, (iv) investing in public capital ($k_{gt}$) for renewables$^1$, (v) increasing the efficiency of electricity in the production of energy services and (vi) increasing the efficiency of oil in the production of energy services. The government allocates the difference between revenues and expenditure into transfers to households ($TR_t$).

The government budget constraint is given by:

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$^1$ Electricity from renewables is produced according to: $\tilde{R}_t = \frac{A}{1 + \psi(\tilde{E}_t/E_t)} k_{gt}$, where $\psi$ represents the integration cost of renewables into the electricity system.
\[ P_{0t}(\bar{O}_t - O_{E_t} - O_{St}) + \bar{P}_{0t}(O_{E_t} + O_{St}) + \bar{P}_{Rt}\bar{R}_t + \bar{P}_{Qt}\bar{G}_t = i_{gt} + TR_t + \zeta_0(\bar{O}_t - \bar{O}_{SS}) + \zeta_G(\bar{G}_t - \bar{G}_{SS}) + \zeta_B(\beta - \bar{\beta}_{SS}) + \zeta_{A1}(A_1 - 1) - \zeta_{A2}(A_2 - 1) \]  

(A1)

where parameters \( \zeta_0, \zeta_G, \zeta_B, \zeta_{A1}, \zeta_{A2} \) represent the cost of a unitary change in the production of oil, the production of natural gas, a change in the calorific efficiency of natural gas power plants, a change in the productivity of electricity and a change in the productivity of oil, respectively. The variables \( \bar{O}_{SS}, \bar{G}_{SS} \) and \( \bar{\beta}_{SS} \) are the variables in the initial steady state, prior to any policy change, and \( \mu \) is the depreciation rate of public capital.

**Electricity sector**

Electricity \( (E_t) \) is generated by a public company with a linear technology using oil, gas and renewables as inputs:

\[ E_t = \alpha O_{E_t} + \beta \bar{G}_t + \bar{R}_t, \]  

(A2)

with \( \alpha \) and \( \beta \) representing the technical efficiency of oil and gas power plants.

The firm chooses the amount of oil to maximize profits, but the amount of gas and renewables is exogenous to the firm. In order to avoid profits or losses in the electricity company, the prices of gas and renewables are equal to the respective marginal productivity.

**Energy services sector**

Energy services are produced by a competitive firm that combines electricity and oil into a Constant Elasticity of Substitution (CES) production function:

\[ S_t = \left[ a(A_1 E_t)^\lambda + (1 - a)(A_2 O_{St})^\lambda \right]^{1/\lambda}, \]  

(A3)

where \( A_1 \) and \( A_2 \) reflect productivity of electricity and oil in the production of energy services. Both parameter values are initially set to 1. The firm takes decisions that maximize profits.
Energy services are demanded by both households ($S_{H_t}$) and final goods firms ($S_{F_t}$), so the market equilibrium condition is:

$$S_t = S_{H_t} + S_{F_t}$$  \hfill (A4)

**Final goods sector**

Final goods and services are produced by using labor, capital and energy services according to:

$$Y_t = A_3 n_t^\phi [(1 - b) k_t^\psi + b S_{F_t}^\phi]^{\frac{1-\theta}{\nu}}$$  \hfill (A5)

$A_3$ represents total factor productivity and is initially set to 1. The firm maximizes profits, so equilibrium conditions make input prices equal to marginal productivities.

**Households**

The household derives welfare from consuming final goods ($c_t$) and energy services ($S_{H_t}$). The household maximizes an intertemporal discounted utility flow subject to the budget constraint:

$$\max_{c_t,S_{Ht},k_{t+1},b_{t+1}} \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^\sigma + d S_{Ht}^\phi}{1-\sigma} \right)$$

subject to

$$b_{t+1} + c_t + P_{S_t} S_{Ht} + (1 - \delta) k_t + \frac{\phi}{2} \left( \frac{k_{t+1} - k_t}{k_t} \right)^2 = w_t n_t + r_t k_t + (1 + r^*_t) b_t + T R_t ,$$

where $\beta$ is the discount factor, $b_t$ is a one-year maturity foreign bond with a yield given by an exogenous international interest rate $r^*_t$. $w_t$ are wages and $n_t$ is labor, which is normalized to 1. $k_t$ represents capital stock in the private sector, while $r_t$ is the return of capital and $\delta$ is the depreciation rate. We assume quadratic adjustment costs in investment. As a way to introduce the trade balance into the small open economy model, we assumed that households have access to a perfectly competitive international capital market where they can buy and sell international bonds.
We induce stationarity in the model assuming that the interest rate of the bond depends on the deviations of the foreign bond from its steady state: \( r_t^* = r^* + (e^{b_t-b_{ss}} - 1) \).

**International oil market**

We assume a price reaction function that links Saudi exports and the international price of oil, as any change in Saudi oil production can have an impact on the international price. We assume that a change in oil exports, \( \Delta X_t = \Delta (\bar{O}_t - O_{b_t} - O_{s_t}) \), leads to a change in oil revenues according to:

\[
\varepsilon = \frac{\Delta X_t}{X_t p_{o_t}} = \frac{\Delta P_{o_t}}{P_{o_t}} = \left[ \varepsilon_D - (1 - \rho)\varepsilon_R \right] / \rho \tag{A7}
\]

where \( \varepsilon_D \) and \( \varepsilon_R \) represent the price elasticity of global demand and non-Saudi supplies, and \( \rho \) is the Saudi market share of global oil output.

We consider long-run elasticities since we compare steady state solutions of the model. The two elasticity values \( \varepsilon_D \) and \( \varepsilon_S \) are not directly observable and, in addition, the literature offers no consensus on these values. Furthermore, as stressed by Hamilton (2009), these values have changed throughout time, with the recent light tight oil revolution increasing the elasticity of supply. For our purposes, we consider the values discussed and used by Pierru et al. (2017): \( \varepsilon_D = -0.3, \varepsilon_R = 0.3 \), and \( \rho = 11.7\% \). This means that increasing oil exports by one barrel in period \( t \) would generate an incremental revenue equal to 79% of the international market price.