Petroleum Prospect Valuation:
The Option to Drill Again

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We examine the value of an exploration prospect that is to be exploited via a series of possibly dependent trials. Failure on any particular trial is assumed to convey bad news, but also provides an option to try again. The pattern and strength of dependence among trials determines the value of this option, and therefore also influences the value of the underlying prospect. We describe the solution to this valuation problem, examine the behavior of the option premium, and characterize potential errors that are inherent in two ad hoc procedures that are often used to estimate prospect value. We demonstrate that the impact of dependence among trials is monotonic: each increase in the degree of dependence results in a further reduction in expected value of the prospect. We also characterize the particular pattern of dependence that is implied by a plausible model of exploratory risk.

1. INTRODUCTION

Sometimes exploration succeeds only after failure. The East Texas oil field, the largest and most prolific producer in the lower-48 states, was discovered on the third attempt. The first two exploration wells, located just 100 feet apart, were dry. The third (successful) attempt was located just 375 feet from the second dry hole. Spindletop, also in Texas and the most prolific oil field of its time, was discovered on the fifth attempt, after four consecutive dry holes had already been

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drilled on the same structure.[1] These examples illustrate the potential value of the option to drill again.

Newendorp (1975) clearly recognized this aspect of the exploration problem. Consider his summary of the situation that confronts a wildcatter:

> If he drills an exploratory well that is dry or only a 'teaser' he has another decision whether to drill another wildcat test or drop the acreage, etc. It becomes fairly obvious that the initial decision is but a link in the chain of future decision options and contingencies. And all of these future options must be considered when evaluating the feasibility of bidding. The simple decision whether to bid has suddenly become quite complex and fraught with future decision options and 'what ifs' at nearly every step of the anticipated exploitation of the lease or tract.[2]

Among the major treatises on exploration economics, only Newendorp (1975, pp. 115-122) and Lerche and MacKay (1999, pp. 287-294) have analyzed the option to drill again in any detail. In both cases, the analysis is limited to the presentation of a single numerical example, and neither study looks beyond the worked example in search of more general principles that may govern the value of this type of option. The goal of this paper, therefore, is to model Newendorp's "chain of future decision options" in a way that sheds new light on the factors that impact the value of the option to drill again, and to provide a framework that allows the full value of such a prospect to be fairly assessed.

It is customary to evaluate petroleum prospects in terms of three parameters: the probability of success (p), the expected gross profit conditional on success (V), and the cost of the drilling trial (C).[3] Usually, as in Newendorp (1975), Megill (1988), and Lerche and MacKay (1999), the expected reward associated with the trial is set against the cost of conducting that trial to obtain the expected economic value of the prospect:[4]

$$EV_a = \mathrm{p} \bullet V - C.$$

1. The discovery of Spindletop oil field is described in Goodwin (1996, pp. 10-25). The history of the East Texas oil field can be found at the Texas Railroad Commission website: http://rra.dst.tx.us/c_t/History1/EAST%20TEXAS%20OILFIELD.cfm.

2. Newendorp (1975, p. 107).

3. Throughout the analysis, we take the view of a risk-neutral investor because the fundamental issues are seen most clearly in that case and the quantitative analysis is tractable. A risk-averse decision maker might be more reluctant to "drill again" or at least to drill again as many times, but the qualitative aspects of our results should still apply. To simplify the argument, we also suppress explicit discounting for time in all equations, although it will become apparent that the effect of discounting can be incorporated implicitly without changing the results.

4. See Newendorp (1975, pp. 64-83), Megill (1988, p. 163) and Lerche and MacKay (1999, p. 18). Kemp and Rose (1984), Hendricks and Kovenock (1989), Pickles and Smith (1993), and Laughton (1998) are among the many applications that have followed this approach.

Alternatively, as in Grayson (1960), the evaluator may recognize the operator's ability to follow up an unsuccessful trial with further attempts. If $n = 1/p$ independent trials were attempted, then the expected number of successes would be given by $np = 1$, which suggests a different calculation of expected economic value:[5]

$$EV_b = V - C/p,$$

in which the full reward for achieving a drilling success is set against the expected cost of a sufficient number of trials to obtain that success.

The relationship between these two estimates depends upon the value of p: $EV_b = EV_a/p$. They tend to converge if the probability of success is high and to diverge otherwise. If the probability of success were judged to be only 1/10, which is entirely plausible for many exploration prospects, the two estimates of value diverge by an order of magnitude. Understanding and resolving this discrepancy is important not only at the prospect acquisition and management stage, it may also figure into the sort of damage claims that typically arise when ongoing drilling operations are involuntarily aborted. The case of Gardes Energy Services v. G&S Oil Company provides one example where these two estimates (EV_a and EV_b) diverged significantly and therefore variously served the legal and financial interests of the adversaries.[6] Where the two methods of valuation diverge, it is important to look more closely at the underlying assumptions.

Which of the two methods is correct? The answer, in most cases, will be: "neither." Both approaches ignore the influence of dependence among trials. By dependence among trials we mean that the conditional probability of success depends on the outcome of earlier trials. In this paper, we proceed under the general assumption that dry holes convey "bad news." Therefore, our analysis is based on the assumption that the conditional probability of success may decline (and certainly will not increase) with the completion of each successive unsuccessful well.[7] Subject to this condition, we describe the solution to the prospect valuation problem, examine the behavior of the option premium, and demonstrate that it is impossible to obtain an unbiased estimate of prospect value without accounting for the degree of dependence among trials. We also show that the two *ad hoc* formulations introduced above (EV_a and EV_b) provide lower and upper bounds, respectively, on the expected economic value of the prospect regardless of the degree of dependence, and that the expected value declines monotonically as the

5. See Grayson (1960, pp. 160-165).

6. U.S. District Court, Southern District of Mississippi, Civil Action No. 3:99CV438WS.

7. Conditional on whatever drilling has gone before, the correlation between results of the next two exploratory wells is assumed to be non-negative. Other forms of dependence are clearly possible. In the extreme, for example, a prospector may be convinced that at least one of two known drilling locations would meet with success on a given prospect, in which case failure on the first would "guarantee" success on the second. If dry holes convey "good news," then the value of the option to drill again will be even larger than in the present model.

degree of dependence among trials increases. Finally, we characterize the particular pattern of dependence that is implied by a plausible model of exploratory risk.

The current study is related to the approach introduced by Paddock, Siegel, and Smith (1988), who examined the option value inherent in petroleum exploration and development prospects. Whereas they focused on the value of the option to *delay drilling* pending the arrival of updated price information (through the value of V, which they treated as stochastic), we focus on the value of the option to *drill again* pending the outcome of previous drilling attempts (where we treat V as non-stochastic). Although any given prospect potentially holds both types of option value, our results suggest that the two are substitutes to some degree: prospects most likely to benefit substantially from the option to drill again are least likely to benefit substantially from the option to delay drilling, and vice versa.

The present study considers sequential exploration of a *single* prospect, and is not directly related to the literature on sequential exploration of *multiple* prospects, of which Harris (1990, chs. 11 and 13) provides a good example. Whereas both subjects deal with Bayesian learning during the exploration process, the nature and impact of information spillovers is decidedly different between the two. For example, dependence among prospects adds value when there are multiple prospects, but subtracts value when there is only one.[8]

2. DEPENDENT TRIALS

Let the sequence $\{p_1, p_2, p_3, \ldots\}$ represent the conditional probability of success on the first, second, third, ... trials, given no previous success. We will assume:

$$p_1 \geq p_2 \geq p_3 \geq \ldots$$

and note that the case of *independent* trials is characterized by strict equalities throughout. For notational convenience, it will also be useful to define $p_0 = 0$.

The strength of dependence between trials is measured, for our purposes, by the extent to which a failure at one trial diminishes the probability of success on the next. Thus, $d_t = (p_t - p_{t+1})/p_t = 1 - p_{t+1}/p_t$ represents the strength of dependence at the t^{th} stage in the sequence. The probability of initial success, p_1, and the set $\{d_t\}$ are sufficient to describe any pattern of dependence among trials. For the case of independent trials, $d_t = 0$ for all t.[9] Another useful benchmark is the case

8. Smith and Thompson (2005) consider the impact of dependence among multiple prospects.

9. If the strength of dependence remains constant throughout the sequence, probabilities evolve according to a geometric series:

$$p_{t+1} = \lambda p_t, \quad \text{for all } t, \text{ with } 0 \leq \lambda < 1;$$

and the probability of success converges to zero. Constant dependence is perhaps not the most likely case, however. Consider, for example, a sequence whereby the probability of success *conditional on the presence of oil in the prospect* does not change from trial to trial. We show later (see Section 5) that, in such cases, the degree of dependence steadily increases throughout the sequence; i.e., $d_{t+1} > d_t$.

of "complete dependence," by which we mean that the probability of success falls immediately to zero after the first failure: $d_1 = 1$.[10]

Each trial is assumed to cost C, and success on any trial brings expected value V, which is assumed constant across trials. Therefore, for $t \geq 1$, the marginal expected profit generated by the t^{th} trial is $\pi_t = p_t V - C$.[11] For convenience, we define $\pi_0 = 0$. The sequence of trials is assumed to be truncated at the first success, or when marginal expected profit becomes negative, whichever comes first. Truncation due to negative marginal profit, if it does occur, will come after trial T, where:

$$p_T \geq C / V > p_{T+1}.$$

The truncation point (T) may not be finite (trials may continue indefinitely if the probabilities converge to a number that exceeds C / V), but if it is finite, then T must be unique due to the monotonic behavior of the probabilities of success. We note that $\pi_t \geq 0$ for all $1 \leq t \leq T$, and $\pi_t < 0$ for all $t > T$. To avoid degenerate cases, we will assume that $p_1 > C / V$; i.e., the marginal value of the first trial is positive.

Finally, we represent by q_t the probability of reaching the t^{th} trial, which happens if and only if there has been no previous success: [12]

$$q_t = \prod_{j=0}^{t-1} (1 - p_j), \quad t = 1, 2, ...$$

3. PROSPECT VALUATION

We start by formulating the expected value of the prospect; i.e., the expression that properly captures the impact of whatever dependence exists among successive trials:

$$EV \equiv \sum_{t=0}^{T} \pi_t \, q_t. \tag{1}$$

We wish to develop a simple, yet exact expression for EV, and to establish that: (a) EV_a and EV_b (defined previously) provide bounds on EV; (b) any increase in the degree of dependence among trials decreases EV; and (c) the bounds established

10. Wang, Kokolis, Rapp, and Litvak (2000, p. 1) refer to complete dependence as the case of "shared risks."

11. Our model is admittedly a gross simplification. Modern exploration is an information-rich process that generates, in addition to confirmatory evidence of petroleum in a given prospect, valuable information regarding the true values of V and C (which at best represent the expected values of unobservable variables within our framework), as well as spillover information regarding the possible presence and/or quality of petroleum deposits within other distinct but geologically-related prospects. Some implications of these additional factors are discussed in the concluding section.

12. If the delay between trials is of sufficient length to warrant discounting the value of future trials, the appropriate discount factor would be subsumed within the denominator of the $\{q_t\}$.

in part (a) are exact: $EV = EV_a$ in the case of complete dependence and $EV = EV_b$ in the case of independence.

It will be useful to consider a measure that recognizes the dependence created by failures among only the first τ trials (and ignores any further dependence created by subsequent failures). This measure is denoted EV_τ, where for all $\tau \geq 0$ for which $p_{\tau+1} > 0$:

$$\begin{aligned} EV_\tau &= \sum_{t=0}^{\tau} \pi_t\, q_t + \pi_{\tau+1}\, q_{\tau+1} [1 + (1 - p_{\tau+1}) + (1 - p_{\tau+1})^2 + \ldots] \\ &= \sum_{t=0}^{\tau} \pi_t\, q_t + \pi_{\tau+1}\, q_{\tau+1} / p_{\tau+1}. \end{aligned} \quad (2)$$

This formula deviates from equation (1) in that all p_t and q_t beyond $t=\tau+1$ are held constant at their previous values.

The benchmark valuation that ignores the impact of dependence altogether, EV_b , is obtained from equation (2) as the special case where $\tau = 0$:

$$EV_b = EV_0 = \pi_1\, q_1 / p_1 = V - C / p_1. \quad (3)$$

Advancing the index in equation (2) shows the impact of dependence through τ+1 trials (again we assume that $p_{\tau+2} > 0$) and therefore reflects the incremental impact of whatever dependence exists at the $\tau+1^{st}$ stage:

$$\begin{aligned} EV_{\tau+1} &= \sum_{t=0}^{\tau+1} \pi_t\, q_t + \pi_{\tau+2}\, q_{\tau+2} [1 + (1 - p_{\tau+2}) + (1 - p_{\tau+2})^2 + \ldots] \\ &= \sum_{t=0}^{\tau+1} \pi_t\, q_t + \pi_{\tau+2}\, q_{\tau+2} / p_{\tau+2}. \end{aligned} \quad (4)$$

To compare EV_τ and $EV_{\tau+1}$, it is convenient to rewrite (2) by grouping the first term of the infinite series with the first summation, which yields the equivalent expression:

$$\begin{aligned} EV_\tau &= \sum_{t=0}^{\tau+1} \pi_t\, q_t + \pi_{\tau+1}\, q_{\tau+1} [(1 - p_{\tau+1}) + (1 - p_{\tau+1})^2 + \ldots] \\ &= \sum_{t=0}^{\tau+1} \pi_t\, q_t + \pi_{\tau+1}\, q_{\tau+1} \left(\frac{1}{p_{\tau+1}} - 1 \right) \\ &= \sum_{t=0}^{\tau+1} \pi_t\, q_t + \pi_{\tau+1}\, q_{\tau+2} / p_{\tau+1}. \end{aligned} \quad (5)$$

Subtracting (5) from (4) then shows the incremental impact of dependence associated with the $\tau+1^{st}$ trial:

$$
\begin{aligned}
EV_{\tau+1} - EV_{\tau} &= q_{\tau+2}\left(V - \frac{C}{p_{\tau+2}}\right) - q_{\tau+2}\left(V - \frac{C}{p_{\tau+1}}\right) \\
&= q_{\tau+2}\, C\left(\frac{1}{p_{\tau+1}} - \frac{1}{p_{\tau+2}}\right) \\
&= -\, q_{\tau+2}\, C \,\frac{d_{\tau+1}}{p_{\tau+2}} \qquad (6) \\
&\leq 0 \text{ (since } d_{\tau+1} \geq 0\text{)}.
\end{aligned}
$$

Upper Bound for Prospect Value

The benchmark valuation EV_b , as defined by equation (3), gives the expected value of a prospect that affords independent trials. If trials are actually dependent, then this benchmark provides an estimate of value that is biased upwards by ignoring the impact of dependence.[13] To see this, we consider first the case where economic truncation would never occur; i.e., where dependence is sufficiently weak that the probability of success converges to a number greater than C / V. In this case, we may express the expected value of the prospect as the sum:

$$EV \equiv EV_0 + (EV_1 - EV_0) + (EV_2 - EV_1) + (EV_3 - EV_2) + \ldots \qquad (7)$$

But, by (6) we have shown each component in this series after the first to be non-positive. Thus, EV_0 (the estimate that ignores dependence altogether) is an upper bound on the expected value of any prospect for which dry holes convey bad news.

Now we consider the possibility that dependence is strong enough to force economic truncation after a finite number of trials. In this event, the expected value of the prospect may be written:

$$EV \equiv \sum_{t=0}^{T} \pi_t\, q_t = EV_{T-1} + \pi_T\, q_T - \pi_T\, q_T / p_T \qquad (8)$$

where we have used equation (2) to obtain the expression on the right. Thus:

$$EV - EV_{T-1} = -\frac{\pi_T\, q_{T+1}}{p_T} \leq 0, \qquad (9)$$

13. Although this result may be intuitively clear, there is some complexity to the proof owing mainly to the fact (cf. equation 1) that while a reduced probability of success at trial t tends to diminish the marginal value of any future trial (π_{t+1}), it also tends to increase the probability of continuing on to reap those marginal profits (q_{t+1}). The proof is a formal demonstration that the former effect outweighs the latter.

since $q_{T+1} = q_T(1\text{-}p_T)$ and $\pi_T \geq 0$ by definition if truncation occurs after trial *T*.

Where dependence forces economic truncation after trial *T*, we can express the expected value of the prospect as the sum (cf. equation 7):

$$EV = EV_0 + (EV_1 - EV_0) + (EV_2 - EV_1) + \ldots + (EV - EV_{T-1}), \qquad (10)$$

where all terms in this summation after the first have been shown via equations (6) and (9) to be non-positive. The first term, EV_0, which is the valuation that ignores dependence altogether, therefore serves as an upper bound for the expected value of the prospect.

Lower Bound for Prospect Value

Recall from equation (1) that the expected value of the prospect, *EV*, may be written:

$$EV \equiv \sum_{t=1}^{T} \pi_t q_t \,;$$

from which we observe:

$$\begin{aligned} \text{EV} &= \pi_1, \quad \text{if } T = 1, \\ &= \pi_1, + \sum_{t=2}^{T} \pi_t q_t, \quad \text{otherwise.} \end{aligned} \qquad (11)$$

Whether the point of economic truncation (*T*) is finite or infinite, each term in the summation on the right is non-negative by construction, which implies:

$$\text{EV} \geq \pi_1 = p_1 V - C = EV_a. \qquad (12)$$

This result establishes a lower bound for the expected value of the prospect, regardless of the strength and pattern of dependence. That bound, EV_a, corresponds to the expected value of the prospect if trials are completely dependent, in which case only one trial is attempted regardless of the outcome. Thus, the results so far have demonstrated that the benchmark cases of *independence* and *complete dependence* provide upper and lower bounds, respectively, for the expected value of any sequence of dependent trials.

An Exact Expression for Prospect Value

We consider first the case without economic truncation. Successive substitution from equation (6) into (7) yields:

$$EV = EV_0 + Cq_2\left(\frac{1}{p_1} - \frac{1}{p_2}\right) + Cq_3\left(\frac{1}{p_2} - \frac{1}{p_3}\right) + \ldots \qquad (13)$$

which becomes, after rearrangement:

$$EV = EV_0 + \frac{Cq_2}{p_1} + \frac{C(q_3 - q_2)}{p_2} + \frac{C(q_4 - q_3)}{p_3} + \ldots \qquad (14)$$

This expression can be simplified by using the identity: $q_t - q_{t-1} \equiv - q_{t-1}p_{t-1}$, plus the facts that $EV_0 = V - C/p_1$ and $q_2 = (1 - p_1)$, which after substitution in (14) yields:

$$EV = V - C \sum_{t=1}^{\infty} q_t = V - \bar{n}C, \qquad (15)$$

where $\bar{n}$ represents the expected number of trials to be attempted. (Recall that q_t corresponds to the probability of reaching trial t, at which point one additional well will be drilled).

Thus, in the case where economic truncation would never occur, the expected value of the prospect is given, and not surprisingly, by the expected gross value of the item less the cost per trial times the expected number of trials to reach first success. This expression holds generally for any pattern of dependence among trials, as long as the residual probability of success never falls below the economic threshold for truncation.

In the other case, where economic truncation would occur after trial T, an analogous expression describes the expected value of the prospect. It is obtained by substituting from equations (6) and (9) into (10), which yields:

$$EV = EV_0 + Cq_2\left(\frac{1}{p_1} - \frac{1}{p_2}\right) + Cq_3\left(\frac{1}{p_2} - \frac{1}{p_3}\right) + \ldots$$

$$\ldots + Cq_T\left(\frac{1}{p_{T-1}} - \frac{1}{p_T}\right) - \frac{\pi_T\, q_{T+1}}{p_T},$$

which after simplification via the same procedure used above reduces to:

$$EV = (1 - q_{T+1})V - \mathrm{C} \sum_{t=1}^{T} q_t = (1 - q_{T+1})V - \bar{n}C. \qquad (16)$$

In other words, regardless of the particular pattern and strength of dependence among trials, the expected value of the prospect is given by the expected gross value of the prospect times the probability it is discovered before giving up, less the cost per trial times the expected number of wells to be drilled.

Using equations (15) and (16), we can easily confirm that dependence among trials has a monotonic impact: any increase in the degree of dependence must decrease the expected value of the prospect. For the case where economic truncation would never occur, this result is immediately apparent. Holding all else constant, an increase in any one of the $\{d_t\}$ will cause one or more of the $\{q_t\}$ in equation (15) to increase, which increases the expected number of trials, $\bar{n}$, and thereby reduces the expected value.

Where economic truncation would occur after trial T, there is something more to the argument. If the increase in dependence does not alter the truncation point, then the situation is again quite simple; the first term in equation (16) must fall (due to the rise in q_{T+1}), while the second term must rise, and the expected value of the prospect is diminished. In addition, we must allow for the possibility that the point of economic truncation will itself decline, say from T to T-1. The proof for this case can be sketched very briefly. Since T was the optimal point of economic truncation before dependence was increased, we know that $EV(T\text{-}1) < EV(T)$, where $EV(t)$ represents the expected value of the original prospect (before the increase in dependence) if the sequence is to be truncated after trial t. But we also know (by the argument in the preceding paragraph) that $EV'(T\text{-}1) < EV(T\text{-}1)$, where the EV' notation represents the expected value of the revised prospect (based on the $\{p_t\}$ that result from the increase in dependence). By transitivity we must then have $EV'(T\text{-}1) < EV(T)$ for any increase in dependence among trials. The same argument works by extension if the optimal truncation point is reduced by more than one step.

4. AN ILLUSTRATION OF THE POTENTIAL OPTION PREMIUM

Although expected value of the prospect varies directly with the initial probability of success (through its impact on the $\{q_t\}$), the distance between our bounds on prospect value varies inversely with this parameter. The relative size of the interval within which the actual value must lie is: $EV_b / EV_a = 1 / p_1$.

If the probability of success were small, say 10%, then the two bounds would differ by an order of magnitude. Such cases merit the most detailed examination of the extent of dependence among trials simply because the potential error from ignoring that dependence is the greatest. The economic lower limit on p_1 is given by C / V. If we define the inverse, V / C, as the “unrisked return” or “gross margin” of the project, it then follows that prospects with the highest gross margins are the ones potentially most prone to mis-estimation of value as a result of ignoring or misstating the extent of dependencies (since those are the prospects that admit the lowest probabilities of success). Prospects with relatively low margins do not admit low success probabilities and therefore the bounds on project value will necessarily be tighter. Consequently, detailed examination of the nature of dependencies for projects with low gross margins would have less impact on the assessment of prospect value.

These relations between probability of success, unrisked return, and width of the valuation interval are illustrated in Figures 1 and 2, below. For convenience, the prospect in each figure is characterized by $V = 100$, which serves only as a scaling factor with no effect on the geometrical shape of the diagrams.

It is evident from the figures that the value of the option to drill again, which accounts for the spread between the two curves, is of greater potential significance for high-margin prospects than for low-margin prospects. In light of the fact that exploration wells, especially in frontier areas or immature plays, are

Figure 1. Bounds on Prospect Value (High Margin: C/V = 10%)

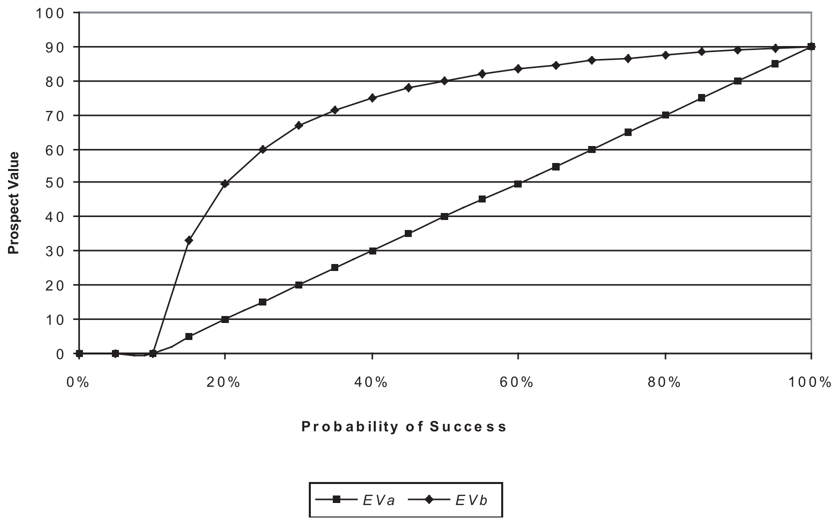
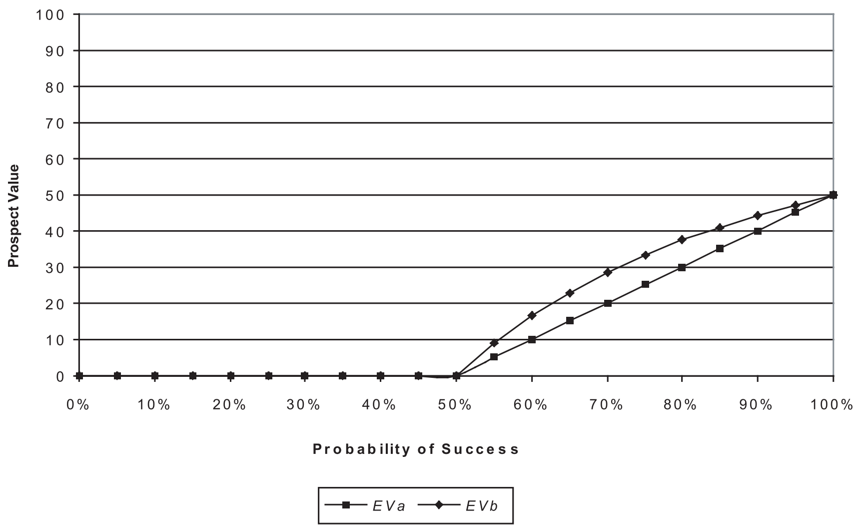


Figure 2. Bounds on Prospect Value (Low Margin: C/V = 50%)



particularly high-risk gambles with large rewards (relative to costs) if successful, exploration prospects would as a general rule be more likely than development wells to benefit from the option to drill again.

There is also a potential impact of dependence among trials on the option to delay drilling, as considered by Paddock, Siegel, and Smith (1988). They showed that the option to delay drilling is of highest value for low-margin prospects. That is, prospects for which the reward exceeds the cost by a sufficiently wide margin should be drilled immediately because the potential gain from favorable future price developments is outweighed by the fact that undeveloped oil reserves appreciate more slowly than money in the bank. It follows that dependence among trials (which tends to decrease the value of the exploration prospect, and therefore also to decrease the gross margin in the Paddock-Siegel-Smith framework) would tend to magnify the value of the option to delay drilling.[14]

5. PETROLEUM EXPLORATION: INCREASING DEPENDENCE

In this section we investigate the implications for dependence and prospect value of a plausible structure of geological uncertainty and its resolution. We begin with the postulate that the conditional probability of success (S_t) at trial t, given the presence of an oil-bearing structure (O) in the prospect, is independent of t:

$$P(S_t \mid O) = \alpha\ (>0) \quad \text{for } t = 1, 2, \ldots$$

We also assume that the probability of success at any trial, conditional on the absence of an oil-bearing structure, is zero:

$$P(S_t \mid \bar{O}) = 0 \qquad \text{for } t = 1, 2, \ldots$$

Finally, we take the a priori probability of an oil-bearing structure to be $P(O) = \beta$, where $0 < \beta < 1$. We can then write the probability of success at the first trial, p_1, as follows:

$$\begin{aligned} p_1 &= P(S_1) = P(S_1 \mid O)P(O) + P(S_1 \mid \bar{O})P(\bar{O}) \\ &= P(S_1 \mid O) \bullet P(O) = \alpha\beta \end{aligned}$$

Moreover, we can write the conditional probability of success on the second trial, given failure on the first, as:

$$p_2 = P(S_2 \mid \bar{S}_1) = P(S_2 \mid O) \bullet P(O \mid \bar{S}_1)$$

In general, the conditional probabilities at each stage in the sequence take the form:

$$p_t = P(S_t \mid \bar{S}_1 \cap \ldots \cap \bar{S}_{t-1}) = P(S_t \mid O) \bullet P(O \mid \bar{S}_1 \cap \ldots \bar{S}_{t-1}). \quad (17)$$

The last term in (17) can be written equivalently, per Bayes Rule, as:

14. The author is grateful to an anonymous referee for suggesting this interpretation.

$$P(O \mid \bar{S}_1 \cap \ldots \bar{S}_{t-1}) = \frac{P(\bar{S}_1 \cap \ldots \cap \bar{S}_{t-1} \mid O) \bullet P(O)}{P(\bar{S}_1 \cap \ldots \cap \bar{S}_{t-1})}$$

The numerator in this last expression equals $(1\text{-}\alpha)^{t\text{-}1}\beta$. The denominator is evaluated by rewriting in the form:

$$P(\bar{S}_1 \cap \ldots \cap \bar{S}_{t-1}) = P(\bar{S}_1 \cap \ldots \cap \bar{S}_{t-1} \mid O) \bullet P(O) + P(\bar{S}_1 \cap \ldots \cap \bar{S}_{t-1} \mid \bar{O}) \bullet P(\bar{O})$$

$$= (1-\alpha)^{t-1}\beta + (1-\beta).$$

Substituting these results back into (17) yields:

$$p_t = \frac{\alpha(1-\alpha)^{t-1}\beta}{(1-\alpha)^{t-1}\beta + (1-\beta)} \tag{18}$$

The degree of dependence at trial t is then determined from (18) by the ratio p_{t+1}/p_t, $= 1 - d_t = \lambda_t$, which after simplification can be expressed as:

$$\lambda_t = \frac{(1-\alpha)^t\beta + (1-\alpha)(1-\beta)}{(1-\alpha)^t\beta + (1-\beta)} < 1 \quad \text{for all } t. \tag{19}$$

The fact that $\lambda_t < 1$ signifies that trials are dependent. That fact that dependence is increasing as trials continue follows from the fact that $\lambda_t/\lambda_{t\text{-}1} < 1$, a result which can be confirmed by using (19) to evaluate the ratio: $\lambda_t/\lambda_{t\text{-}1}$:

$$\frac{\lambda_t}{\lambda_{t\text{-}1}} = \frac{\beta^2(1-\alpha)^{2t-2} + 2\beta(1-\beta)(1-\alpha)^{t-1} + (1-\beta)^2}{\beta^2(1-\alpha)^{2t-2} + \beta(1-\beta)(1-\alpha)^{t-2} + \beta(1-\beta)(1-\alpha)^t + (1-\beta)^2}$$

$$= \frac{\beta^2(1-\alpha)^{2t-2} + 2\beta(1-\beta)(1-\alpha)^{t-1} + (1-\beta)^2}{\beta^2(1-\alpha)^{2t-2} + \beta(1-\beta)(1-\alpha)^{t-1}[(1-\alpha)^{-1} + (1-\alpha)] + (1-\beta)^2}$$

Only the middle term differs between numerator and denominator of the last expression. Thus:

$$\frac{\lambda_t}{\lambda_{t\text{-}1}} < 1 \text{ if and only if: } (1-\alpha)^{-1} + (1-\alpha) > 2, \text{ which is true for all } \alpha \in (0,1)$$

In summary, under the maintained hypothesis that the conditional probability of success is constant *given the presence of an oil-bearing structure,* it follows that successive failures have increasing negative impacts on the relative probability of success.

The evolution of success probabilities over successive trials, and the associated decline in λ_t, are illustrated below in Figures 3 and 4, respectively. We present two cases: Case 1 has $\alpha = 30\%$ and $\beta = 60\%$ (a strong a priori probability of an oil-bearing structure but a relatively weak test), whereas Case 2 has $\alpha = 60\%$ and $\beta = 30\%$ (a weaker prior accompanied by a more powerful test). Clearly, the evolution of success probabilities varies significantly as these parameters are changed. What we have established in this section is that, subject to the structure of uncertainty described above, all such curves are monotonically decreasing.

The figures illustrate a further, rather intuitive lesson about the option to drill again. If there is a strong *a priori* belief in the presence of oil but the power of the drilling trial to confirm that belief is low, then the degree of dependence among trials is reduced. In the extreme, this situation would approximate the case of independent trials, wherein drilling continues until the original presumption of a deposit is proven correct. On the other hand, if the *a priori* probability of a deposit is low and the power of the drilling trial to detect that deposit is high, the situation would approximate the case of completely dependent trials in which the first well tells the whole story, and the value of the option to drill again would diminish.

6. CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

We have demonstrated, within a highly simplified model of the exploration process, that the value of the option to drill again can be significant—in some cases greatly exceeding the expected value of the initial trial considered in isolation. The value of this option is diminished, however, by the influence of positive dependence among trials. Dependence essentially reduces the volatility of outcomes, and therefore also reduces the value of the option, by reducing the upside potential of successive trials. We have provided exact formulas by which the expected value of any such prospect can be computed. We have also demonstrated that the impact of dependence is monotonic: any increase in the degree of dependence among trials must further reduce the expected value of the prospect.

Two simple and familiar estimation procedures provide bounds on the actual value of the prospect. The most common indicator of potential value, EV_a, errs by ignoring the option component completely. The other common indicator, EV_b, recognizes the option component but ignores the impact of dependence among trials. The gap between these two approaches is potentially very wide if the initial probability of success is not high.

The model presented here necessarily abstracts from several important aspects of the exploration process. Exploration wells produce a multitude of information—going well beyond the mere confirmation of a deposit. Whether "successful" or not, an exploration well may produce indications regarding the likely existence, size, and/or quality of deposits in related geological structures, and provide more accurate estimates of drilling costs and operating conditions in the general vicinity—all information that is valuable for planning purposes. We have given no weight to these aspects in considering the value of the option

Figure 3. Success Probability Declines at Each Trial

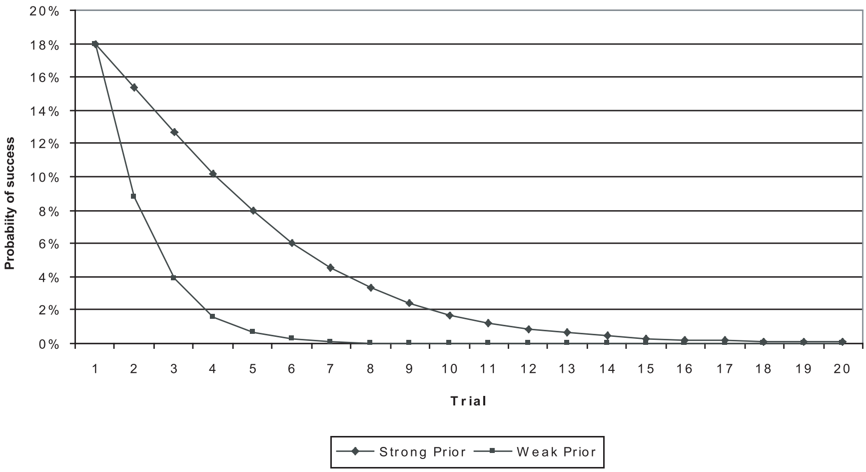
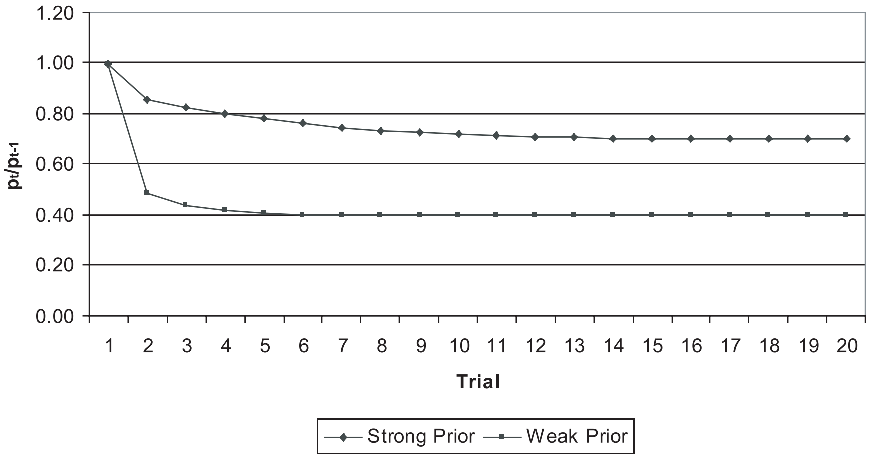


Figure 4. Lambda Declines (Dependence Increases) at Each Trial



to drill again. This limitation could be addressed, but not without elaborating substantially on the underlying information structure and the nature and capabilities of exploration technologies. Even without introducing such extensions to the model, however, it seems clear that the influence of these neglected benefits of exploratory drilling could only be to enhance the value of the option to drill again. By taking full account of the range of potential information spillovers, we suspect that drilling options would become, if anything, even more paramount to the overall value of any given prospect, but this is a question for future research.

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