

An energy demand model with a random trend

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Order of presentation

Introduction : motivation of the paper

1- The econometric model

2- The empirical results

- **Parameter estimates**
- **Price and income elasticities**
- **Estimated trend level**
- **Within sample forecast**

Conclusion

The model:

$$\ln X_t = \mu_t + a_1 \ln X_{t-1} + a_2 \ln P_t + a_3 \ln Y_t + a_4 \ln HDD_t - a_1 a_4 \ln HDD_{t-1} + \varepsilon_t \quad (1)$$

where X_t = total energy demand,

P_t = real price of energy,

Y_t = real income,

HDD_t = heating degree days,

μ_t = random trend,

ε_t = random error term which is assumed $NID (0, \sigma_\varepsilon^2)$

a_1, \dots, a_4 are structural parameters of interest.

The random trend:

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \quad \text{and} \quad \eta_t \sim NID (0, \sigma_\eta^2) \quad (2)$$

$$\beta_t = \beta_{t-1} + \xi_t \quad \text{and} \quad \xi_t \sim NID (0, \sigma_\xi^2) \quad (3)$$

TABLE 1

**TOTAL ENERGY DEMAND:
NO TREND**

	RESIDENTIAL	COMMERCIAL	INDUSTRIAL
Constant	<i>1.784</i> <i>(0.025)</i>	<i>1.457</i> <i>(0.000)</i>	<i>0.556</i> <i>(0.305)</i>
Lagged dependant variable	<i>0.820</i> <i>(0.000)</i>	<i>0.391</i> <i>(0.000)</i>	<i>0.759</i> <i>(0.000)</i>
Real energy price	<i>-0.025</i> <i>(0.134)</i>	<i>-0.334</i> <i>(0.000)</i>	<i>-0.063</i> <i>(0.109)</i>
Real disposable income per household	<i>0.050</i> <i>(0.691)</i>	–	–
Real commercial GDP	–	<i>0.583</i> <i>(0.000)</i>	–
Real industrial GDP	–	–	<i>0.254</i> <i>(0.002)</i>
Heating degree-days	<i>0.356</i> <i>(0.000)</i>	<i>0.288</i> <i>(0.070)</i>	–
σ_e^2	<i>0.0853</i>	<i>0.0033</i>	<i>0.0371</i>
Log-likelihood	<i>81.70</i>	<i>57.64</i>	<i>66.20</i>

Note: p-values appear in parentheses.

TABLE 2

**TOTAL ENERGY DEMAND:
RANDOM TREND**

	RESIDENTIAL	COMMERCIAL	INDUSTRIAL
Lagged dependant variable	<i>0.267</i> <i>(0.076)</i>	<i>0.122</i> <i>(0.277)</i>	<i>0.404</i> <i>(0.009)</i>
Real energy price	<i>-0.294</i> <i>(0.102)</i>	<i>-0.403</i> <i>(0.001)</i>	<i>-0.212</i> <i>(0.000)</i>
Real disposable income per household	<i>0.914</i> <i>(0.000)</i>	–	–
Real commercial GDP	–	<i>1.139</i> <i>(0.000)</i>	–
Real industrial GDP	–	–	<i>0.754</i> <i>(0.000)</i>
Heating degree-days	<i>0.318</i> <i>(0.000)</i>	<i>0.443</i> <i>(0.011)</i>	–
$\sigma_{\eta}^2 \times 10^{-12}$	<i>2.699</i>	<i>0.031</i>	<i>82.5</i>
σ_{ξ}^2	<i>0.0012</i>	<i>0.0052</i>	<i>0.0021</i>
$\sigma_e^2 \times 10^{-12}$	<i>1.88 \times 10^7</i>	<i>1.45</i>	<i>5.06</i>
Log-likelihood	<i>82.81</i>	<i>58.0</i>	<i>69.45</i>
LR statistic	<i>2.17</i>	<i>0.72</i>	<i>6.51</i>

Note: p-values appear in parentheses.

TABLE 3

DEMAND ELASTICITIES:

	RESIDENTIAL		COMMERCIAL		INDUSTRIAL	
	Short ^a run	Long ^b run	Short ^a run	Long ^b run	Short ^a run	Long ^b run
NO TREND						
Price	<i>-0.02</i> <i>(-0.06 : 0.01)</i>	<i>-0.14</i> <i>(-1.89 : 0.04)</i>	<i>-0.33</i> <i>(-0.49 : - 0.18)</i>	<i>-0.55</i> <i>(-0.70 : - 0.37)</i>	<i>-0.06</i> <i>(-0.14 : 0.01)</i>	<i>-0.26</i> <i>(-1.01 : 0.05)</i>
Income	<i>0.05</i> <i>(-0.20 : 0.30)</i>	<i>0.28</i> <i>(-9.75 : 1.12)</i>	<i>0.58</i> <i>(0.34 : 0.83)</i>	<i>0.95</i> <i>(0.76 : 1.10)</i>	<i>0.25</i> <i>(0.09 : 0.42)</i>	<i>1.05</i> <i>(0.61 : 1.69)</i>
RANDOM TREND						
Price	<i>-0.29</i> <i>(-0.65 : 0.06)</i>	<i>-0.40</i> <i>(-1.09 : 0.08)</i>	<i>-0.40</i> <i>(-0.63 : - 0.17)</i>	<i>-0.46</i> <i>(-0.72 : - 0.21)</i>	<i>-0.21</i> <i>(-0.32 : -0.11)</i>	<i>-0.36</i> <i>(-0.89 : -0.14)</i>
Income	<i>0.91</i> <i>(0.46 : 1.37)</i>	<i>1.25</i> <i>(0.87 : 1.59)</i>	<i>1.14</i> <i>(1.11 : 1.16)</i>	<i>1.30</i> <i>(1.05 : 1.69)</i>	<i>0.75</i> <i>(0.39 : 1.12)</i>	<i>1.26</i> <i>(0.95 : 1.74)</i>

a: 95 % confidence interval based on standard error used to compute p-value.

b: 95 % confidence set based on the Fieller method.

FIGURE 1

TREND LEVEL

RESIDENTIAL

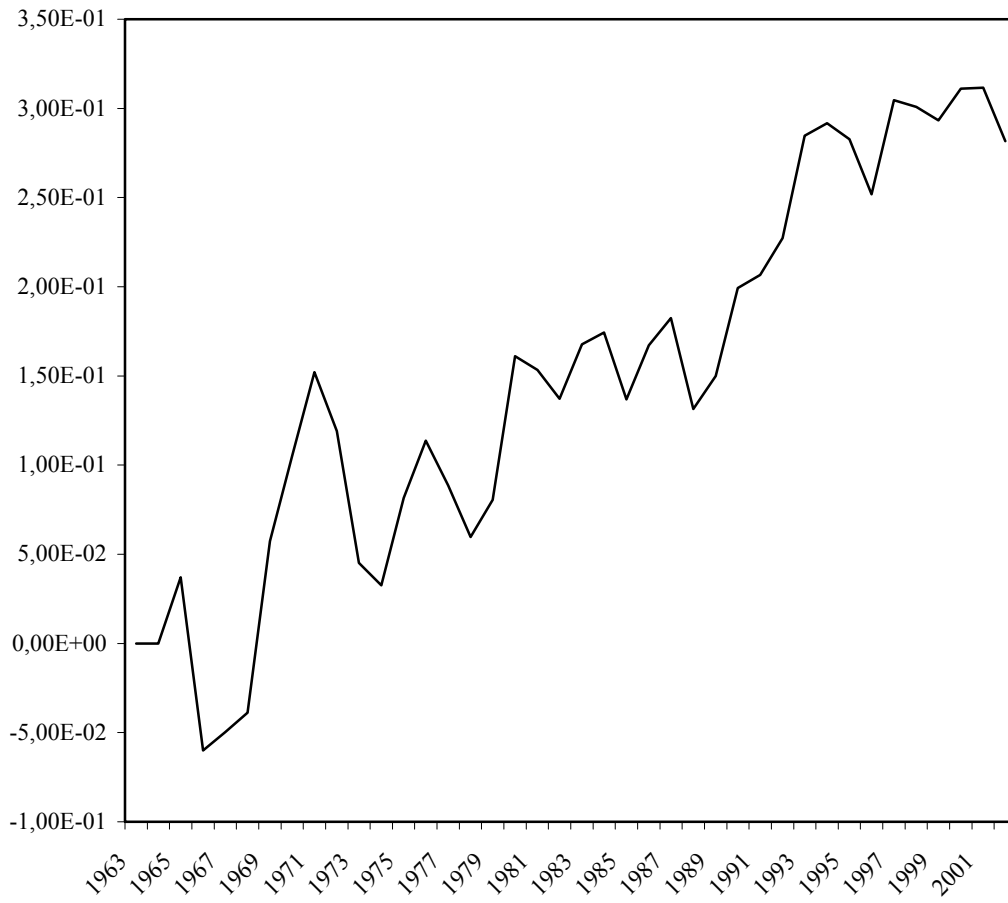


FIGURE 2

TREND LEVEL

COMMERCIAL

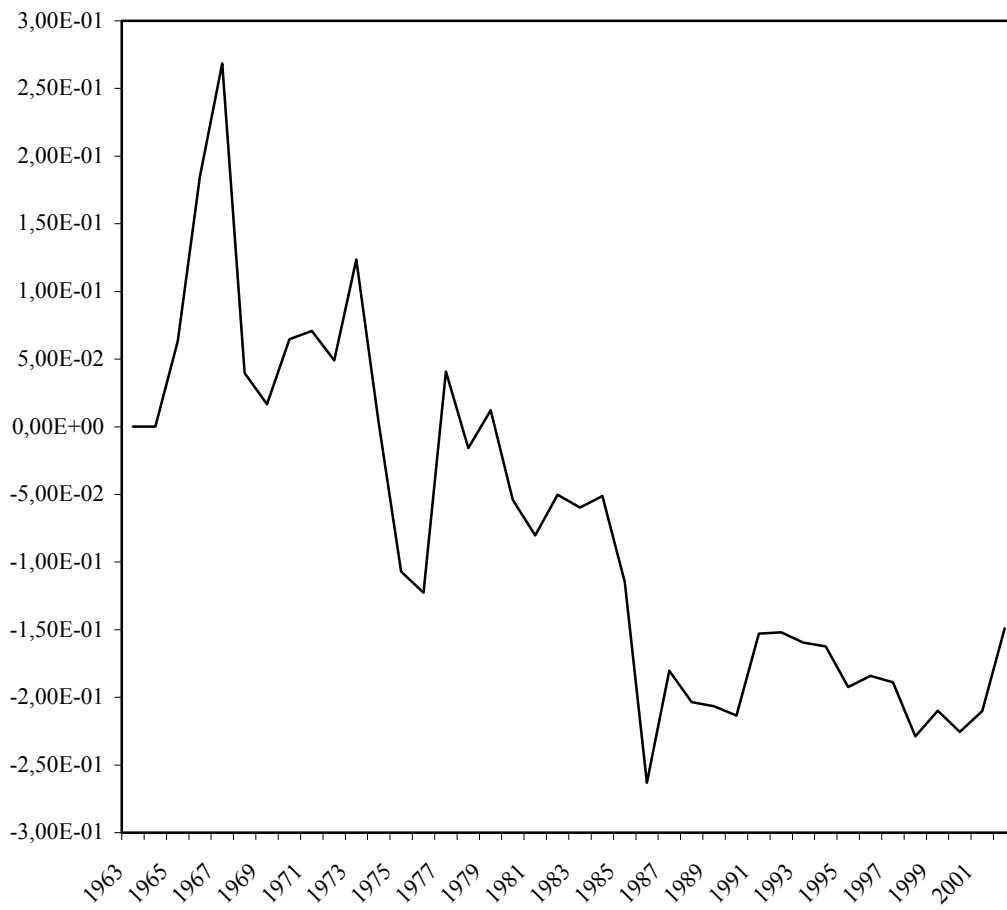


FIGURE 3

TREND LEVEL

INDUSTRIAL

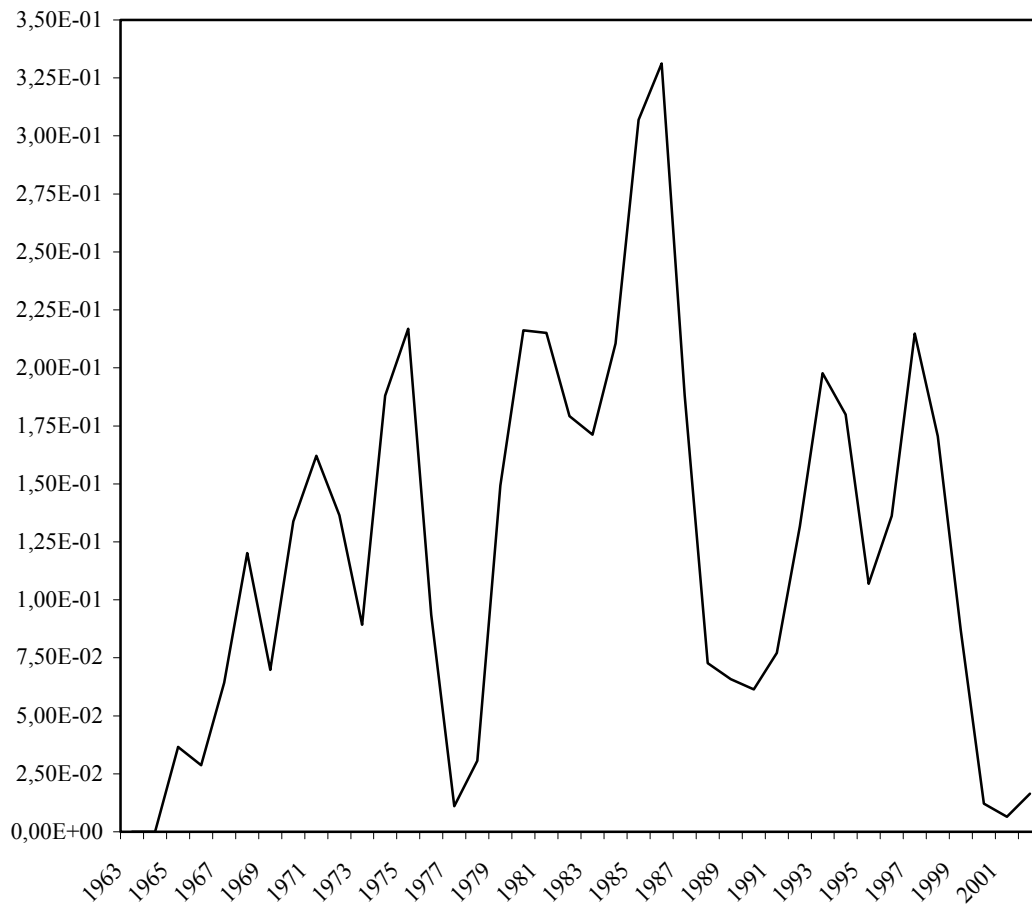


FIGURE 4

TOTAL ENERGY DEMAND

RESIDENTIAL

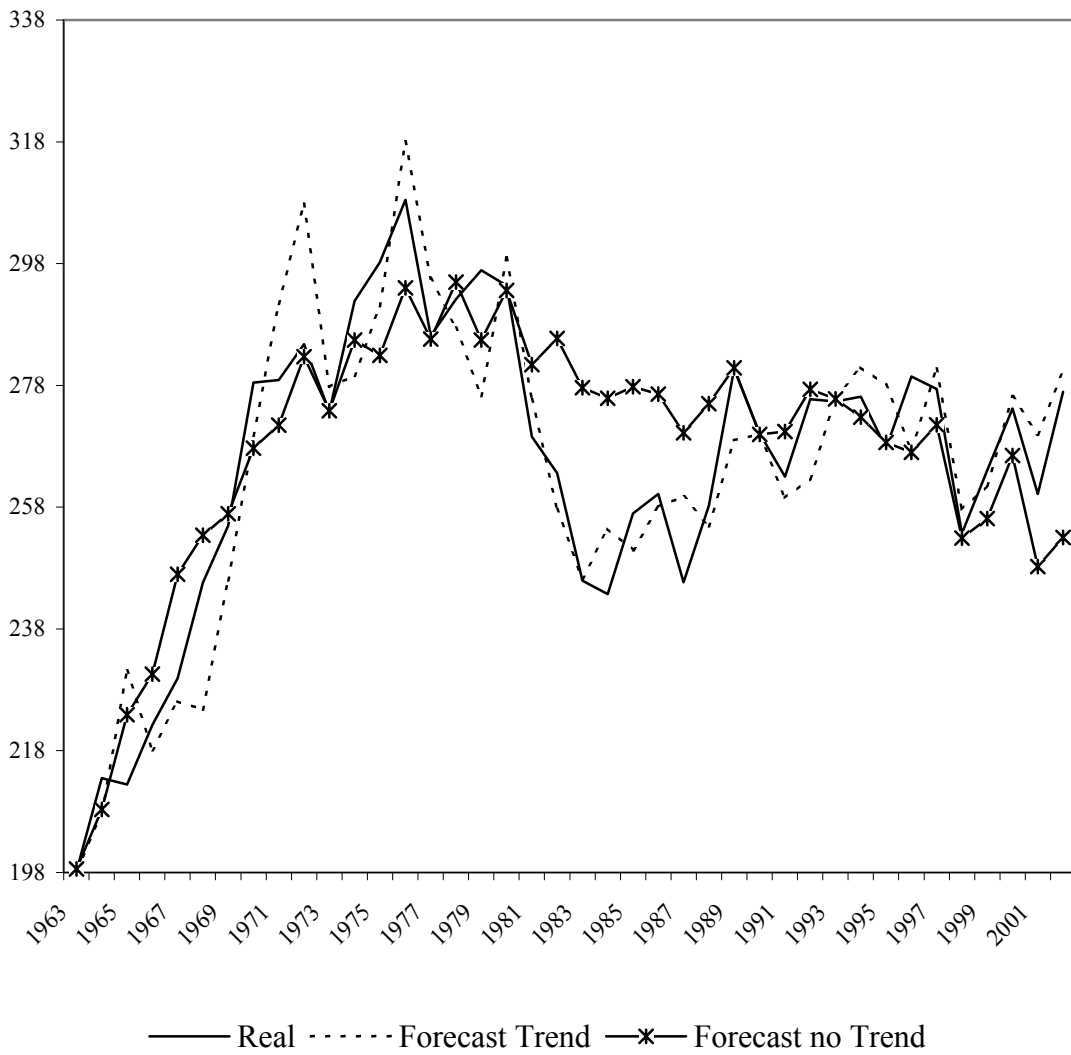


FIGURE 5

TOTAL ENERGY DEMAND

COMMERCIAL

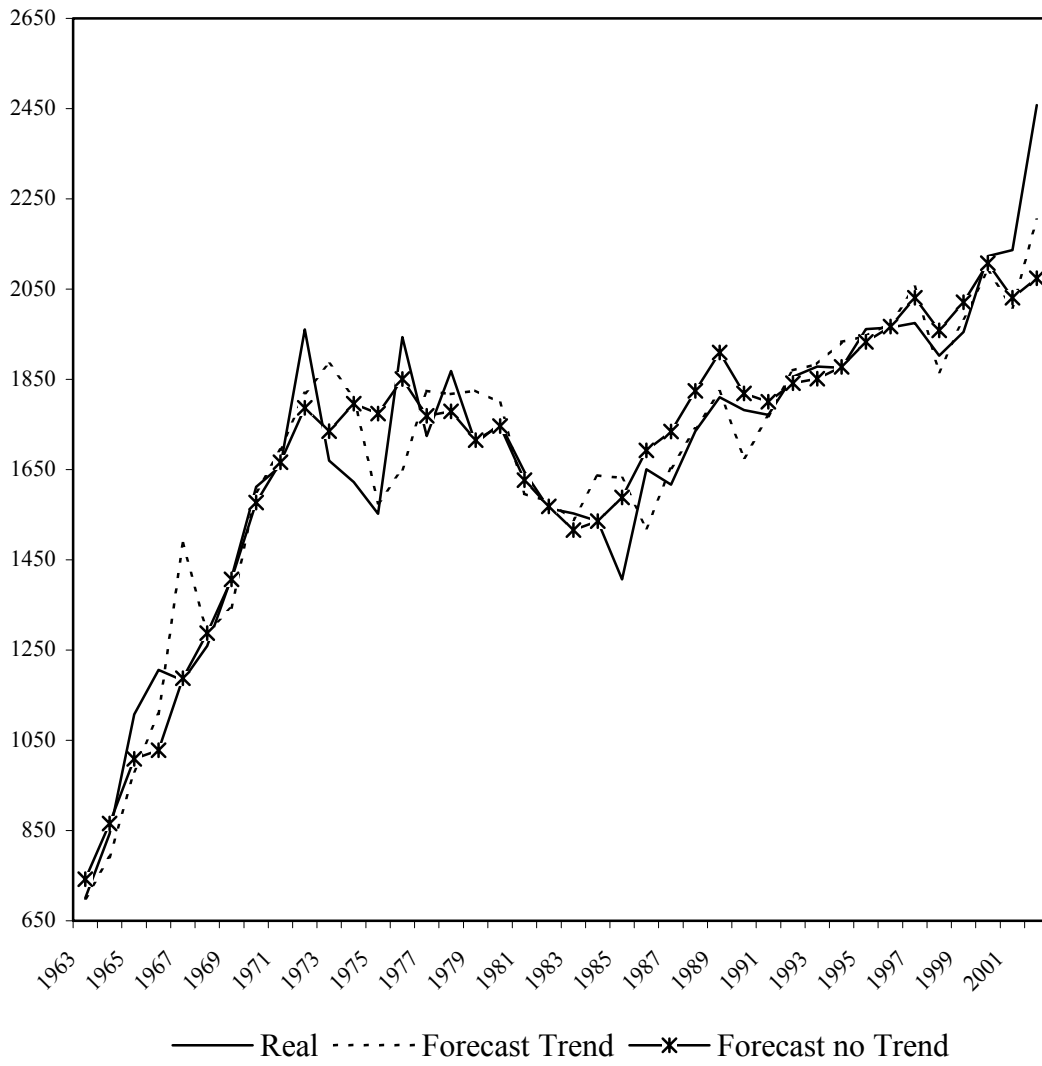


FIGURE 6

TOTAL ENERGY DEMAND

INDUSTRIAL

